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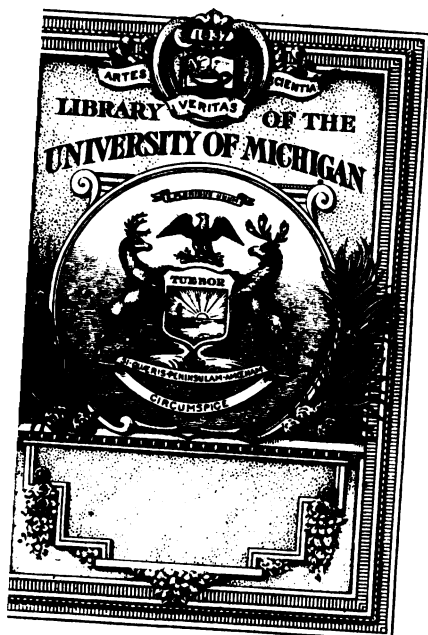
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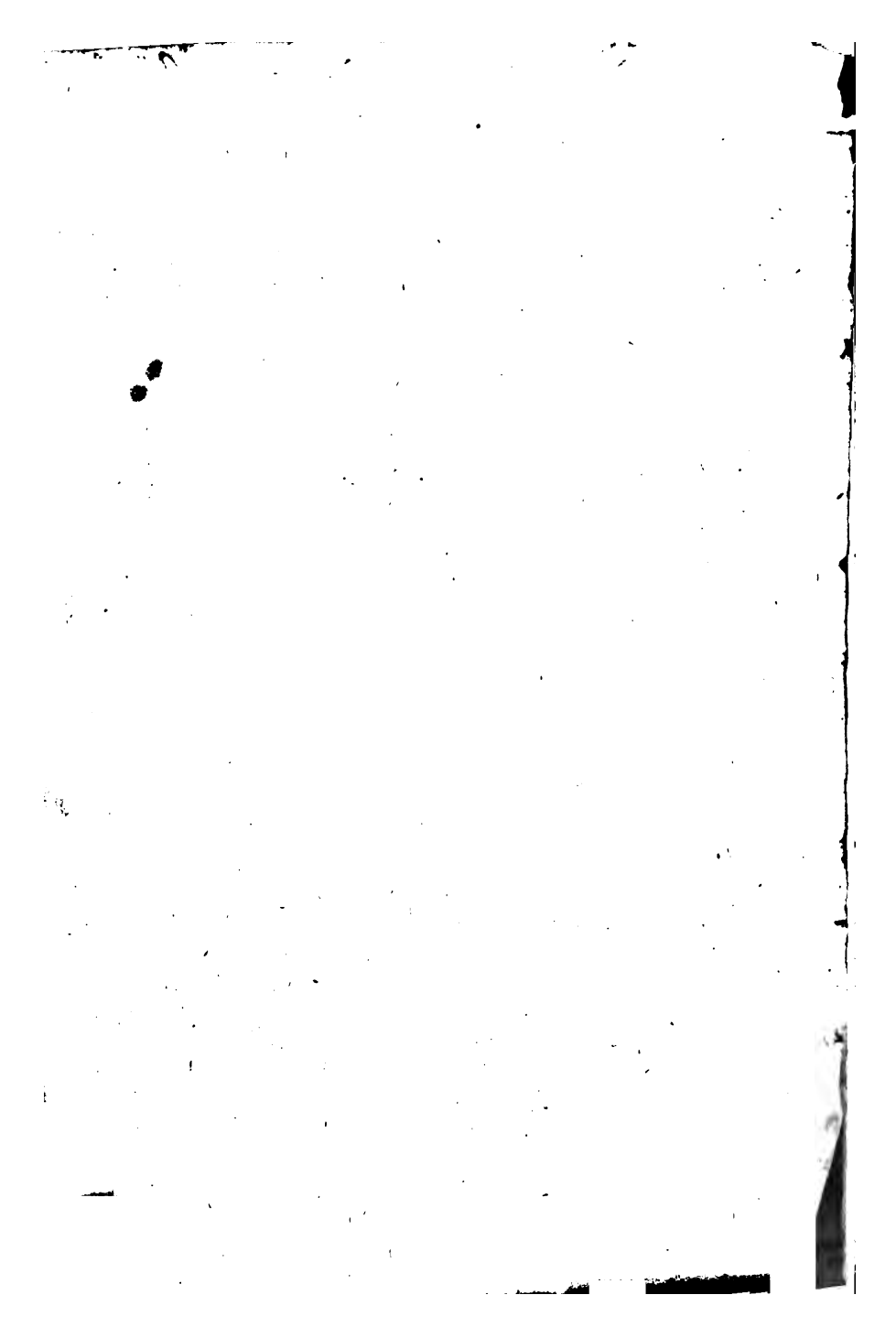
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THE
Gaugers Magazine

WHEREIN THE
FOUNDATION
OF HIS
ART

Is briefly Explain'd and Illustrated
WITH SUCH
FIGURES,

As may render the Whole intelligible
to a mean Capacity.

By *William Hunt*, GAUGER.

Litera Scripta manet.

L O N D O N, Printed by *Mary Clark* for
the Author 1687.

THE UNIVERSITY OF CHICAGO

PHYSICS DEPARTMENT

PHYSICS 311

LECTURE 1

1.1.1

1.1.2

1.1.3

1.1.4

1.1.5

1.1.6

1.1.7

1.1.8

1.1.9

1.1.10

To the Right Worshipful

Sir Denny Ashburnham, Bar.

Sir John Freind, Kt.

Francis Parry, Esq;

Charles Davenant, Dr. of Laws.

Felix Calver,

Nath. Horneby, } Esq;

Richard Graham, }

Chief Commissioners, and Governours for
the Management and Receipt of his Ma-
jesties Revenue of EXCISE, and HEARTH-
MONEY within the Kingdom of England,
&c.

SIRS,

Considering the nearness of your Worships
relation to this Subject, and your Profi-
ciency in these affairs; to You therefore as the
most proper Judges, encourag'd by your Worships
condescention to accept, do I humbly make this
Dedication; my Beseech therefore is that it may
have your Approbation, and under your Favour
and Power be commended to the World, well

The Epistle Dedicatory.

knowing that it will make its passage easie, and procure it a more ready and cheerful entertainment with all those imploy'd in the Revenue, always hoping that the diligent perusal of this small Tract will so enlighten the industrious Officer, and beget such a Resolution in him, that he shall be able to maintain the just Prerogative of the Crown in this Branch against all private In-croachments; part of it, especially that for finding the Vacuity of a Spheroidal or Parabolical Cask, having been much sought after by many curious Enquirers and learned Geometers, but never attain'd, or not Publisht before now: I shall add no more but my Prayers for the Encrease of your Worships Honour, and Felicity, and beg leave to acquaint you that the grand Design of its Publication is to express the Zeal I bear to the Revenue, and the real Affection and Duty, wherewith in all Humility I subscribe my self

Your Worships

most Humble and Oblig'd

Servant and Officer

W. Hunt.

A

P R E F A C E T O T H E R E A D E R.

EVery small Author in this Age concludes it necessary to court his Reader with an Apology, not thinking himself secure, unless he be first perswaded to be good Condition'd, and in my opinion, there was never more need, considering the variety of Humors and Interests that are in the World.

A poor Scribler in these times must like an offending Souldier run the Gauntlet, and give every illiterate fellow leave to claim the privilege of a Last, one fault or slip of a Pen being a sufficient reason without further inspection to condemn a whole Book; I would willingly say something to excuse my own mistakes, but however it is I can by no means admit every man to be a competent Judge: for——

Rough and crooked are the ways that lead to the Practice of solid Geometry, and though many very accurately have endeavour'd to conduct the willing through those Meanders, yet few or none hitherto have arriv'd at their intended Perfection.

'Tis an easie matter to bear the Name, but very difficult to be an expert Gauger; this Magazine con-

A Preface to the Reader.

tains such Directions as may guide the industrious through many of these Intricacies.

Now the Utility of this Art being so well known, to commend it were but to light a Candle before the Sun; And as my utmost endeavours have not been wanting in the Collections, so as to what oversights, or omissions may have escap'd for want of that Vacancy which is necessarily requir'd in a work of this nature; I submit to the Censure of all impartial judicious ARTISTS, ever accounting my self happy, and that my Labour hath its full Reward, if some of them taking advantage by my Imperfections shall gratifie the Revenue with a more compleat Epitome; and as for the Errors of the Press I have Corrected onely the most material, being very sensible how easie it is for any man who is not blinded by prejudice, to distinguish between a Printers and an Authors mistakes, so wishing you profit in the study of these delightful Speculations: I remain

A true Friend to all that are

Mathematically affected

W. HUNT.

THE

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ERRATA.

Pag.	Line	Read thus.	Pag.	Line	Read thus.	Pag.	Line	Read thus.
39	14	05177	72	10	2259009	114	16	79210000
ibid.		05213	76	3	4494400	152	18	Theor. 19.
40	24	20729	76	5	4579600	163	28	110190
42	19	31421	76	26	5579044	163	31	12.0401
43	22	32517	79	10	6487209	164	30	27.5178
49	13	0.5764	79	26	7879249	164	31	28.0184
50	16	9.0077	80	6	8139609	166	14	39.0431
50	23	11.6835	80	32	9672100	166	34	51.2735
50	34	16.3972	82	16	10936249	167	32	51.8927
51	5	5.5890	84	13	13118884	168	6	58.7993
53	11	23.2086	89	28	20061441	170	34	120.9596
55	25	59.7448	91	17	2250089	175	16	0.8596
55	32	66.4596	94	8	28323684	189	6	1.253314
56	31	102.7840	94	17	29268100	201	35	484720
58	21	135.2557	94	27	30393169	203	8	794517
59	13	125.2001	95	32	29877156	203	9	795632
59	18	131.8151	95	35	559	210	13	Square equal
64	25	57600	103	36	49112064	222	6	LO the Axe
65	12	13225	110	33	70073641	229	6	GV
66	15	241081	111	8	66015625	234	1	9 times CD
67	22	320356	112	33	76108176	236	4	.0027851
67	33	459684	113	28	75255625	248	7	07.01.03.2.63
71	35	1948816	113	29	75429225	251	4	Pages.

NAME	AGE	SEX	REL	STATUS	REMARKS
JOHN DOE	35	M	H	W	
JANE DOE	32	F	W	W	
JOHN DOE	30	M	H	W	
JANE DOE	28	F	W	W	
JOHN DOE	25	M	H	W	
JANE DOE	22	F	W	W	
JOHN DOE	20	M	H	W	
JANE DOE	18	F	W	W	
JOHN DOE	15	M	H	W	
JANE DOE	12	F	W	W	
JOHN DOE	10	M	H	W	
JANE DOE	8	F	W	W	
JOHN DOE	6	M	H	W	
JANE DOE	4	F	W	W	
JOHN DOE	2	M	H	W	
JANE DOE	0	F	W	W	

A

A C A T A L O G U E

of the *Authors* consulted in the
following Collections.

<i>Euclid.</i>	<i>Mr. Oughtred.</i>
<i>Diophantus.</i>	<i>Dr. Barrow.</i>
<i>Vieta.</i>	<i>Mr. Newton.</i>
<i>Jacquet.</i>	<i>Dr. Wallis.</i>
<i>Vin. Scoten.</i>	<i>Sir Jonas Moore.</i>
<i>Ramus.</i>	<i>Mr. Ker.</i>
<i>Metius.</i>	<i>Dr. Pell.</i>
<i>De Chales.</i>	<i>Mr. Collins.</i>
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<i>Mr. Walker.</i>	<i>Mr. Clerke.</i>
<i>Mr. Mayne.</i>	<i>Mr. Everard.</i>

THE

THE GAUGERS MAGAZINE.

Introduction.

HE that intends to make any Progress in *Practical Gauging*, ought first to understand the true nature and manner of *Decimals*, which of all *Fractions* are the most *Homogeneous*; *Fractions* here whether *alone*, or joyn'd with *whole Numbers* being wrought as *whole Numbers* without any manner of preparatory operation, which in *Vulgar Fractions* is absolutely requir'd, all our *English Weights, Coyns, and Measures*, being divided and subdivided into so many *Heterogeneous Fractions*, whereas if they were done *Decimally*, all *Operations* would be perform'd with ease and pleasure.

CHAP.

C H A P. I.

§. I. Notation of Decimals.

A *Decimal Fraction* hath its *Numerator* only exprest, (the *Denominator* being an *Unit* with as many *Cyphers* postpon'd as there are places of *Decimals* in the *Numerator*;) and is distinguished whether it stand alone or be joyn'd with whole *Numbers* by preposing a *Point*, or *Comma*:

Thus the *Denominator* of .5 is 10; of .05, is 100; of .015 is 1000, and so on, every *Cypher* or *Figure* prepon'd decreasing its value in a *Decuple* proportion.

Wherefore the postponing of *Cyphers* to a *Decimal Fraction* alter not its value.

§. II. Addition and Subduction of Decimals.

In *Addition*, or *Subduction* of *Decimals*, whether alone or joyn'd with whole *Numbers*, having plac'd every *Figure* under that of the like value, draw a line under them, and *Add*, or *Subduct* as in whole *Numbers* of one *Denomination*, separating as many *Figures* as there are places of *Decimals* in any of the given *Numbers*.

And if the given *Decimals* consist not of the same number of places, postpone as many *Cyphers* as will make them equal.

Examples in Addition.

<p>To 25.725</p> <p>add { 46.6375</p> <p> 33.9421</p> <hr style="width: 100%;"/> <p>Sum—106.3046</p>	<p>32.056</p> <p>7.07</p> <p>9</p> <hr style="width: 100%;"/> <p>40.026</p>
--	---

Proof

14. Multiplication of Decimals.

Proof.

Divide the *Parcels*, and if the *Sum* of the *Parts* be equal to the *Whole*, the operation is true.

Examples in Subduction.

$$\begin{array}{r}
 \text{From} \text{---} 24.0438 \\
 \text{Subduct} \text{---} .65 \\
 \hline
 \text{Remains} \text{ } 23.3938
 \end{array}
 \qquad
 \begin{array}{r}
 37 \\
 .104 \\
 \hline
 36.896
 \end{array}$$

Proof.

If the *Number* to be *subducted*, and *Remainder* are equal to the *Number* from which the *Subduction* was made, the work is right.

6. III. Multiplication of Decimals.

In *Multiplication of Decimals* whether alone, or joyn'd with whole *Numbers*, having plac'd the given *Factors* one under another most commodiously for your purpose, draw a line under them, and *Multiply* as in whole *Numbers* of one *Denomination*.

Then to find the *Value* of the *Product* take this ---

General Rule.

The places of *Decimals* in the *Product* must be equal to those in both the *Factors*.

And if they are not so many, *prepone* as many *Cyphens* as will make them equal.

Examples.

$$\begin{array}{r}
 \text{Factors } \left\{ \begin{array}{l} 1.395 \\ 56.3 \end{array} \right. \\
 \hline
 3915 \\
 7830 \\
 6525 \\
 \hline
 \text{Product} \text{---} 73.4715
 \end{array}
 \qquad
 \begin{array}{r}
 .0375 \\
 .75 \\
 \hline
 1875 \\
 2625 \\
 \hline
 .028125
 \end{array}$$

In *Multiplication of Decimals*, the *Product* is always less than either of the given *Factors*, as in the last Example.

Proof

Multiplication of Decimals.

25

Proof by the Cross.

Cast away the 9th out of both the *Factors*, and set the *Remainders*, one on the *left*, and the other on the *right* side of the *Cross*, multiply these *Remainders* together, and casting away the 9th place the *Remainder* over the *Cross*: Lastly, cast away the 9th out of the *Product*, and if this last *Remainder* be equal to that over the *Cross*, the operation is true.

$$\begin{array}{r} \text{Factors } \left\{ \begin{array}{l} .0375 \\ .75 \end{array} \right. \\ \hline \text{Product } .028125 \end{array} \quad \begin{array}{c} 6 \times 3 \\ 0 \end{array}$$

But the most certain Proof of *Multiplication* is by *Division*; for the *Product* being divided by either of the *Factors*, the *Quotient* is the other.

A necessary Contraction in Multiplication.

Under that Place which you would have secur'd, set the *Units* place of the *Multiplier*, and write the rest in the *Inverse* order; then let each *Figure* of the *Multiplier* begin to multiply that of the *Multiplicand* which is just over it, (but so as to have due regard to the increase that would be brought thither from the following *Figures*) setting every respective *Product* equal with the *Multiplicand* towards the *right* hand, and the *Sum* of these gives you so much as was to be secur'd.

Example.

$\begin{array}{r} 246.914 \\ 35.27 \\ \hline 1728398 \\ 493828 \\ \hline 1234570 \\ 740742 \\ \hline 8708.65678 \end{array}$	$\begin{array}{r} 246.914.. \\ 72.53 \\ \hline 74074200 \\ 12345700 \\ \hline 493828 \\ 172840 \\ \hline 8708.6568 \end{array}$
--	---

S. IV.

S. IV. Division of Decimals.

In Division of Decimal Fractions whether alone, or join'd with whole Numbers, having postpon'd Cyphers sufficient (if need be) to the Dividend that it may have more places of Decimals than the Divisor, the operation is the same as in whole Numbers: Then to find the Value of the Quotient note well this—

General Rule.

The places of Decimals in the Divisor, and Quotient must be equal to those in the Dividend.

And if they are not so many, prepone as many Cyphers to the Quotient as will make them equal.

Example.

Let the Dividend be 78, and the Divisor 47.3764.

Here Note, that although the Dividend exceeds the Divisor, yet before there can be any operation you must postpone 4 Cyphers, and you may take as many more as you please according to the requir'd accuracy; if you postpone 7, the Example will stand thus:

$$47.3764 \overline{) 78.0000000}$$

Make a Point over the fourth Cypher (for so far the Divisor would extend, if plac'd orderly under the Dividend) and demand how many times 47.3764 in 78.0000? or rather how many times 4 in 7? the Answer is once, place 1 in the Quotient, and multiply the Divisor by 1, subtracting the Product out of the Dividend, and the Remainder is 219776 to which postpone the

Division of Decimals.

17

the next Cypher for a new *Dividend*, and the *Example* will stand thus:

$$\begin{array}{r} 47.3764 \overline{) 78.0000000} \quad (1 \\ \underline{30 \ 62360} \end{array}$$

Repeat the work again, and ask how many times 4 in 30? you will find 6 times, set 6 in the *Quotient*, and multiply the *Divisor* by 6; thus 6 times 4 is 24 from 30, and there Remains 6; 6 times 6 is 36 and 3 you borrow'd is 39 from 46, and there rests 7; 6 times 7 is 42 and 4 you borrow'd is 46 from 53, and there remains 7; 6 times 2 is 18 and 5 you borrow'd is 23 from 32, and there rests 9; 6 times 7 is 42 and 3 you borrow'd is 45 from 46 and there remains 1; 6 times 4 is 24 and 4 you borrow'd is 28 from 30, and there rests 2; to which postpone the next Cypher for a new *Dividend*, and the *work* will stand thus:

$$\begin{array}{r} 47.3764 \overline{) 78.0000000} \quad (16 \\ \underline{3062360} \\ 2197760 \end{array}$$

Again, inquire how many times 4 in 21? the Answer is 4 times, place 4 in the *Quotient*, and multiply the *Divisor* by 4 subtracting the *Product* out of the *Dividend*, and the *Remainder* is 302704; the whole operation will stand thus:

$$\begin{array}{r} 47.3764 \overline{) 78.0000000} \quad (1.646 \\ \underline{30 \ 62360} \\ 2197760 \\ \underline{3027040} \\ 184456 \end{array}$$

Reduction of Decimals.

Now there being 7 places of *Decimals* in the *Dividend* and but 4 in the *Divisor*, you must cut off 3 in the *Quotient* to make them equal.

The *Points* above serve to inform you how far you have proceeded in the operation, and must be removed *one place* farther towards the *right hand* at every *Division*, till you come to the last figure in the *Dividend*.

And after all the *Figures* or *Cyphers* are brought down, as long as there is any *Remainder*, if you postpone *Cyphers* to it and work thus, you may have as many places of *Decimals* as you please.

Proof by the Cross.

Cast away the 9th out of the *Dividend*, and place the *Remainder* over the *Cross*, as also out of the *Divisor* and *Quotient*, and set the *Remainders*, one on the left, and the other on the right side of the *Cross*, multiply these *Remainders* together, adding to the *Product* the *Remainder* after the *Division* is ended, (if any) from the *Sum* cast away the 9th. and if this last *Remainder* be equal to that in the upper part of the *Cross*, the operation is true.

<i>Dividend</i>	78		6
<i>Divisor</i>	47.3764		
<i>Quotient</i>	1.646	4	X 8
<i>Remainder</i>	184456		6

But *Division* is best prov'd by *Multiplication*, for if you multiply the *Quotient* by the *Divisor*, the *Product* and *Remainder* (if any) will be equal to the *Dividend*.

§. V. Reduction of Decimals.

A *Vulgar Fraction* will be reduc'd to a *Decimal*, by postponing to the *Numerator* what number of *Cyphers* you

Reduction of Decimals.

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you please, so near as you think necessary to make your approach, and dividing it by the Denominator.

Examples.

$\frac{1}{2}$ reduc'd to a Decimal is .6; 45 Minutes which is $\frac{3}{4}$ of an Hour will be .75; 9 Inches being $\frac{3}{4}$ of a Foot will be .75; 1 Penny being $\frac{1}{240}$ of a Pound is .0041667; and 1 Farthing being $\frac{1}{960}$ of a Pound will be .0010417.

And after this manner by supposing any Divisor a Vulgar Fraction, you may make a Factor equal thereto.

Note, that the Aliquot or Even parts only of a whole Number can be exactly reduc'd to a Decimal Fraction, in all such Numbers there will be still some Remainder, therefore when there is need of Mathematical exactness, you must work by Vulgar Fractions.

A Compendious way to find the Value of any Decimal Fraction of a Pound may be this,

For the first Figure account twice as many Shillings as it contains Units, the next two Figures signifie so many Farthings (lels. 1 in every 24) as they contain Units; and if the second Figure be 5 or above 5, then add 1 to the Shillings before found, the other Figures after the three foremost (if any) being only the Fraction of a Farthing are inconsiderable in Practice.

Example.

In the Decimal .7770833 the first Figure 7 doubled is 14 which are Shillings, now because the second Figure is above 5, subtract 5 from it, and account 1 Shilling more, and the .027 are so many Farthings abating 1, so the Value of .7770833 is 15 s. 6 d. 2 f.

Note, that Half the number of Shillings is the Decimal for Shillings.

Reduction of Decimals.

But it being troublesome to reduce *Vulgar Fractions* this way to *Decimals*, it will be convenient for the *Practitioner* to have *Tablets* ready prepared of the most eminent known parts of *Money, Weights, Measure, &c.* such as are most futable to his occasions, two whereof follow, whose uses are so easie that *Examples* are needless.

Integer 1 Pound		Integer 1 Foot	
Pen.	Decimals.	Inch.	Decimals.
11	.0458333	11	.9166667
10	.0416667	10	.8333333
9	.0375	9	.75
8	.0333333	8	.6666667
7	.0291667	7	.5833333
6	.025	6	.5
5	.0208333	5	.4166667
4	.0166667	4	.3333333
3	.0125	3	.25
2	.0083333	2	.1666667
1	.0041667	1	.0833333
Far.	Decimals.	Quar.	Decimals.
3	.003125	3	.0625
2	.0020833	2	.0416667
1	.0010417	1	.0208333

A *Decimal Fraction* is reduc'd to a *Vulgar*, by multiplying the given *Decimal* by the next lower number of known parts of the *Integer*, for the *Product* separating as many *Figures* towards the right hand as there are places in the given *Decimal* is the *Value* in that *Denomination*, and so proceed to the least known part of the *Integer*.

Examples.

Extraction of the Square-root.

21

Examples.

.7770033

.777803 *Decimal of a* } *a Pound.*
20 *Shillings in*

Shill. 15.5416660

12 *Pence in a Shilling.*

Pence. .64999920

4 *Farthings in a Penny.*

Farth. 1.9999680

.75 *Decimal of* } *a Foot.*
12 *Inches in*

.75 *Decimal of* } *an Hour.*
60 *Minutes in*

9.00 = *Inches.*

45.00 = *Minutes.*

§. VI. Extraction of the Square-root.

Defin.

A Square Number is produc'd by the *Multiplication* of any Number into its *Self*; as 36 is a Square Number whose *Side* or *Root* is 6, for 6 times 6 is 36.

Square Numbers are either *Single*, or *Compound*.

Single are those whose *Roots* consist only of one *Figure*, as in the following *Tablet*.

<i>Square Numbers.</i>	1	4	9	16	25	36	49	64	81
<i>Sides or Roots.</i>	1	2	3	4	5	6	7	8	9

Compound are all those above 100, and before you can begin the *Extraction*, they must be prepar'd by *Punctuation* after this manner:

C 3

Make

Extraction of the Square-root.

Make a *Point* always over the place of *Unity*, and over every *second Figure* after thus 35344 the Number of these *Points* shews the *Root*, or how many *Figures* will be in the *Quotient*.

But in *Pointing* the places of *Decimals* you must proceed after the same manner from the *left hand* towards the *right*.

The *Reason* why you *Point* every *second Figure* is because the *Square* of the greatest Number under 10 can consist but of *two places*.

Having thus *Pointed* the given Number, on the *right hand* thereof draw a *crooked line* as you do for a *Quotient* in *Division* behind which to place the *Root*: then proceed, and

1. Inquire what is the greatest *Root* contain'd in 3? the first *Member* towards the *left hand*, which you will find to be 1, and its *Square* 1, which subtract from 3, placing 1 the *Root* in the *Quotient*, and the *Remainder* underneath:

[This is the first work, and is no more to be repeated.]

To this *Remainder* postpone the next *Member*, or the two next *Figures* for a *Dividend*, and the *Example* will stand thus:

$$\begin{array}{r} 35344(1 \\ 253 \end{array}$$

2. Double the *Quotient* and it makes 2, which set before 253 for a *Divisor*, drawing a *crooked line* before it towards the *left hand* as in *Division*.

[This work is to be repeated for finding every *Figure*.]

Then inquire how many times 2 in 25? (which is one place short of the place of *Unity* according to Note II) you will find 8 times, place 8 in the *Quotient*, and also after the *Divisor* 2, and multiply 28 by 8, subtracting the *Products* as in *Division*, thus 8 times 8 is 64 from 73 and there remains 9; 8 times 2 is 16, and 7 you carried is

Extraction of the Square-root. 23

23 from 25 and there rests 2; to which *postpone* 44 the next Member for a new Dividend, and the Example will stand thus:

$$\begin{array}{r} 35344 \text{ (18)} \\ 28 \overline{) 253} \\ \underline{2944} \end{array}$$

3. Double the Quotient 18 it makes 36 for a new Divisor, which will be found 8 times in 294, set 8 in the Quotient, and after the Divisor 36, and multiply 368 by 8 subtracting the Products as in Division, and there rests 0, which shews it is an exact Square Number: See the whole work.

$$\begin{array}{r} 35344 \text{ (188)} \\ 18 \overline{) 253} \\ \underline{368} \quad 2944 \end{array}$$

If the given Number be not an exact Square, *postpone* so many Binaries or Pairs of Cyphers as 00, 0000, &c. according to the desired accuracy, and extract its Root according to the former Rules, and so many Points as were plac'd over the Fraction, so many Decimals will be in the Root;

But in the Extraction of the Cube-root you must *postpone* so many Ternaries of Cyphers as 000, 000000, &c.

1. Note, that in seeking how often the Divisor is contain'd in the Dividend, you must consider that what you place in the Quotient must be also plac'd after the Divisor as a part of it, wherefore having multipl'd the Divisor by the last Figure in the Quotient, if the Product exceeds the Dividend, or the Remainder the next Divisor the work is erroneous, and a lesser, or greater Figure must be plac'd in the Quotient.

2.4 Extraction of the Square-root.

2. Note, that the *Divisor* when multipl'd must not reach by one place so far as the place of *Unity* in the *Dividend*; for in every *Power* each part is to have so many places left vacant as there are *Cyphers* understood to be wanting, and consequently each of the *intermediate Species* to stand one place more towards the right hand, as having one *Cypher* less than that next above it.

And if the *Divisor* exceeds the particular *Dividend*, you must place a *Cypher* in the *Quotient*, and after the *Divisor* also, then *postpone* the next *Member*, and proceed as before.

Proof by the Cross.

Cast away the 9th out of the given *Number*, and place the *Remainder* over the *Cross*, and also out of the *Root* placing the *Remainder* on either side of the *Cross*, which *Square*, and to the *Product* add the *Figures* remaining after the *Extraction* is ended (if any) out of the *Sum* cast away the 9th and this last *Remainder* will be equal to that above the *Cross*, if there be error in the operation.

Given Number	35344	
Root	188	
Remainder	0	

$$\begin{array}{c}
 \text{I} \\
 8 \times 8 \\
 \text{I}
 \end{array}$$

But the most certain *Proof* is by *Recomposition*, or multiplying 188 by 188, for the *Product*, and *Remainder* (if any) will be equal to the given *Number*.

The *Reason* of this *Rule* is clear by *Algebra*, for let a *Number* be made *Binomial*, and divided into any two *Parts*, put (a) for the greater, and (b) for the lesser then by *Multiplication*.

$$a+b$$

Demonstration of the Square-root. 25

$$\begin{array}{r}
 a+b \\
 a+b \\
 \hline
 aa+ab \\
 \quad +ab+bb \\
 \hline
 aa+2ab+bb
 \end{array}$$

In Words thus:

The Square of the Sum of the Parts will be equal to the Squares of both the Parts, more two Rectangles comprehended under the same Parts.

And thus Synthetically demonstrated 4 Eucl. 2.

Let the Line AB in FIG. 1. = 188 be divided } 180 8
 into two parts, viz. CB 180 and AC 8—} 180 8

Greater part CB Squar'd = HE ————— 32400

Rectangle of AC and CB = CI ————— 1440

Repeat'd = HD ————— 1440

Lesser part AC Squar'd = AH ————— 64

Sum is the intire Square = AE ————— 35344

Again FIG. 2.

Let the given Number be 35344 represented by the Square EF whose Square-root EG is requir'd, and because it is known that the Root will consist of three Figures, suppose EG to be divided into H and I, so that the first Part may be equal to EH, the second to EI, and the third to IG then:

1. Inquire the greatest Root in 3 or 30000, which is 1 or 100, and its Square 1 or 10000 = EK which subtract from 3 or 35344, and there remains the Gnomon HPF = 25344.

2. Double 1 the first Figure in the Quotient = EH, and you have SK = 2 or 200, and the Rectangle SL = to the Gnomon PLH 253, or 25300, therefore if you divide

26 Demonstration of the Square-root.

divide 25300 by 200, or 25 by 2 (*i.e.*) SL by SK the Quotient will be KT 8 or 80, which added to SK the Sum is 28, or 280 = ST which multipl'd by 8 or 80, and the Product subducted from 25300 the Remainder is 29 or 2900.

3. Double 18 or 180 the two first Figures in the Quotient = NL, and you have VL = 36 or 360, and the Rectangle VF = to the Gnomon NFI 2944, therefore if you divide 2944 by 36 (*i.e.*) VF by VL, the Quotient is LX 8, which added to VL the Sum = VX 368, which multipl'd by 8 and the Product subducted from 2944, the Remainder is 0, therefore 188 is the Line EG, the Side of Square EF, and the Square-root of 35344 the Number first given.

S. VII Extraction of the Cube-root.

Defin.

A Cube Number is produc'd by the Multiplication of any Number into its Self, and that Product again by the Root; as 216 is a Cube Number whose Side or Root is 6, for 6 times 6 is 36, and 6 times 36 is 216.

Cube Numbers are either Single, or Compound.

Single are those whose Roots consist only of one Figure as in the annexed Table.

Cube Numbers.	1	8	27	64	125	216	343	512	729
Sides or Roots.	1	2	3	4	5	6	7	8	9

Compound are all those above 1000, and before you can begin the Extraction they must be prepar'd by Partition after this manner:

Make

Extraction of the Cube-root.

27

Make a *Point* always over the place of *Unity*, and over every *third Figure* after thus 6644672 the number of those *Points* shews the *Root*, or how many *Figures* will be in the *Quotient*.

But in *Decimals* you must *Point* from the *left hand*.

The *Reason* why you *Point* every *third Figure* is, because the *Cube* of the *greatest Number* under 10 will consist but of *three places*.

And thus you may proceed in *Pointing* the places of any higher *Power*.

Having thus *Pointed* the given *Number*, draw a *crook-ed Line* on the *right hand* thereof behind which to place the *Root* as you do for a *Quotient* in *Division*; then proceed, and

1. Inquire what is the *greatest Root* contained in 6? the first *Member* towards the *left hand*, which you will find to be 1, and its *Cube* 1, which subduct from 6, placing 1 the *Root* in the *Quotient*, and the *Remainder* underneath:

[*This is the first operation, and is no more to be renew'd.*]

To this *Remainder* postpone the next *Member*, or the three next *Figures* calling it the *Resolvend* (i.e.) that part of the given *Number* which is yet to be *resolv'd*, and the *Example* will stand thus:

$$\begin{array}{r} 6644672 \text{ (1)} \\ 5644 \end{array}$$

2. Square the *Quotient*, and triple the *Product*; thus once 1 is 1, and 3 times 1 is 3, which set before 5644 for a *Divisor*, drawing a *crook-ed line* before it towards the *left hand* as in *Division*:

[*This*

[*This work is to be repeated for finding every Figure.*]

Then inquire how many times 3 in 56? the answer is 8 times, set 8 in the *Quotient*, and multiply 3 by 8, setting 24 the *Product* two places short of the place of *Unity*; (*according to Note II.*)

Next triple the *first Figure* in the *Quotient*, and multiply it by the *Square* of 8 the *last Figure*; thus 3 times 1 is 3, and 64 times 3 is 192 which set one place nearer the place of *Unity*; Lastly *Cube* 8 which is 512, and set it under 4 the place of *Unity*, then add these three *Products* together and they make 4832; which subducted from 5644 there remains 812; to which *postpone* the next *Member*, or the three next *Figures* for a new *Resolvend*, and the *Example* will stand thus;

$$\begin{array}{r}
 6644672 \text{ (18} \\
 5644 \text{ ————— } \text{Resolvend} \\
 \hline
 24 \dots \\
 192 \dots \\
 512 \dots \\
 \hline
 4832 \text{ ————— } \text{Subducent} \\
 812672 \text{ ————— } \text{Resolvend.}
 \end{array}$$

3, Square the *Quotient*, and triple the *Product*; as 18 times 18 is 324, and 3 times 324 is 972 which place before 812672 for a new *Divisor*, drawing a *crooked line* before it towards the *left hand*:

Then Inquire how many times 972 in 8126 the answer is 8 times, set 8 in the *Quotient*, and multiply 972 by 8, placing 7776 the *Product* two places short of the place of *Unity*; next triple the two *first Figures* in the *Quotient* and multiply the *Product* by the *Square* of 8 the

Extraction of the Cube-root.

29

the last Figure, thus 3 times 18 is 54, and 64 times 54 is 3456 which set one place nearer the place of *Unity*; Lastly, Cube 8, which is 512, and subscribe it under the place of *Unity*, then add these three *Products* together, and the *Total* is 812672, which subducted from the *Resolvend* the *Remainder* is 0, because it is an exact *Cube Number*; See the whole operation:

$$\begin{array}{r}
 6644672 \text{ (188=Root)} \\
 3) 5644 \text{ --- Resolvend} \\
 \hline
 24 \dots \\
 192 \dots \\
 512 \dots \\
 \hline
 4832 \text{ --- Subducend} \\
 \hline
 972) 812672 \text{ --- Resolvend} \\
 \hline
 7776 \dots \\
 3456 \dots \\
 312 \dots \\
 \hline
 812672 \text{ --- Subducend} \\
 \hline
 \dots\dots
 \end{array}$$

1. Note, that when the *Subducend* exceeds the *Resolvend*, or the *Remainder* the next *Divisor*, the work is erroneous and a lesser or greater *Figure* must be plac'd in the *Quotient*.

2. Note, that the *Divisor* when multipli'd must not reach by two places so far as the place of *Unity* in the *Resolvend*: And if the *Divisor* exceeds the particular *Resolvend*, you must place a *Cypher* in the *Quotient*, and two *Cyphers* after the *Divisor*, then postpone the next *Member*, and proceed as before.

Proof

30 Demonstration of the Cube-root.

Proof by the Cross.

Cast away the 9th. out of the *given Number*, and set the *Remainder* over the *Cross*, and also out of the *Root*, and place the *Remainder* on either side of the *Cross*, which *Square* and cast away the 9th. out of the *Product*, setting the *Remainder* on the other side of the *Cross*, then multiply these two *Remainders* together, and to the *Product*, add the *Figures* remaining after the *Extraction* is finished (if any) out of the *Sum* cast away the 9th. and this last *Remainder* will be equal to that over the *Cross*, if the *work* be true.

Given Number 6644672	8
Root ————— 188	8 X I
Remainder — 0	8

But it is most certainly prov'd by *Recampssition*, or multiplying 188 by 188, and that *Product* again by 188, for this last *Product* and *Remainder* (if any) will be equal to the *given Number*.

The *Reason* of this *Rule* is evident from *Algebra*, for let a *Number*, or *right Line* be cut into any two *Segments*, put (a) for the greater, and (b) for the lesser, then ———

$$a + b$$

$$\begin{array}{r}
 a+b \\
 a+b \\
 \hline
 aa+ab \\
 +ab+bb \\
 \hline
 aa+2ab+bb \\
 \quad a+b \\
 \hline
 aaa+2aab+abb \\
 \quad +aab+2abb+bbb \\
 \hline
 aaa+3aab+3abb+bbb
 \end{array}$$

In Words thus:

The Cube of the Sum of the Segments will be equal to the Cubes of both the Segments, more 3 times the Square of the greater multipl'd by the lesser, more 3 times the Square of the lesser multipl'd by the greater.

And thus Synthetically Demonstrated by *Ramus*,
Lib. 24.

Let the Line CK = 188 in FIG. 3. be made Binomial, and cut into two Segments, viz. CI 180 and IK 8:

Greater Segment CI Cub'd	_____	5832000
CI Squar'd = 32400 multipl'd by IK	} _____	777600
= 259200, which tripl'd is		
IK Squar'd = 64 multipl'd by CI	} _____	34560
= 11520 which tripl'd is		
Lesser Segment IK Cub'd	_____	512

Sum is the intire Cube	_____	6644672
------------------------	-------	---------

§. Or

§. Or thus.

If a Number be divided into any two Parts, the Cube of the whole is equal to the Cubes of the Parts, more 3 times a Solid comprehended under the Whole and the Parts.

Demonstration.

A . . . C . . B
D G . . . K M

Let DM be the Square of AB, which is equal to the Squares of AC and CB, more the double Rectangle comprehended under AC and CB; Let DG be the Square of AC, GK the Square of CB, and KM the double Rectangle; now it is evident by the definition of a Cube that if you multiply AB by DM, the Product will be the Cube of AB, therefore the respective parts of AB drawn into the respective parts of DM will produce the same Cube, viz. AC drawn into its Square DG gives the Cube of AC, and CB drawn into its Square GK produces the Cube of CB; then multiply AC by GK, and CB by DG, also AC and CB by KM: And forasmuch as KM is equal to the double Rectangle comprehended under the parts AC and CB, it is manifest that to multiply AC and CB by KM is the same as to draw AB into KM: And to multiply AC by GK, and CB by DG is the same as to draw AB into the Rectangle; wherefore the Sum of the Products of all these Multiplications (or the Cube of the whole AB) is equal to the Cubes of the Parts AC and CB more three times a Solid comprehended under the Whole, and the Parts, which was to be Demonstrated:

And the Algebraical Proof is as follows, put (a) equal to the greater part (b) equal to the lesser part then by Multiplication—

$a + b$

Demonstration of the Cube-root. 33

$$\begin{array}{r}
 a+b \\
 \underline{a} \\
 aa+ab \\
 \underline{b} \\
 aab+abb \\
 \underline{3} \\
 aaa+3aab+3abb+bbb
 \end{array}$$

And of this I suppose there may be made a further improvement.

Note, That the Ancients made great use of Algebra in their Inventions tho they seem to take little notice of it in their Writings, and wave it in their Demonstrations, which caused Nonius a learned Spaniard thus to complain:

O how well had it been if those Authors who have written in Mathematicks had delivered to us their Inventions in the same way, and with the same discourse as they were found out, and not as Aristotle saith of Artificers in Mechanicks, who shew us the Engines they have made, but conceal the Artifice to make them the more admired!

The method of Invention in divers Arts is very different from that of Tradition wherein they were delivered; not are we to think that all these Propositions in Euclid, and Archimedes were in the same way found out, as they are, now delivered to us.

34 Geometrical Definitions.

Hoping that the diligent *Gauger* doth by this time understand *Decimal fractions*, I shall lead him a step farther and desire him to take notice that in *Geometry* there are three kinds of Magnitudes, viz. *Lines*, *Surfaces*, and *Solids*, nature not admitting of any more; Length, Breadth, and Depth taking up the whole of space.

A Line hath only *Length*, whose *Boundaries* are *Points*.

A Surface hath *Length*, and *Breadth*, whose *Boundaries* are *Lines*.

A Solid hath *Length*, *Breadth*, and *Depth* or *Thickness*, whose *Boundaries* are *Surfaces*.

Every of these three kinds of *Magnitudes* is measured by some known kind of *Magnitude* that is *Homogeneous* or like to its self.

$\left. \begin{array}{l} \text{Line} \\ \text{Surface} \\ \text{Solid} \end{array} \right\} \begin{array}{l} \text{is mea-} \\ \text{fured} \\ \text{by a} \end{array} \left\{ \begin{array}{l} \text{Line} \\ \text{Surface} \\ \text{Solid} \end{array} \right\} \begin{array}{l} \text{as one} \\ \text{Square} \\ \text{Solid} \end{array} \left\{ \begin{array}{l} \text{Lineal} \\ \text{Square} \\ \text{Solid} \end{array} \right\} \begin{array}{l} \text{Inch} \\ \text{Square} \\ \text{Solid} \end{array}$

And when it is $\left\{ \begin{array}{l} \text{Lineal} \\ \text{Square} \\ \text{Solid} \end{array} \right\}$ Inches are con- $\left\{ \begin{array}{l} \text{Line} \\ \text{Surface} \\ \text{Solid} \end{array} \right\}$
 tained in a

Then is the *Quantity* or *Content* of either of these *Magnitudes* said to be known.

Now the common *Measure* for *Gauging* is the *Gallon*, and that is of several *Dimensions*, of which there are now allowed and used in *England* three sorts viz.

The $\left\{ \begin{array}{l} \text{Ale or Beer} \\ \text{Wine} \\ \text{Corn} \end{array} \right\} \left\{ \begin{array}{l} \text{Gallon con-} \\ \text{taining} \end{array} \right\} \left\{ \begin{array}{l} 282 \\ 231 \\ 272.25 \end{array} \right\} \left\{ \begin{array}{l} \text{Cubiok} \\ \text{Inches.} \end{array} \right\}$

Given AB the Side of a Square; to find the Area.

Defin.

A Square is a Figure contained under four equal Sides, and four right Angles as ABCD.

Theor.

Multiply the Side into its self (which is called Squaring a number) the Product is the Area in Inches, which divided by 282, or multiplied by .0035461 (the Quotient of 1 with Cyphers divided by 282) gives the Area or Content upon one Inch in Ale-Gallons, &c.

S. Or by the Table of Recti-lineal Figures.

Enter the Table with the Inches of the Side in the first Column towards the left hand, and the Tenths (if any) at the Top, under which in that Column directly against the Side, stands a number which multiplied by the Side produces the Area in Ale-Gallons, separating as many Figures as there are Decimals in both the Factors.

Note, That the Numbers in this Table are found by a continual addition, therefore if the Inches given exceed the Table, take the number against half, and doubling it the sum is the number required.



**A
TABLE
TO
GAUGE**

Recti-lineal Figures.

**IN
Ale-Gallons,**

**AS
*Squares, Triangles, and Trapezias.***

38 Recti-lineal Figures in Ale-Gallons.

Side	0	.1	.2	.25	.3	.4
1	00355	00390	00425	00448	00461	00496
2	00709	00745	00780	00798	00816	00851
3	01064	01099	01135	01152	01170	01206
4	01418	01454	01490	01507	01525	01560
5	01778	01808	01844	01862	01879	01915
6	02128	02163	02199	02216	02234	02270
7	02482	02518	02553	02571	02589	02624
8	02837	02872	02908	02925	02943	02979
9	03191	03227	03262	03280	03298	03333
10	03546	03582	03617	03635	03652	03688
11	03901	03936	03972	03989	04007	04043
12	04255	04291	04326	04344	04362	04398
13	04610	04645	04681	04699	04716	04752
14	04965	05000	05035	05053	05071	05106
15	05319	05355	05390	05408	05426	05461
16	05674	05709	05745	05762	05780	05816
17	06028	06064	06099	06117	06135	06170
18	06383	06418	06454	06472	06489	06525
19	06738	06773	06809	06826	06844	06879
20	07092	07128	07163	07181	07199	07234
21	07447	07482	07518	07535	07553	07589
22	07801	07837	07872	07890	07908	07943
23	08156	08192	08227	08245	08262	08298
24	08511	08546	08582	08599	08617	08652
25	08865	08901	08936	08954	08972	09007
26	09220	09255	09291	09308	09326	09362
27	09574	09610	09645	09663	09681	09716
28	09880	09915	10000	10018	10035	10071
29	10284	10319	10355	10372	10390	10426
30	10638	10674	10709	10727	10745	10780
31	10993	11028	11064	11081	11099	11135
32	11348	11383	11418	11436	11454	11489
33	11702	11738	11773	11791	11809	11844
34	12057	12092	12128	12145	12163	12199
35	12411	12447	12482	12500	12518	12553

Recti-linear Figures in Ale-Gallons. 39

Side	.5	.6	.7	.75	.8	.9
1	00532	00567	00603	00621	00638	00657
2	00887	00922	00957	00975	00993	01028
3	01242	01277	01312	01330	01348	01383
4	01596	01631	01667	01684	01702	01737
5	01950	01986	02021	02039	02057	02092
6	02395	02430	02476	02494	02511	02547
7	02660	02695	02730	02748	02766	02801
8	03014	03050	03085	03103	03121	03156
9	03369	03404	03440	03457	03475	03511
10	03715	03750	03794	03812	03830	03865
11	04078	04113	04149	04167	04184	04220
12	04433	04468	04504	04521	04539	04574
13	04787	04823	04858	04876	04894	04929
14	05142	05177	05213	05230	05248	05284
15	05496	05532	05567	05585	05603	05638
16	05851	05887	05922	05940	05957	05993
17	06206	06241	06277	06294	06312	06348
18	06599	06635	06671	06689	06707	06742
19	06985	07020	07056	07073	07091	07126
20	07270	07305	07340	07358	07376	07411
21	07624	07660	07695	07713	07730	07766
22	07979	08014	08050	08067	08085	08121
23	08333	08369	08404	08422	08440	08475
24	08688	08723	08759	08776	08794	08830
25	09043	09078	09113	09132	09149	09184
26	09397	09433	09468	09486	09504	09539
27	09752	09787	09823	09840	09858	09894
28	10106	10142	10177	10195	10213	10248
29	10461	10496	10532	10549	10567	10603
30	10816	10851	10886	10905	10922	10957
31	11170	11206	11241	11259	11277	11302
32	11525	11560	11596	11613	11631	11669
33	11879	11915	11950	11968	11986	12021
34	12234	12270	12305	12322	12340	12376
35	12589	12624	12660	12678	12695	12730

40 Recti-lineal Figures in Ale-Gallons.

Side	0	.1	.2	.25	.3	.4
36	12766	12801	12837	12854	12872	12908
37	13121	13156	13191	13210	13226	13262
38	13475	13510	13546	13564	13582	13617
39	13830	13865	13901	13918	13936	13972
40	14184	14220	14255	14273	14291	14326
41	14539	14574	14610	14627	14645	14681
42	14893	14929	14965	14982	15000	15035
43	15248	15284	15319	15337	15355	15390
44	15603	15638	15674	15691	15709	15745
45	15957	15993	16028	16046	16064	16099
46	16312	16348	16383	16400	16418	16454
47	16667	16702	16738	16755	16773	16809
48	17021	17057	17092	17110	17128	17163
49	17376	17411	17447	17464	17482	17518
50	17730	17766	17801	17819	17837	17872
51	18085	18121	18156	18173	18191	18227
52	18439	18475	18511	18528	18546	18582
53	18794	18830	18865	18883	18901	18936
54	19149	19184	19220	19237	19255	19291
55	19503	19539	19574	19592	19610	19645
56	19858	19894	19929	19946	19965	20001
57	20212	20248	20284	20301	20319	20355
58	20567	20603	20638	20656	20674	20709
59	20922	20957	20993	21011	21028	21064
60	21277	21312	21348	21365	21383	21418
61	21631	21667	21702	21719	21738	21773
62	21986	22021	22057	22074	22092	22128
63	22340	22376	22411	22429	22447	22482
64	22695	22730	22765	22783	22801	22837
65	23050	23085	23121	23138	23156	23191
66	23404	23444	23475	23492	23511	23546
67	23759	23794	23830	23847	23865	23901
68	24113	24150	24184	24202	24220	24255
69	24468	24504	24539	24556	24574	24610
70	24823	24858	24894	24911	24929	24965

Recti-lineal Figures in Ale-Gallons. 41

Side	.5	.6	.7	.75	.8	.9
36	12943	12979	13014	13032	13050	13085
37	13298	13333	13369	13386	13404	13440
38	13652	13688	13723	13741	13759	13794
39	14007	14043	14078	14095	14113	14149
40	14362	14397	14433	14450	14468	14504
41	14716	14752	14787	14805	14823	14858
42	15071	15106	15141	15159	15177	15213
43	15426	15461	15496	15514	15532	15567
44	15780	15816	15851	15868	15887	15922
45	16135	16171	16206	16223	16241	16277
46	16489	16525	16560	16578	16596	16631
47	16844	16879	16915	16932	16950	16986
48	17199	17233	17270	17289	17305	17340
49	17553	17589	17624	17641	17660	17695
50	17908	17943	17979	17997	18014	18050
51	18262	18298	18333	18351	18369	18404
52	18617	18652	18688	18705	18723	18759
53	18972	19007	19043	19060	19078	19113
54	19326	19362	19398	19415	19433	19468
55	19681	19716	19752	19770	19787	19823
56	20035	20071	20106	20124	20142	20177
57	20390	20426	20461	20478	20496	20532
58	20745	20780	20816	20833	20851	20887
59	21099	21135	21170	21187	21206	21241
60	21454	21489	21525	21543	21560	21596
61	21809	21844	21879	21900	21915	21950
62	22163	22199	22234	22251	22270	22305
63	22518	22553	22589	22606	22624	22660
64	22872	22908	22943	22960	22979	23014
65	23227	23262	23298	23316	23333	23369
66	23582	23617	23652	23671	23688	23723
67	23936	23972	24007	24024	24043	24078
68	24291	24326	24362	24380	24397	24433
69	24645	24681	24716	24733	24752	24787
70	25000	25035	25071	25089	25106	25142

42 Recti-lineal Figures in Ale-Gallons.

Side	0	.1	.2	.25	.3	.4
71	25177	25213	25248	25265	25284	25319
72	25532	25567	25603	25620	25638	25674
73	25887	25922	25957	25976	25993	26028
74	26241	26277	26312	26329	26348	26383
75	26596	26613	26667	26684	26702	26738
76	26950	26986	27021	27039	27057	27092
77	27305	27340	27376	27394	27411	27447
78	27660	27695	27730	27749	27766	27801
79	28014	28050	28085	28103	28121	28156
80	28369	28404	28440	28457	28475	28511
81	28723	28759	28794	28812	28830	28865
82	29078	29113	29149	29167	29184	29220
83	29433	29468	29504	29521	29539	29574
84	29787	29823	29858	29876	29894	29929
85	30142	30177	30213	30230	30248	30284
86	30496	30532	30567	30585	30603	30638
87	30851	30887	30922	30940	30957	30993
88	31206	31241	31277	31294	31312	31348
89	31560	31596	31631	31649	31667	31702
90	31915	31950	31986	32004	32021	32057
91	32270	32305	32340	32358	32376	32411
92	32624	32660	32695	32713	32730	32766
93	32979	33014	33050	33067	33085	33121
94	33333	33369	33404	33422	33440	33475
95	33688	33723	33759	33777	33794	33830
96	34043	34078	34113	34131	34149	34184
97	34397	34433	34463	34486	34504	34539
98	34752	34787	34823	34840	34858	34894
99	35106	35142	35177	35195	35213	35248
100	35461	35497	35532	35550	35567	35603

Recti-linear Figures in Ale-Gallons. 43

Side	.5	.6	.7	.75	.8	.9
71	25355	25390	25426	25443	25461	25496
72	25709	25745	25780	25798	25816	25851
73	26064	26099	26135	26153	26170	26206
74	26418	26454	26489	26507	26525	26560
75	26773	26809	26844	26862	26879	26915
76	27128	27163	27199	27217	27234	27270
77	27482	27518	27553	27571	27589	27624
78	27837	27872	27908	27926	27943	27979
79	28191	28227	28262	28280	28298	28333
80	28546	28582	28617	28635	28652	28688
81	28901	28936	28972	28989	29007	29043
82	29255	29291	29326	29344	29362	29397
83	29610	29645	29681	29699	29716	29752
84	29965	30000	30035	30053	30071	30106
85	30319	30355	30390	30408	30426	30461
86	30674	30709	30745	30762	30780	30816
87	31028	31064	31099	31117	31135	31170
88	31383	31418	31454	31472	31489	31529
89	31738	31773	31809	31826	31844	31879
90	32092	32128	32163	32181	32199	32234
91	32447	32482	32512	32535	32553	32589
92	32801	32837	32872	32890	32908	32943
93	33156	33191	33227	33245	33262	33298
94	33511	33546	33582	33599	33617	33652
95	33865	33901	33936	33954	33972	34007
96	34220	34255	34291	34309	34326	34362
97	34574	34610	34645	34663	34681	34716
98	34929	34965	35000	35018	35035	35071
99	35284	35319	35355	35372	35390	35426
100	35638	35674	35709	35727	35745	35780

5. Or by the Tetragonical Table.

Against the Side in the first Column, and under the Tenths (if there be any) at the Top stands the Content upon one Inch without any more trouble.

Note, That the Square of any Side divided by 282 or multiplied by .0035461 gives the several numbers in this Table.

But to calculate it after this manner would require much time, therefore seeing the *Second Differences are equal* it may be made by an easie Collection, yet it will be necessary to examine the work at every 5 or 10 Inches, or as often as any doubt ariseth, the following example will make it plain and easie.

Here

Here I would desire you to take notice of what Mr. Oughtred saith in his *Circles of Proportion*, viz. That the dividing the Foot into 12 Inches, and those Inches into Halves, and Quarters is in artificial, and was done before *Decimal Arithmetick* was well understood: for (saith he) if each Foot were divided into 10 equal parts, (which may as well be called Inches as now it is divided into 12 parts) and each of these parts again divided into 10 other equal parts, then the mensuration of all manner of Surfaces, and Solids, would be much easier than now it is, and in all cases exact enough for Practice.

Side.	Area.	Differen.
1	.0035461 106383	106383
2	.0141844 177305	70922 177305
3	.0319149 248227	70922 248227
4	.0567376 319149	70922 319149
5	.0886525 390071	70922 390071
6	.1276596 460993	70922 460993
7	.1737589 531915	70922 531915
8	.2269504 602837	70922 602837
9	.2872341 673759	70922 673759
10	.3546100	

A TE

Date	Description	Amount
1900	Jan 1	100.00
	Feb 1	100.00
	Mar 1	100.00
	Apr 1	100.00
	May 1	100.00
	Jun 1	100.00
	Jul 1	100.00
	Aug 1	100.00
	Sep 1	100.00
	Oct 1	100.00
	Nov 1	100.00
	Dec 1	100.00
1901	Jan 1	100.00
	Feb 1	100.00
	Mar 1	100.00
	Apr 1	100.00
	May 1	100.00
	Jun 1	100.00
	Jul 1	100.00
	Aug 1	100.00
	Sep 1	100.00
	Oct 1	100.00
	Nov 1	100.00
	Dec 1	100.00
1902	Jan 1	100.00
	Feb 1	100.00
	Mar 1	100.00
	Apr 1	100.00
	May 1	100.00
	Jun 1	100.00
	Jul 1	100.00
	Aug 1	100.00
	Sep 1	100.00
	Oct 1	100.00
	Nov 1	100.00
	Dec 1	100.00
1903	Jan 1	100.00
	Feb 1	100.00
	Mar 1	100.00
	Apr 1	100.00
	May 1	100.00
	Jun 1	100.00
	Jul 1	100.00
	Aug 1	100.00
	Sep 1	100.00
	Oct 1	100.00
	Nov 1	100.00
	Dec 1	100.00
1904	Jan 1	100.00
	Feb 1	100.00
	Mar 1	100.00
	Apr 1	100.00
	May 1	100.00
	Jun 1	100.00
	Jul 1	100.00
	Aug 1	100.00
	Sep 1	100.00
	Oct 1	100.00
	Nov 1	100.00
	Dec 1	100.00
1905	Jan 1	100.00
	Feb 1	100.00
	Mar 1	100.00
	Apr 1	100.00
	May 1	100.00
	Jun 1	100.00
	Jul 1	100.00
	Aug 1	100.00
	Sep 1	100.00
	Oct 1	100.00
	Nov 1	100.00
	Dec 1	100.00
1906	Jan 1	100.00
	Feb 1	100.00
	Mar 1	100.00
	Apr 1	100.00
	May 1	100.00
	Jun 1	100.00
	Jul 1	100.00
	Aug 1	100.00
	Sep 1	100.00
	Oct 1	100.00
	Nov 1	100.00
	Dec 1	100.00
1907	Jan 1	100.00
	Feb 1	100.00
	Mar 1	100.00
	Apr 1	100.00
	May 1	100.00
	Jun 1	100.00
	Jul 1	100.00
	Aug 1	100.00
	Sep 1	100.00
	Oct 1	100.00
	Nov 1	100.00
	Dec 1	100.00
1908	Jan 1	100.00
	Feb 1	100.00
	Mar 1	100.00
	Apr 1	100.00
	May 1	100.00
	Jun 1	100.00
	Jul 1	100.00
	Aug 1	100.00
	Sep 1	100.00
	Oct 1	100.00
	Nov 1	100.00
	Dec 1	100.00
1909	Jan 1	100.00
	Feb 1	100.00
	Mar 1	100.00
	Apr 1	100.00
	May 1	100.00
	Jun 1	100.00
	Jul 1	100.00
	Aug 1	100.00
	Sep 1	100.00
	Oct 1	100.00
	Nov 1	100.00
	Dec 1	100.00
1910	Jan 1	100.00
	Feb 1	100.00
	Mar 1	100.00
	Apr 1	100.00
	May 1	100.00
	Jun 1	100.00
	Jul 1	100.00
	Aug 1	100.00
	Sep 1	100.00
	Oct 1	100.00
	Nov 1	100.00
	Dec 1	100.00

A
**TETRAGONICAL
TABLE**

Exhibiting the
AREAS of *SQUARES*
IN

Ale-Gallons,

AND
Decimilleffimal Parts.

Calculated to every Tenth part, and
Quarter of an Inch of the Side
from 1 to 210 Inches.

Side	0	.1	.2	.25	.3	.4
1	0.0035	0.0043	0.0051	0.0055	0.0060	0.0070
2	0.0142	0.0156	0.0172	0.0179	0.0188	0.0204
3	0.0319	0.0341	0.0363	0.0374	0.0386	0.0410
4	0.0567	0.0596	0.0626	0.0640	0.0656	0.0687
5	0.0887	0.0922	0.0959	0.0977	0.0996	0.1034
6	0.1276	0.1320	0.1363	0.1385	0.1407	0.1452
7	0.1738	0.1788	0.1838	0.1864	0.1890	0.1942
8	0.2270	0.2327	0.2384	0.2413	0.2443	0.2502
9	0.2873	0.2937	0.3001	0.3034	0.3067	0.3133
10	0.3546	0.3617	0.3689	0.3725	0.3762	0.3835
11	0.4291	0.4369	0.4448	0.4488	0.4528	0.4609
12	0.5106	0.5192	0.5278	0.5321	0.5365	0.5452
13	0.5993	0.6085	0.6179	0.6225	0.6273	0.6367
14	0.6950	0.7050	0.7150	0.7200	0.7251	0.7353
15	0.7979	0.8085	0.8193	0.8246	0.8301	0.8410
16	0.9078	0.9192	0.9306	0.9363	0.9422	0.9538
17	1.0248	1.0369	1.0491	1.0551	1.0613	1.0736
18	1.1489	1.1617	1.1746	1.1810	1.1876	1.2006
19	1.2801	1.2937	1.3072	1.3140	1.3209	1.3346
20	1.4184	1.4327	1.4470	1.4541	1.4613	1.4757
21	1.5638	1.5788	1.5938	1.6012	1.6088	1.6240
22	1.7163	1.7320	1.7477	1.7555	1.7634	1.7793
23	1.8759	1.8922	1.9087	1.9168	1.9251	1.9417
24	2.0426	2.0596	2.0767	2.0853	2.0939	2.1112
25	2.2163	2.2341	2.2519	2.2608	2.2698	2.2878
26	2.3972	2.4156	2.4341	2.4434	2.4528	2.4715
27	2.5851	2.6043	2.6235	2.6332	2.6429	2.6623
28	2.7801	2.8000	2.8200	2.8300	2.8400	2.8601
29	2.9823	3.0029	3.0235	3.0339	3.0443	3.0651
30	3.1915	3.2128	3.2341	3.2449	3.2556	3.2772
31	3.4078	3.4298	3.4519	3.4630	3.4741	3.4963
32	3.6312	3.6539	3.6767	3.6882	3.6996	3.7226
33	3.8617	3.8851	3.9087	3.9204	3.9322	3.9559
34	4.0993	4.1234	4.1477	4.1598	4.1720	4.1963
35	4.3440	4.3688	4.3938	4.4063	4.4188	4.4438

Areas of Squares in Ale-Gallons.

Side	5	6	7	75	8	9
1	0.0080	0.0091	0.0102	0.0108	0.0119	0.0128
2	0.0322	0.0340	0.0359	0.0368	0.0378	0.0398
3	0.0434	0.0460	0.0485	0.0498	0.0512	0.0539
4	0.0718	0.0750	0.0783	0.0800	0.0817	0.0851
5	0.1073	0.1112	0.1153	0.1172	0.1192	0.1234
6	0.1498	0.1545	0.1592	0.1615	0.1640	0.1688
7	0.1991	0.2048	0.2102	0.2129	0.2157	0.2213
8	0.2562	0.2623	0.2684	0.2714	0.2746	0.2809
9	0.3200	0.3268	0.3337	0.3371	0.3406	0.3476
10	0.3910	0.3984	0.4060	0.4097	0.4136	0.4213
11	0.4690	0.4772	0.4854	0.4895	0.4938	0.5032
12	0.5543	0.5630	0.5716	0.5764	0.5810	0.5901
13	0.6463	0.6559	0.6646	0.6704	0.6753	0.6851
14	0.7456	0.7559	0.7663	0.7714	0.7767	0.7873
15	0.8520	0.8630	0.8741	0.8796	0.8852	0.8965
16	0.9654	0.9772	0.9890	0.9949	1.0009	1.0128
17	1.0880	1.0984	1.1110	1.1172	1.1235	1.1362
18	1.2139	1.2268	1.2400	1.2466	1.2533	1.2667
19	1.3484	1.3623	1.3762	1.3832	1.3902	1.4043
20	1.4902	1.5048	1.5195	1.5268	1.5342	1.5490
21	1.6391	1.6545	1.6698	1.6775	1.6852	1.7007
22	1.7952	1.8115	1.8273	1.8353	1.8434	1.8596
23	1.9583	1.9750	1.9918	2.0002	2.0087	2.0256
24	2.1285	2.1460	2.1634	2.1722	2.1810	2.1986
25	2.3059	2.3240	2.3422	2.3512	2.3604	2.3788
26	2.4903	2.5091	2.5280	2.5374	2.5470	2.5660
27	2.6817	2.7013	2.7209	2.7307	2.7406	2.7603
28	2.8803	2.9006	2.9209	2.9310	2.9413	2.9617
29	3.0860	3.1070	3.1200	3.1385	3.1491	3.1702
30	3.2988	3.3204	3.3422	3.3530	3.3640	3.3859
31	3.5186	3.5410	3.5634	3.5747	3.5860	3.6085
32	3.7436	3.7687	3.7918	3.8034	3.8150	3.8383
33	3.9796	4.0064	4.0333	4.0391	4.0512	4.0752
34	4.2208	4.2452	4.2698	4.2811	4.2945	4.3199
35	4.4690	4.4943	4.5193	4.5321	4.5468	4.5703

Areas of Squares in Ale-Gallons.

Side	0	.1	.2	.3	.4	.5
36	4.5957	4.6213	4.6470	4.6598	4.6727	4.6984
37	4.8546	4.8809	4.9072	4.9204	4.9337	4.9601
38	5.1206	5.1476	5.1746	5.1882	5.2017	5.2289
39	5.3936	5.4213	5.4491	5.4630	5.4769	5.5048
40	5.6738	5.7022	5.7306	5.7449	5.7592	5.7878
41	5.9610	5.9901	6.0193	6.0339	6.0485	6.0779
42	6.2553	6.2851	6.3150	6.3300	6.3450	6.3750
43	6.5567	6.5873	6.6179	6.6331	6.6485	6.6793
44	6.8652	6.8965	6.9278	6.9434	6.9592	6.9906
45	7.1809	7.2128	7.2448	7.2608	7.2769	7.3091
46	7.5035	7.5362	7.5689	7.5856	7.6027	7.6346
47	7.8333	7.8667	7.9001	7.9169	7.9337	7.9672
48	8.1702	8.2043	8.2384	8.2556	8.2727	8.3070
49	8.5142	8.5490	8.5838	8.6013	8.6188	8.6538
50	8.8653	8.9007	8.9363	8.9541	8.9720	8.0077
51	9.2234	9.2596	9.2959	9.3141	9.3322	9.3687
52	9.5887	9.6256	9.6626	9.6811	9.6996	9.7367
53	9.9610	9.9986	10.0363	10.0552	10.0741	10.1119
54	10.3404	10.3788	10.4172	10.4364	10.4556	10.4942
55	10.7270	10.7660	10.8051	10.8247	10.8443	10.8835
56	11.1206	11.1603	11.2001	11.2201	11.2400	11.2800
57	11.5213	11.5617	11.6023	11.6226	11.6429	11.6835
58	11.9291	11.9703	12.0115	12.0322	12.0528	12.0942
59	12.3440	12.3859	12.4278	12.4488	12.4698	12.5119
60	12.7660	12.8085	12.8512	12.8725	12.8939	12.9367
61	13.1950	13.2383	13.2817	13.3034	13.3251	13.3687
62	13.6312	13.6752	13.7193	13.7413	13.7634	13.8077
63	14.0745	14.1192	14.1640	14.1864	14.2088	14.2538
64	14.5248	14.5703	14.6157	14.6386	14.6613	14.7070
65	14.9823	15.0284	15.0746	15.0998	15.1209	15.1671
66	15.4468	15.4937	15.5406	15.5641	15.5876	15.6346
67	15.9184	15.9660	16.0136	16.0375	16.0613	16.1091
68	16.3972	16.4454	16.4938	16.5180	16.5422	16.5906
69	16.8830	16.9320	16.9810	17.0056	17.0301	17.0793
70	17.3759	17.4256	17.4753	17.5002	17.5251	17.5750

Areas of Squares in Ale-Gallons.

Side	.5	.6	.7	.75	.8	.9
36	4.7243	4.7502	4.7762	4.7892	4.8023	4.8284
37	4.9867	5.0133	5.0400	5.0534	5.0668	5.0937
38	5.2562	5.2835	5.3110	5.3247	5.3384	5.3660
39	5.5328	5.5609	5.5890	5.6031	5.6172	5.6454
40	5.8165	5.8452	5.8741	5.8886	5.9030	5.9420
41	6.1073	6.1367	6.1663	6.1811	6.1959	6.2256
42	6.4051	6.4353	6.4656	6.4807	6.4959	6.5261
43	6.7101	6.7410	6.7720	6.7874	6.8030	6.8341
44	7.0222	7.0538	7.0854	7.1012	7.1172	7.1490
45	7.3413	7.3736	7.4060	7.4223	7.4384	7.4710
46	7.6676	7.7006	7.7337	7.7507	7.7668	7.8000
47	8.0009	8.0346	8.0684	8.0853	8.1023	8.1362
48	8.3413	8.3757	8.4102	8.4275	8.4448	8.4795
49	8.6888	8.7240	8.7592	8.7768	8.7945	8.8298
50	9.0434	9.0793	9.1152	9.1332	9.1512	9.1873
51	9.4051	9.4417	9.4783	9.4967	9.5150	9.5518
52	9.7739	9.8112	9.8485	9.8673	9.8860	9.9234
53	10.1498	10.1878	10.2259	10.2449	10.2640	10.3022
54	10.5328	10.5715	10.6103	10.6297	10.6491	10.6880
55	10.9229	10.9623	11.0017	11.0215	11.0413	11.0809
56	11.3200	11.3601	11.4003	11.4205	11.4405	11.4809
57	11.7243	11.7651	11.8060	11.8165	11.8470	11.8880
58	12.1356	12.1772	12.2188	12.2396	12.2604	12.3022
59	12.5541	12.5963	12.6386	12.6597	12.6810	12.7234
60	12.9796	13.0226	13.0656	13.0871	13.1087	13.1518
61	13.4122	13.4559	13.4996	13.5215	13.5434	13.5873
62	13.8520	13.8963	13.9407	13.9623	13.9853	14.0298
63	14.2988	14.3438	14.3890	14.4116	14.4342	14.4795
64	14.7527	14.7984	14.8443	14.8673	14.8902	14.9362
65	15.2137	15.2601	15.3067	15.3301	15.3533	15.4000
66	15.6817	15.7289	15.7762	15.7999	15.8235	15.8710
67	16.1569	16.2048	16.2528	16.2769	16.3009	16.3490
68	16.6392	16.6877	16.7364	16.7609	16.7853	16.8331
69	17.1285	17.1779	17.2273	17.2520	17.2767	17.3263
70	17.6250	17.6750	17.7251	17.7502	17.7753	17.8258

Areas of Squares in Ale-Gallons.

Side	.0	.1	.2	.25	.3	.4
71	17.8759	17.9263	17.9767	18.0080	18.0273	18.0779
72	18.3830	18.4341	18.4853	18.5108	18.5365	18.5878
73	18.8972	18.9490	18.9909	19.0267	19.0428	19.0948
74	19.4184	19.4710	19.5236	19.5497	19.5762	19.6289
75	19.9468	20.0000	20.0533	20.0801	20.1067	20.1610
76	20.4823	20.5362	20.5902	20.6172	20.6443	20.6984
77	21.0248	21.0795	21.1342	21.1616	21.1890	21.2438
78	21.5745	21.6298	21.6853	21.7130	21.7407	21.7963
79	22.1312	22.1873	22.2434	22.2715	22.2996	22.3559
80	22.6950	22.7518	22.8087	22.8371	22.8656	22.9226
81	23.2660	23.3234	23.3810	23.4100	23.4386	23.4963
82	23.8440	23.9022	23.9604	23.9895	24.0188	24.0772
83	24.4291	24.4880	24.5470	24.5764	24.6060	24.6651
84	25.0213	25.0809	25.1406	25.1705	25.2003	25.2601
85	25.6206	25.6809	25.7413	25.7714	25.8017	25.8623
86	26.2270	26.2880	26.3491	26.3796	26.4103	26.4715
87	26.8404	26.9022	26.9640	26.9948	27.0259	27.0878
88	27.4610	27.5234	27.5860	27.6172	27.6486	27.7112
89	28.0887	28.1518	28.2150	28.2467	28.2783	28.3417
90	28.7234	28.7873	28.8512	28.8821	28.9152	28.9793
91	29.3653	29.4298	29.4945	29.5268	29.5592	29.6240
92	30.0142	30.0795	30.1448	30.1775	30.2103	30.2758
93	30.6702	30.7362	30.8023	30.8353	30.8684	30.9346
94	31.3333	31.4000	31.4668	31.5001	31.5337	31.6006
95	32.0036	32.0710	32.1384	32.1723	32.2060	32.2736
96	32.6809	32.7490	32.8172	32.8502	32.8854	32.9538
97	33.3653	33.4341	33.5030	33.5375	33.5720	33.6410
98	34.0567	34.1263	34.1959	34.2307	34.2656	34.3353
99	34.7553	34.8256	34.8959	34.9310	34.9663	35.0367
100	35.4610	35.5320	35.6030	35.6385	35.6741	35.7453
101	36.1738	36.2454	36.3172	36.3530	36.3890	36.4609
102	36.8936	36.9660	37.0384	37.0746	37.1110	37.1836
103	37.6206	37.6937	37.7668	37.8034	37.8400	37.9133
104	38.3546	38.4284	38.5023	38.5392	38.5762	38.6502
105	39.0958	39.1703	39.2448	39.2822	39.3195	39.3942

Areas of Squares in Ale-Gallons.

Side	.5	.6	.7	.75	.8	.9
71	18.1285	18.1793	18.2301	18.2555	18.2810	18.3320
72	18.6392	18.6906	18.7422	18.7679	18.7938	18.8454
73	19.1569	19.1991	19.2513	19.2873	19.3036	19.3560
74	19.6817	19.7346	19.7876	19.8140	19.8406	19.8937
75	20.2137	20.2672	20.3209	20.3480	20.3746	20.4284
76	20.7527	20.8070	20.8613	20.8886	20.9157	20.9703
77	21.2988	21.3538	21.4088	21.4366	21.4640	21.5192
78	21.8520	21.9077	21.9634	21.9915	22.0193	22.0752
79	22.4122	22.4687	22.5251	22.5534	22.5817	22.6383
80	22.9796	23.0367	23.0939	23.1227	23.1512	23.2086
81	23.5541	23.6119	23.6698	23.6990	23.7278	23.7859
82	24.1356	24.1942	24.2528	24.2821	24.3115	24.3703
83	24.7243	24.7836	24.8429	24.8725	24.9023	24.9618
84	25.3200	25.3799	25.4400	25.4700	25.5001	25.5603
85	25.9229	25.9839	26.0443	26.0746	26.1051	26.1660
86	26.5328	26.5942	26.6556	26.6863	26.7172	26.7788
87	27.1498	27.2119	27.2741	27.3051	27.3363	27.3986
88	27.7740	27.8367	27.8996	27.9310	27.9626	28.0256
89	28.4051	28.4687	28.5322	28.5640	28.5959	28.6596
90	29.0434	29.1077	29.1720	29.2041	29.2363	29.3008
91	29.6888	29.7538	29.8188	29.8512	29.8838	29.9496
92	30.3413	30.4070	30.4727	30.5055	30.5384	30.6043
93	31.0009	31.0672	31.1337	31.1668	31.2001	31.2667
94	31.6676	31.7346	31.8017	31.8353	31.8689	31.9362
95	32.3413	32.4091	32.4769	32.5116	32.5448	32.6128
96	33.0222	33.0906	33.1592	33.1934	33.2278	33.2965
97	33.7101	33.7793	33.8486	33.8833	33.9179	33.9973
98	34.4051	34.4750	34.5450	34.5799	34.6150	34.5851
99	35.1073	35.1779	35.2486	35.2838	35.3193	35.3901
100	35.8165	35.8878	35.9592	35.9949	36.0306	36.1022
101	36.5328	36.6048	36.6769	36.7130	36.7491	36.8213
102	37.2562	37.3289	37.4017	37.4381	37.4746	37.5476
103	37.9867	38.0601	38.1337	38.1704	38.2072	38.2809
104	38.7243	38.7984	38.8727	38.9098	38.9470	39.0213
105	39.4690	39.5438	39.6188	39.6662	39.6938	39.7688

Areas of Squares in Ale-Gallons.

Side	0	.1	.2	.25	.3	.4
106	39.8440	39.9192	39.9945	40.0321	40.0693	40.1053
107	40.5993	40.6752	40.7512	40.7893	40.8273	40.9034
108	41.3617	41.4383	41.5150	41.5534	41.5918	41.6687
109	42.1313	42.2086	42.2860	42.3247	42.3634	42.4410
110	42.9078	42.9859	43.0640	43.1030	43.1422	43.2204
111	43.6915	43.7703	43.8491	43.8885	43.9280	44.0070
112	44.4823	44.5617	44.6413	44.6811	44.7209	44.8006
113	45.2802	45.3603	45.4406	45.4807	45.5209	45.6013
114	46.0851	46.1660	46.2470	46.2874	46.3290	46.4101
115	46.8972	46.9788	47.0604	47.1012	47.1422	47.2240
116	47.7163	47.7986	47.8810	47.9222	47.9634	48.0460
117	48.5426	48.6256	48.7087	48.7502	48.7918	48.8750
118	49.3759	49.4596	49.5434	49.5853	49.6273	49.7112
119	50.2163	50.3008	50.3853	50.4275	50.4698	50.5545
120	51.0638	51.1490	51.2342	51.2768	51.3195	51.4048
121	51.9185	52.0043	52.0902	52.1332	52.1762	52.2623
122	52.7802	52.8667	52.9533	52.9966	53.0400	53.1268
123	53.6489	53.7362	53.8236	53.8671	53.9110	53.9985
124	54.5248	54.6128	54.7009	54.7449	54.7890	54.8771
125	55.4078	55.4963	55.5853	55.6296	55.6741	55.7630
126	56.2979	56.3873	56.4767	56.5215	56.5663	56.6559
127	57.1950	57.2852	57.3753	57.4204	57.4656	57.5559
128	58.0993	58.1901	58.2810	58.3264	58.3720	58.4630
129	59.0107	59.1022	59.1938	59.2396	59.2854	59.3772
130	59.9291	60.0213	60.1136	60.1599	60.2060	60.2985
131	60.8546	60.9476	61.0406	61.0871	61.1337	61.2268
132	61.7872	61.8809	61.9746	62.0215	62.0684	62.1623
133	62.7270	62.8213	62.9158	62.9630	63.0103	63.1048
134	63.6738	63.7688	63.8640	63.9116	63.9592	64.0545
135	64.6277	64.7235	64.8193	64.8672	64.9152	65.0112
136	65.5887	65.6852	65.7817	65.8300	65.8783	65.9750
137	66.5568	66.6539	66.7512	66.7998	66.8486	66.9460
138	67.5319	67.6298	67.7278	67.7768	67.8259	67.9240
139	68.5143	68.6128	68.7115	68.7609	68.8103	68.9091
140	69.5036	69.6029	69.7023	69.7520	69.8018	69.9013

Areas of Squares in Ale-Gallons.

Side	.5	.6	.7	.75	.8	.9
106	40.2208	40.2963	40.3710	40.4098	40.4477	40.5234
107	40.9796	41.0559	41.1322	41.1704	41.2087	41.2852
108	41.7456	41.8226	41.8996	41.9382	41.9767	42.0539
109	42.5186	42.5963	42.6741	42.7130	42.7519	42.8298
110	43.2988	43.3772	43.4556	43.4949	43.5342	43.6128
111	44.0860	44.1651	44.2443	44.2840	44.3236	44.4029
112	44.8803	44.9602	45.0400	45.0800	45.1200	45.2000
113	45.6817	45.7623	45.8429	45.8831	45.9236	46.0043
114	46.4903	46.5725	46.6538	46.6934	46.7352	46.8166
115	47.3059	47.3878	47.4698	47.5188	47.5519	47.6341
116	48.1286	48.2112	48.2940	48.3354	48.3767	48.4596
117	48.9583	49.0417	49.1252	49.1668	49.2087	49.2922
118	49.7952	49.8793	49.9634	50.0055	50.0477	50.1320
119	50.6392	50.7240	50.8088	50.8512	50.8938	50.9788
120	51.4903	51.5758	51.6603	51.7040	51.7470	51.8327
121	52.3484	52.4346	52.5210	52.5640	52.6072	52.6937
122	53.2137	53.3006	53.3876	53.4311	53.4746	53.5617
123	54.0860	54.1736	54.2613	54.3052	54.3491	54.4370
124	54.9654	55.0538	55.1422	55.1864	55.2306	55.3192
125	55.8520	55.9410	56.0301	56.0747	56.1193	56.2086
126	56.7456	56.8353	56.9252	56.9700	57.0150	57.1050
127	57.6463	57.7367	57.8273	57.8726	57.9179	58.0086
128	58.5541	58.6453	58.7365	58.7822	58.8278	58.9192
129	59.4690	59.5609	59.6528	59.6989	59.7448	59.8369
130	60.3910	60.4836	60.5762	60.6226	60.6689	60.7617
131	61.3200	61.4133	61.5067	61.5534	61.6002	61.6937
132	62.2562	62.3502	62.4443	62.4914	62.5385	62.6327
133	63.1995	63.2942	63.3890	63.4364	63.4838	63.5788
134	64.1498	64.2453	64.3408	64.3885	64.4363	64.5320
135	65.1073	65.2034	65.2996	65.3477	65.3959	65.4922
136	66.0718	66.1687	66.2656	66.3140	66.3626	66.4595
137	67.0433	67.1410	67.2386	67.2874	67.3363	67.4341
138	68.0122	68.1204	68.2188	68.2679	68.3172	68.4157
139	69.0080	69.1070	69.2060	69.2555	69.3051	69.4043
140	70.0009	70.1006	70.2003	70.2502	70.3002	70.4000

Areas of Squares in Ale-Gallons.

Side	0	.1	.2	.25	.3	.4
141	70.5000	70.6000	70.7002	70.7502	70.8603	70.9006
142	71.5036	71.6043	71.7051	71.7555	71.8060	71.9070
143	72.5142	72.6157	72.7172	72.7679	72.8188	72.9204
144	73.5319	73.6341	73.7363	73.7875	73.8386	73.9410
145	74.5568	74.6596	74.7626	74.8140	74.8656	74.9687
146	75.5887	75.6922	75.7959	75.8477	75.8996	76.0034
147	76.6277	76.7320	76.8363	76.8885	76.9408	77.0453
148	77.6738	77.7788	77.8838	77.9364	77.9890	78.0942
149	78.7270	78.8327	78.9385	78.9914	79.0443	79.1502
150	79.7873	79.8937	80.0002	80.0534	80.1067	80.2133
151	80.8546	80.9618	81.0690	81.1225	81.1762	81.2836
152	81.9291	82.0369	82.1448	82.1987	82.2528	82.3609
153	83.0107	83.1192	83.2278	83.2821	83.3365	83.4453
154	84.0993	84.2086	84.3179	84.3713	84.4273	84.5368
155	85.1951	85.3050	85.4151	85.4702	85.5252	85.6353
156	86.2979	86.4086	86.5193	86.5748	86.6301	86.7416
157	87.4078	87.5192	87.6307	87.6865	87.7422	87.8538
158	88.5248	88.6369	88.7491	88.8053	88.8613	88.9736
159	89.6490	89.7618	89.8746	89.9311	89.9876	90.1006
160	90.7802	90.8937	91.0073	91.0641	91.1209	91.2346
161	91.9185	92.0327	92.1470	92.2042	92.2613	92.3758
162	93.0638	93.1788	93.2938	93.3514	93.4088	93.5240
163	94.1163	94.2320	94.3477	94.4056	94.4635	94.6793
164	95.3759	95.4923	95.6087	95.6670	95.7252	95.8417
165	96.5426	96.6596	96.7768	96.8355	96.8940	97.0112
166	97.7163	97.8341	97.9519	98.0108	98.0698	98.1878
167	98.8972	99.0157	99.1342	99.1935	99.2528	99.3715
168	100.0851	100.2043	100.3236	100.3832	100.4429	100.5623
169	101.2805	101.4001	101.5200	101.5800	101.6401	101.7602
170	102.4823	102.6029	102.7236	102.7840	102.8443	102.9651
171	103.6915	103.8128	103.9342	103.9949	104.0557	104.1772
172	104.9078	105.0298	105.1519	105.2130	105.2741	105.3963
173	106.1312	106.2540	106.3768	106.4381	106.4996	106.6226
174	107.3617	107.4852	107.6087	107.6704	107.7323	107.8559
175	108.5993	108.7235	108.8477	108.9098	108.9720	109.0963

Areas of Squares in Ale-Gallons.

Side	.5	.6	.7	.75	.8	.9
141	71.0009	71.1013	71.2018	71.2520	71.3023	71.4029
142	72.0080	72.1091	72.2103	72.2609	72.3112	72.4113
143	73.0222	73.1240	73.2259	73.2569	73.3278	73.4298
144	74.0435	74.1460	74.2486	74.2999	74.3512	74.4540
145	75.0718	75.1751	75.2783	75.3300	75.3817	75.4852
146	76.1073	76.2112	76.3152	76.3672	76.4193	76.5235
147	77.1498	77.2545	77.3592	77.4116	77.4640	77.5688
148	78.1995	78.3048	78.4103	78.4630	78.5158	78.6213
149	79.2562	79.3623	79.4684	79.5215	79.5746	79.6809
150	80.3201	80.4268	80.5337	80.5870	80.6406	80.7476
151	81.3910	81.4985	81.6060	81.6598	81.7136	81.8213
152	82.4690	82.5772	82.6854	82.7393	82.7938	82.9022
153	83.5541	83.6630	83.7720	83.8261	83.8810	83.9901
154	84.6463	84.7559	84.8656	84.9204	84.9753	85.0852
155	85.7456	85.8559	85.9663	86.0216	86.0768	86.1873
156	86.8520	86.9630	87.0741	87.1297	87.1853	87.2965
157	87.9654	88.0772	88.1890	88.2450	88.3009	88.4128
158	89.0860	89.1985	89.3110	89.3673	89.4236	89.5362
159	90.2137	90.3268	90.4401	90.4968	90.5534	90.6667
160	91.3484	91.4623	91.5762	91.6333	91.6902	91.8043
161	92.4903	92.6048	92.7195	92.7796	92.8342	92.9490
162	93.6392	93.7545	93.8698	93.9276	93.9853	94.1008
163	94.7952	94.9112	95.0273	95.0854	95.1434	95.2598
164	95.9584	96.0751	96.1918	96.2504	96.3087	96.4256
165	97.1286	97.2460	97.3635	97.4223	97.4810	97.5986
166	98.3059	98.4240	98.5422	98.6013	98.6604	98.7788
167	99.4903	99.6091	99.7280	99.7874	99.8470	99.9660
168	100.6818	100.8013	100.9209	100.9807	101.0406	101.1603
169	101.8803	102.0006	102.1209	102.1802	102.2413	102.3618
170	103.0860	103.2070	103.3280	103.3885	103.4491	103.5703
171	104.2988	104.4204	104.5422	104.6030	104.6640	104.7859
172	105.5186	105.6410	105.7635	105.8247	105.8860	106.0086
173	106.7456	106.8687	106.9918	107.0534	107.1151	107.2384
174	107.9796	108.1034	108.2273	108.2891	108.3512	108.4752
175	109.3208	109.3453	109.4698	109.5321	109.5945	109.7192

Areas of Squares in Ale-Gallons.

Side	0	.1	.2	.25	.3	.4
176	109.8440	109.9689	110.0938	110.1562	110.2188	110.3439
177	111.0958	111.2213	111.3470	111.4098	111.4727	111.5985
178	112.3546	112.4709	112.5973	112.6704	112.7339	112.8602
179	113.6206	113.7476	113.8746	113.9382	114.0018	114.1290
180	114.8936	115.0213	115.1491	115.2131	115.2769	115.4048
181	116.1738	116.3022	116.4307	116.4949	116.5592	116.6878
182	117.4610	117.5901	117.7193	117.7839	117.8486	117.9779
183	118.7553	118.8852	119.0151	119.0799	119.1450	119.2751
184	120.0568	120.1873	120.3179	120.3833	120.4486	120.5793
185	121.3653	121.4965	121.6278	121.6935	121.7592	121.8907
186	122.6809	122.8128	122.9448	123.0009	123.0769	123.2091
187	124.0036	124.1362	124.2690	124.3353	124.4018	124.5346
188	125.3334	125.4667	125.6001	125.6669	125.7337	125.8673
189	126.6702	126.8043	126.9385	127.0055	127.0727	127.2070
190	128.0142	128.1490	128.2839	128.3513	128.4188	128.5538
191	129.3653	129.5008	129.6363	129.7042	129.7720	129.9077
192	130.7234	130.8596	130.9959	131.0641	131.1323	131.2687
193	132.0887	132.2256	132.3626	132.4311	132.4996	132.6362
194	133.4610	133.5986	133.7363	133.8051	133.8741	134.0119
195	134.8405	134.9788	135.1172	135.1854	135.2557	135.3942
196	136.2270	136.3660	136.5051	136.5746	136.6443	136.7836
197	137.6206	137.7603	137.9002	137.9701	138.0401	138.1800
198	139.0213	139.1618	139.3023	139.3726	139.4429	139.5836
199	140.4291	140.5703	140.7115	140.7822	140.8528	140.9942
200	141.8440	141.9859	142.1278	142.1988	142.2699	142.4119
201	143.2660	143.4086	143.5512	143.6225	143.6940	143.8368
202	144.6951	144.8384	144.9817	145.0534	145.1252	145.2687
203	146.1312	146.2752	146.4193	146.4913	146.5633	146.7077
204	147.5745	147.7192	147.8640	147.9364	148.0089	148.1538
205	149.0249	149.1703	149.3158	149.3886	149.4613	149.6070
206	150.4823	150.6284	150.7746	150.8477	150.9209	151.0673
207	151.9468	152.0937	152.2406	152.3141	152.3876	152.5346
208	153.4185	153.5660	153.7136	153.7876	153.8613	154.0091
209	154.8972	155.0455	155.1938	155.2677	155.3422	155.4907
210	156.3830	156.5320	156.6810	156.7555	156.8301	156.9791

Areas of Squares in Ale-Gallons.

Side	.5	.6	.7	.75	.8	.9
176	110.4690	110.5942	110.7195	110.7822	110.8448	110.9703
177	111.7243	111.8504	111.9762	112.0392	112.1023	112.2284
178	112.9867	113.1134	113.2401	113.3034	113.3668	113.4937
179	114.2561	114.3836	114.5110	114.5747	114.6385	114.7660
180	115.5328	115.6609	115.7890	115.8530	115.9172	116.0454
181	116.8165	116.9453	117.0741	117.1385	117.2030	117.3320
182	118.1073	118.2368	118.3663	118.4310	118.4959	118.6256
183	119.4052	119.5353	119.6656	119.7307	119.7959	119.9263
184	120.7101	120.8410	120.9720	121.0375	121.1030	121.2341
185	122.0222	122.1538	122.2854	122.3513	122.4172	122.5490
186	123.3413	123.4736	123.6060	123.6722	123.7385	123.8710
187	124.6676	124.8006	124.9337	125.0003	125.0668	125.2001
188	126.0009	126.1346	126.2684	126.3353	126.4023	126.5362
189	127.3413	127.4758	127.6103	127.6758	127.7448	127.8795
190	128.6889	128.8240	128.9592	129.0168	129.0949	129.2298
191	130.0435	130.1793	130.3152	130.3833	130.4512	130.5873
192	131.4052	131.5417	131.6784	131.7467	131.8151	131.9518
193	132.7740	132.9112	133.0486	133.1173	133.1860	133.3235
194	134.1498	134.2878	134.4259	134.4948	134.5640	134.7022
195	135.5328	135.6715	135.8103	135.8796	135.9491	136.0880
196	136.9299	137.0623	137.2018	137.2716	137.3413	137.4809
197	138.3201	138.4602	138.6003	138.6705	138.7406	138.8809
198	139.7243	139.8651	139.9060	140.0764	140.1470	140.2880
199	141.1357	141.2772	141.4188	141.4896	141.5605	141.7022
200	142.5541	142.6963	142.8386	142.9098	142.9810	143.1235
201	143.9796	144.1226	144.2656	144.3371	144.4087	144.5518
202	145.4123	145.5559	145.6996	145.7615	145.8434	145.9873
203	146.8520	146.9963	147.1408	147.2130	147.2853	147.4299
204	148.2988	148.4439	148.5890	148.6616	148.7342	148.8795
205	149.7527	149.8985	150.0443	150.1172	150.1902	150.3363
206	151.2137	151.3602	151.5067	151.5780	151.6534	151.8001
207	152.6818	152.8290	152.9762	153.0500	153.1236	153.2710
208	154.1569	154.3049	154.4528	154.5270	154.6009	154.7490
209	155.6392	155.7878	155.9365	156.0102	156.0853	156.2341
210	157.1286	157.2779	157.4273	157.5018	157.5768	157.7263

...of ... to ...

[illegible]

Probl. 2.

Given the Area of the Square ABCD in Ale-Gallons; to find the side.

FIG. 4.

Theor.

Multiply the Area by 282, the Product is the Area in Inches, whose Square Root is the side.

Probl. 3.

Given AB, and BC, the sides of an Oblong; to find the Area.

FIG. 5.

Defin.

An Oblong is, a Figure contained under four right-Angles, and four sides, any two of which being opposite are parallel, and of one length.

Theor.

Multiply AB, by BC, the Product is the Area in Inches, which divided by 282, or multiplied by 0035461 gives the Area, or content at one Inch deep in Ale-Gallons.

S. Or by the Table of Rectilineal Figures.

• Multiply the Tabular number agreeing to either of the sides, by the other, the Product is the Area.

S. Or

§. Or by the Tetragonical Table.

Multiply AB by BC, the Square Root of the Product is the side of a Square equal to the Oblong, against which in the proper place of meeting stands the Area.

§. Or more readily by the Table of Squares thus,

With the Product enter the Table, and having found it in any of the Angles against it in the side, and over it at Top stands the Root:

And though the Product consist of Decimal fractions which the Table gives not expressly, yet implicitly and virtually it doth to such as understand Decimal Multiplication, for you may look them as if they were whole numbers, and then separate half so many Figures towards the right hand of the Root as there were Decimals in the given Product.

Such a Table I gave you as far as 2000 in the first Edition, and have now continued it to 10000, because of its admirable use in the speedy solution of any Question that requires the extraction of a Square-root.

**A
T A B L E
O F
S Q U A R E N U M B E R S**

By which the

S I D E O R R O O T

**Of any Square Number between 0 and 100
Millions may be readily and
exactly discovered.**

Square Numbers and their Roots.

Root	0	1	2	3	4
1	100	121	144	169	196
2	400	441	484	529	576
3	900	961	1024	1089	1156
4	1600	1681	1764	1849	1936
5	2500	2601	2704	2809	2916
6	3600	3721	3844	3969	4096
7	4900	5041	5184	5329	5476
8	6400	6561	6724	6889	7056
9	8100	8281	8464	8649	8836
10	10000	10201	10404	10609	10816
11	12100	12321	12544	12769	12996
12	14400	14641	14884	15129	15376
13	16900	17161	17424	17689	17956
14	19600	19881	20164	20449	20736
15	22500	22801	23104	23409	23716
16	25600	25921	26244	26569	26896
17	28900	29241	29584	29929	30276
18	32400	32761	33124	33489	33856
19	36100	36481	36864	37249	37636
20	40000	40401	40804	41209	41616
21	44100	44521	44944	45369	45796
22	48400	48841	49284	49729	50176
23	52900	53361	53824	54289	54756
24	57600	58081	58564	59049	59536
25	62500	63001	63504	64009	64516
26	67600	68121	68644	69169	69696
27	72900	73441	73984	74529	75076
28	78400	78961	79524	80089	80656
29	84100	84681	85264	85849	86436
30	90000	90601	91204	91809	92416
31	96100	96721	97344	97969	98596
32	102400	103041	103684	104329	104976
33	108900	109561	110224	110889	111556
34	115600	116281	116964	117649	118336
35	122500	123201	123924	124609	125316

Square Numbers and their Roots.

69

Root	5	6	7	8	9
1	225	256	289	324	361
2	625	676	729	784	841
3	1225	1296	1369	1444	1521
4	2025	2116	2209	2304	2401
5	3025	3136	3249	3364	3481
6	4225	4356	4489	4624	4761
7	5625	5776	5929	6084	6241
8	7225	7396	7569	7744	7921
9	9025	9216	9409	9604	9801
10	11025	11236	11449	11664	11881
11	23225	13456	13689	13924	14161
12	15625	15876	16129	16384	16641
13	18225	18496	18769	19044	19321
14	21025	21316	21609	21904	22201
15	24025	24336	24649	24964	25281
16	27225	27556	27889	28224	28561
17	30625	30976	31329	31684	32041
18	34225	34596	34969	35344	35721
19	38025	38416	38809	39204	39601
20	42025	42436	42849	43264	43681
21	46225	46656	47089	47524	47961
22	50625	51076	51529	51984	52441
23	55225	55696	56169	56644	57121
24	60025	60516	61009	61504	62001
25	65025	65536	66049	66564	67081
26	70225	70756	71289	71824	72361
27	75625	76176	76729	77284	77841
28	81225	81796	82369	82944	83521
29	87025	87616	88209	88804	89401
30	93025	93636	94249	94864	95481
31	99225	99856	100489	101124	101761
32	105625	106276	106929	107584	108241
33	112225	112896	113569	114244	114921
34	119025	119716	120409	121104	121801
35	126025	126736	127449	128164	128881

Root	0	1	2	3	4
36	129600	130321	131044	131769	132496
37	136900	137641	138384	139129	139876
38	144400	145161	145924	146689	147456
39	152100	152881	153664	154449	155236
40	160000	160801	161604	162409	163216
41	168100	168921	169744	170569	171396
42	176400	177241	178084	178929	179776
43	184900	185761	186624	187489	188356
44	193600	194481	195364	196249	197136
45	202500	203401	204304	205209	206116
46	211600	212521	213444	214369	215296
47	220900	221841	222784	223729	224676
48	230400	231361	232324	233289	234256
49	240100	241081	242064	243049	244036
50	250000	251001	252004	253009	254016
51	260100	261121	262144	263169	264196
52	270400	271441	272484	273529	274576
53	280900	281961	283024	284089	285156
54	291600	292681	293764	294849	295936
55	302500	303601	304704	305809	306916
56	313600	314721	315844	316969	318096
57	324900	326041	327184	328329	329476
58	336400	337561	338724	339889	341056
59	348100	349281	350464	351649	352836
60	360000	361201	362404	363609	364816
61	372100	373321	374544	375769	376996
62	384400	385641	386884	388129	389376
63	396900	398161	399424	400689	401956
64	409600	410881	412164	413449	414736
65	422500	423801	425104	426409	427716
66	435600	436921	438244	439569	440896
67	448900	450241	451584	452929	454276
68	462400	463761	465124	466489	467856
69	476100	477481	478864	480249	481636
70	490000	491401	492804	494209	495616

Square Numbers and their Roots.

67

Root	5	6	7	8	9
36	133225	133956	134689	135424	136161
37	140625	141376	142129	142884	143641
38	148125	148996	149769	150544	151321
39	156025	156816	157609	158404	159201
40	164025	164836	165649	166464	167281
41	172125	173056	173889	174724	175561
42	180425	181376	182329	183184	184041
43	189125	190096	190969	191844	192721
44	198025	198916	199809	200704	201601
45	207025	207936	208849	209764	210681
46	216225	217156	218089	219024	219961
47	225625	226576	227529	228484	229441
48	235225	236196	237169	238144	239121
49	245025	246016	247009	248004	249001
50	255025	256036	257049	258064	259081
51	265225	266256	267289	268324	269361
52	275625	276676	277729	278784	279841
53	286225	287296	288369	289444	290521
54	297025	298116	299209	300304	301401
55	308025	309136	310249	311364	312481
56	319225	320356	321489	322624	323761
57	330625	331776	332929	334084	335241
58	342225	343396	344569	345744	346921
59	354025	355216	356409	357604	358801
60	366025	367236	368449	369664	370881
61	378225	379456	380689	381924	383161
62	390625	391876	393129	394384	395641
63	403225	404496	405769	407044	408321
64	416025	417316	418609	419904	421201
65	429025	430336	431649	432964	434281
66	442225	443556	444889	446224	447561
67	455625	456976	458329	459684	461041
68	469225	470596	471969	473344	474721
69	483025	484416	485809	487204	488601
70	497025	498436	499849	501264	502681

Root	0	1	2	3	4
71	504100	505521	506944	508369	509796
72	518400	519841	521284	522729	524176
73	532900	534361	535824	537289	538756
74	547600	549081	550564	552049	553536
75	562500	564001	565504	567009	568516
76	577600	579121	580644	582169	583696
77	592900	594441	595984	597529	599076
78	608400	609961	611524	613089	614656
79	624100	625681	627264	628849	630436
80	640000	641601	643204	644809	646416
81	656100	657721	659344	660969	662596
82	672400	674041	675684	677329	678976
83	688900	690561	692224	693889	695556
84	705600	707281	708964	710649	712336
85	722500	724201	725904	727609	729316
86	739600	741321	743044	744769	746496
87	756900	758641	760384	762129	763876
88	774400	776161	777924	779681	781456
89	792100	793881	795664	797449	799236
90	810000	811801	813604	815409	817216
91	828100	829921	831744	833569	835396
92	846400	848241	850084	851929	853776
93	864900	866761	868624	870489	872356
94	883600	885481	887364	889249	891136
95	902500	904401	906304	908209	910116
96	921600	923521	925444	927369	929296
97	940900	942841	944784	946729	948676
98	960400	962361	964324	966289	968256
99	980100	982081	984064	986049	988036
100	1000000	1002001	1004004	1006009	1008016
101	1020100	1022121	1024144	1026169	1028196
102	1040400	1042441	1044484	1046529	1048576
103	1060900	1062961	1065024	1067089	1069156
104	1081600	1083681	1085764	1087849	1089936
105	1102500	1104601	1106704	1108809	1110916

Square Numbers and their Roots.

69

Root	5	6	7	8	9
71	511225	512656	514089	515524	516961
72	525625	527076	528529	529984	531441
73	540225	541696	543169	544644	546121
74	555025	556516	558009	559504	561001
75	570025	571536	573049	574564	576081
76	585225	586756	588289	589824	591361
77	600625	602176	603729	605284	606841
78	616225	617796	619369	620944	622521
79	632025	633616	635209	636804	638401
80	648025	649636	651249	652864	654481
81	664225	665856	667489	669124	670761
82	680625	682276	683929	685584	687241
83	697225	698896	700569	702244	703921
84	714025	715716	717409	719104	720801
85	731025	732736	734449	736164	737881
86	748225	749956	751689	753424	755161
87	765625	767376	769129	770884	772641
88	783225	784996	786769	788544	790321
89	801025	802816	804609	806404	808201
90	819025	820836	822649	824464	826281
91	837225	839056	840889	842724	844561
92	855625	857476	859329	861184	863041
93	874225	876096	877969	879844	881721
94	893025	894916	896809	898704	900601
95	912025	913936	915849	917764	919681
96	931225	933156	935089	937024	938961
97	950625	952576	954529	956484	958441
98	970225	972196	974169	976144	978121
99	990025	992016	994009	996004	998001
100	1010025	1012036	1014049	1016064	1018081
101	1030225	1032256	1034289	1036324	1038361
102	1050625	1052676	1054729	1056784	1058841
103	1071225	1073296	1075369	1077444	1079521
104	1092025	1094116	1096209	1098304	1100401
105	1113025	1115136	1117249	1119364	1121481

Root	1	2	3	4
106	1123600	1125701	1127844	1129969
107	1144900	1147041	1149184	1151329
108	1166400	1168561	1170724	1173889
109	1188100	1190281	1192464	1194649
110	1210000	1212201	1214404	1216609
111	1232100	1234301	1236544	1238789
112	1254400	1256641	1258884	1261129
113	1276900	1279161	1281424	1283689
114	1299600	1301881	1304164	1306449
115	1322500	1324801	1327104	1329409
116	1345600	1347921	1350244	1352569
117	1368900	1371241	1373584	1375929
118	1392400	1394761	1397124	1399489
119	1416100	1418481	1420864	1423249
120	1440000	1442401	1444804	1447209
121	1464100	1466521	1468944	1471369
122	1488400	1490841	1493284	1495729
123	1512900	1515361	1517824	1520289
124	1537600	1540081	1542564	1545049
125	1562500	1565001	1567504	1570009
126	1587600	1590121	1592644	1595169
127	1612900	1615441	1617984	1620529
128	1638400	1640961	1643524	1646089
129	1664100	1666681	1669264	1671849
130	1690000	1692601	1695209	1697809
131	1716100	1718701	1721344	1723969
132	1742400	1745041	1747684	1750329
133	1768900	1771561	1774224	1776889
134	1795600	1798281	1800964	1803649
135	1822500	1825201	1827904	1830609
136	1849600	1852321	1855044	1857769
137	1876900	1879641	1882384	1885129
138	1904400	1907161	1909924	1912689
139	1932100	1934881	1937664	1940449
140	1960000	1962801	1965604	1968409

Square Numbers and their Roots.

71

Root	5	6	7	8	9
106	1134225	1136356	1138489	1140624	1142761
107	1155625	1157776	1159929	1162084	1164241
108	1177225	1179396	1181569	1183744	1185921
109	1199025	1201216	1203409	1205604	1207801
110	1221025	1223226	1225449	1227664	1229881
111	1243225	1245456	1247689	1249924	1252161
112	1265625	1267876	1270129	1272384	1274631
113	1288225	1290496	1292769	1295044	1297321
114	1311025	1313316	1315609	1317904	1320201
115	1334025	1336336	1338649	1340964	1343281
116	1357225	1359556	1361889	1364224	1366561
117	1380625	1382976	1385329	1387684	1390041
118	1404225	1406596	1408969	1411344	1413721
119	1428025	1430416	1432809	1435204	1437601
120	1452025	1454436	1456849	1459264	1461681
121	1476225	1478656	1481089	1483524	1485961
122	1500625	1503076	1505529	1507984	1510441
123	1525225	1527696	1530169	1532644	1535121
124	1550025	1552516	1555009	1557504	1560001
125	1575025	1577536	1580149	1582764	1585381
126	1600225	1602756	1605289	1607824	1610361
127	1625625	1628176	1630729	1633284	1635841
128	1651225	1653796	1656369	1658944	1661521
129	1677025	1679616	1682209	1684804	1687401
130	1703025	1705636	1708249	1710864	1713481
131	1729225	1731856	1734489	1737124	1739761
132	1755625	1758276	1760929	1763584	1766241
133	1782225	1784896	1787569	1790244	1792921
134	1809025	1811716	1814409	1817104	1819801
135	1836025	1838726	1841449	1844164	1846881
136	1863225	1865956	1868689	1871424	1874161
137	1890625	1893376	1896129	1898884	1901641
138	1918225	1920996	1923769	1926544	1929321
139	1946025	1948816	1951609	1954404	1957201
140	1974025	1976836	1979649	1982464	1985281

Root	0	1	2	3	4
141	1988100	1990921	1993744	1996569	1999396
142	2016400	2019241	2022084	2024929	2027776
143	2044900	2047761	2050624	2053489	2056356
144	2073600	2076481	2079364	2082249	2085136
145	2102500	2105401	2108304	2111209	2114116
146	2131600	2134521	2137444	2140369	2143296
147	2160900	2163841	2166784	2169729	2172676
148	2190400	2193361	2196324	2199289	2202256
149	2220100	2223081	2226064	2229049	2232036
150	2250000	2253001	2256004	2259049	2262016
151	2280100	2283121	2286144	2289169	2292196
152	2310400	2313441	2316484	2319529	2322576
153	2340900	2343961	2347024	2350089	2353156
154	2371600	2374681	2377764	2380849	2383936
155	2402500	2405601	2408704	2411809	2414916
156	2433600	2436721	2439844	2442969	2446096
157	2464900	2468041	2471184	2474329	2477476
158	2496400	2499561	2502724	2505889	2509056
159	2528100	2531281	2534464	2537649	2540836
160	2560000	2563201	2566404	2569609	2572816
161	2592100	2595321	2598544	2601769	2604996
162	2624400	2627641	2630884	2634129	2637376
163	2656900	2660161	2663424	2666689	2669956
164	2689600	2692881	2696164	2699449	2702736
165	2722500	2725801	2729104	2732409	2735716
166	2755600	2758921	2762244	2765569	2768896
167	2788900	2792241	2795584	2798929	2802276
168	2822400	2825761	2829124	2832489	2835856
169	2856100	2859481	2862864	2866249	2869636
170	2890000	2893401	2896804	2900209	2903616
171	2924100	2927521	2930944	2934361	2937796
172	2958400	2961841	2965284	2968729	2972176
173	2992900	2996361	2999824	3003289	3006756
174	3027600	3031081	3034564	3038049	3041536
175	3062500	3066001	3069504	3073009	3076516

Square Numbers and their Roots.

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Root	5	6	7	8	9
141	2002225	2005056	2007889	2010724	2013561
142	2030625	2033476	2036329	2039184	2042041
143	2059225	2062096	2064969	2067844	2070721
144	2088025	2090916	2093869	2096704	2099601
145	2117025	2119936	2122849	2125764	2128681
146	2146225	2149156	2152089	2155024	2157961
147	2175625	2178576	2181529	2184484	2187441
148	2205225	2208196	2211169	2214144	2217121
149	2235025	2238016	2241009	2244004	2247001
150	2265025	2268036	2271049	2274064	2277081
151	2295225	2298256	2301289	2304324	2307361
152	2325625	2328676	2331729	2334784	2337841
153	2356225	2359296	2362369	2365444	2368521
154	2387025	2390116	2393209	2396304	2399401
155	2418025	2421136	2424249	2427364	2430481
156	2449225	2452356	2455489	2458624	2461761
157	2480625	2483776	2486929	2490084	2493241
158	2512225	2515396	2518569	2521744	2524921
159	2544025	2547216	2550409	2553604	2556801
160	2576025	2579236	2582449	2585664	2588881
161	2608225	2611456	2614689	2617924	2621161
162	2640625	2643876	2647129	2650384	2653641
163	2673225	2676496	2679769	2683044	2686321
164	2706025	2709316	2712609	2715904	2719201
165	2739025	2742336	2745649	2748964	2752281
166	2772225	2775556	2778889	2782224	2785561
167	2805625	2808976	2812329	2815684	2819041
168	2839225	2842596	2845969	2849344	2852721
169	2873025	2876416	2879809	2883204	2886601
170	2907025	2910436	2913849	2917264	2920681
171	2941225	2944656	2948089	2951524	2954961
172	2975625	2979076	2982529	2985984	2989441
173	3010225	3013696	3017169	3020644	3024121
174	3045025	3048516	3052009	3055504	3059001
175	3080025	3083536	3087049	3090564	3094081

Root	0	1	2	3	4
176	3097600	3101121	3104644	3108169	3111696
177	3132900	3136441	3139984	3143529	3147096
178	3168400	3171961	3175524	3179089	3182656
179	3204100	3207681	3211264	3214849	3218436
180	3240000	3243601	3247204	3250809	3254416
181	3276100	3279721	3283344	3286969	3290596
182	3312400	3316041	3319684	3323329	3326976
183	3348900	3352561	3356224	3359889	3363556
184	3385600	3389281	3392964	3396649	3400336
185	3422500	3426201	3429904	3433609	3437316
186	3459600	3463321	3467044	3470769	3474496
187	3496900	3500641	3504384	3508129	3511876
188	3534400	3538161	3541924	3545689	3549456
189	3572100	3575881	3579664	3583449	3587236
190	3610000	3613801	3617604	3621409	3625216
191	3648100	3651921	3655744	3659569	3663396
192	3686400	3690241	3694084	3697929	3701776
193	3724900	3728761	3732624	3736489	3740356
194	3763600	3767481	3771364	3775249	3779136
195	3802500	3806401	3810304	3814209	3818116
196	3841600	3845521	3849444	3853369	3857296
197	3880900	3884841	3888784	3892729	3896676
198	3920400	3924361	3928364	3932329	3936256
199	3960100	3964081	3968064	3972049	3976036
200	4000000	4004001	4008004	4012009	4016016
201	4040100	4044121	4048144	4052169	4056196
202	4080400	4084441	4088484	4092529	4096576
203	4120900	4124961	4129024	4133089	4137156
204	4161600	4165681	4169764	4173849	4177936
205	4202500	4206601	4210704	4214809	4218916
206	4243600	4247721	4251844	4255969	4260096
207	4284900	4289041	4293184	4297329	4301476
208	4326400	4330561	4334724	4338889	4343056
209	4368100	4372281	4376464	4380649	4384836
210	4410000	4414201	4418404	4422609	4426816

Square Numbers and their Roots.

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Root	5	6	7	8	9
176	3115225	3118756	3122289	3125824	3129361
177	3150625	3154176	3157729	3161284	3164841
178	3186225	3189796	3193369	3196944	3200521
179	3222025	3225616	3229209	3232804	3236401
180	3258025	3261636	3265249	3268864	3272481
181	3294225	3297856	3301489	3305124	3308761
182	3330625	3334276	3337929	3341584	3345241
183	3367225	3370896	3374569	3378244	3381921
184	3404025	3407716	3411409	3415104	3418801
185	3441025	3444736	3448449	3452164	3455881
186	3478225	3481956	3485689	3489424	3493161
187	3515625	3519376	3523129	3526884	3530641
188	3553225	3556996	3560769	3564544	3568321
189	3591025	3594816	3598609	3602404	3606201
190	3629025	3632836	3636649	3640464	3644281
191	3667225	3671056	3674889	3678724	3682561
192	3705625	3709476	3713329	3717184	3721041
193	3744225	3748096	3751969	3755844	3759721
194	3783025	3786916	3790809	3794704	3798601
195	3822025	3825936	3829849	3833764	3837681
196	3861225	3865156	3869089	3873024	3876961
197	3900625	3904576	3908529	3912484	3916441
198	3940225	3944196	3948169	3952144	3956121
199	3980025	3984016	3988009	3992004	3996001
200	4020025	4024036	4028049	4032064	4036081
201	4060225	4064256	4068289	4072324	4076361
202	4100625	4104676	4108729	4112784	4116841
203	4141225	4145296	4149369	4153444	4157521
204	4182025	4186116	4190209	4194304	4198401
205	4223025	4227136	4231249	4235364	4239481
206	4264225	4268356	4272489	4276624	4280761
207	4305625	4309776	4313929	4318084	4322241
208	4347225	4351396	4355569	4359744	4363921
209	4389025	4393216	4397409	4401604	4405801
210	4431025	4435236	4439449	4443664	4447881

Root	0	1	2	3	4
211	4452100	4456321	4460544	4464769	4468996
212	4494900	4498641	4502884	4507129	4511376
213	4536900	4541161	4545424	4549689	4553956
214	4570600	4583881	4588164	4592449	4596736
215	4622500	4626801	4631104	4635409	4639716
216	4665600	4669921	4674244	4678569	4682896
217	4708900	4713241	4717584	4721929	4726276
218	4752400	4756761	4761124	4765489	4769856
219	4796100	4800481	4804864	4809249	4813636
220	4840000	4844401	4848804	4853209	4857616
221	4884100	4888521	4892944	4897369	4901796
222	4928400	4932841	4937284	4941729	4946176
223	4972900	4977361	4981824	4986289	4990756
224	5017600	5022081	5026564	5031049	5035536
225	5062500	5067001	5071504	5076009	5080516
226	5107600	5112121	5116644	5121169	5125696
227	5152900	5157441	5161984	5166529	5171076
228	5198400	5202961	5207524	5212089	5216656
229	5244100	5248681	5253264	5257849	5262436
230	5290000	5294601	5299204	5303809	5308416
231	5336100	5340721	5345344	5349969	5354596
232	5382400	5387041	5391684	5396329	5400976
233	5428900	5433561	5438224	5442889	5447556
234	5475600	5480281	5484964	5489649	5494336
235	5522500	5527201	5531904	5536609	5541316
236	5569600	5574321	5578984	5583769	5588496
237	5616900	5621641	5626384	5631129	5635876
238	5664400	5669161	5673924	5678689	5683456
239	5712100	5716881	5721664	5726449	5731236
240	5760000	5764801	5769604	5774409	5779216
241	5808100	5812921	5817744	5822569	5827396
242	5856400	5861241	5866084	5870929	5875776
243	5904900	5909761	5914624	5919489	5924356
244	5953600	5958481	5963364	5968249	5973136
245	6002500	6007401	6012304	6017209	6022116

Square Numbers and their Roots.

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Root	5	6	7	8	9
211	4473225	4477456	4481689	4485924	4490161
212	4515625	4519876	4524129	4528384	4532641
213	4558225	4562496	4566769	4571044	4575321
214	4601025	4605316	4609609	4613904	4618201
215	4644025	4648336	4652649	4656964	4661281
216	4687225	4691556	4695889	4700224	4704561
217	4730625	4734976	4739329	4743684	4748041
218	4774225	4778596	4782969	4787344	4791721
219	4818025	4822416	4826809	4831204	4835601
220	4862025	4866436	4870849	4875264	4879681
221	4906225	4910656	4915089	4919524	4923961
222	4950625	4955076	4959529	4963984	4968441
223	4995225	4999696	5004169	5008644	5013121
224	5040025	5044516	5049009	5053504	5058001
225	5085025	5089536	5094049	5098564	5103081
226	5130225	5134756	5139289	5143824	5148361
227	5175625	5180176	5184729	5189284	5193841
228	5221225	5225796	5230369	5234944	5239521
229	5267025	5271616	5276209	5280804	5285401
230	5313025	5317636	5322249	5326864	5331481
231	5359225	5363856	5368489	5373124	5377761
232	5405625	5410276	5414929	5419584	5424241
233	5452225	5456896	5461569	5466244	5470921
234	5499025	5503716	5508409	5513104	5517801
235	5546025	5550736	5555449	5560164	5564881
236	5593225	5597956	5602689	5607424	5612161
237	5640625	5645376	5650129	5654884	5659641
238	5688225	5692996	5697769	5702544	5707321
239	5736025	5740816	5745609	5750404	5755201
240	5784025	5788836	5793649	5798464	5803281
241	5832225	5837056	5841889	5846724	5851561
242	5880625	5885476	5890329	5895184	5900041
243	5929225	5934096	5938969	5943844	5948721
244	5978025	5982916	5987869	5992704	5997601
245	6027025	6031936	6036849	6041764	6046681

Root	0	1	2	3	4
246	6051600	6056521	6061444	6066369	6071296
247	6100900	6105841	6110784	6115729	6120676
248	6150400	6155361	6160324	6165289	6170256
249	6200100	6205085	6210064	6215049	6220036
250	6250000	6255001	6260004	6265009	6270016
251	6300100	6305121	6310144	6315169	6320196
252	6350400	6355441	6360484	6365529	6370576
253	6400900	6405961	6411024	6416089	6421156
254	6451600	6456681	6461764	6466849	6471936
255	6502500	6507601	6512704	6517809	6522916
256	6553600	6558721	6563844	6568969	6574096
257	6604900	6610041	6615184	6620329	6625476
258	6656400	6661561	6666724	6671889	6677056
259	6708100	6713281	6718464	6723649	6728836
260	6760000	6765201	6770404	6775609	6780816
261	6812100	6817321	6822544	6827769	6832996
262	6864400	6869641	6874884	6880129	6885376
263	6916900	6922161	6927424	6932689	6937956
264	6969600	6974881	6980164	6985449	6990736
265	7022500	7027801	7033104	7038409	7043716
266	7075600	7080921	7086244	7091569	7096896
267	7128900	7134241	7139584	7144929	7150276
268	7182400	7187761	7193124	7198489	7203856
269	7236100	7241481	7246864	7252249	7257636
270	7290000	7295401	7300804	7306206	7311616
271	7344100	7349521	7354944	7360369	7365796
272	7398400	7403841	7409284	7414729	7420176
273	7452900	7458361	7463824	7469289	7474756
274	7507600	7513081	7518564	7524099	7529536
275	7562500	7568001	7573504	7579009	7584516
276	7617600	7623121	7628644	7634169	7639696
277	7672900	7678441	7683984	7689529	7695076
278	7728400	7733961	7739524	7745089	7750656
279	7784100	7789681	7795264	7800849	7806436
280	7840000	7845601	7851204	7856809	7862416

Square Numbers and their Roots.

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Root	5	6	7	8	9
246	6076225	6081156	6086089	6091024	6095961
247	6125625	6130576	6135529	6140484	6145441
248	6175225	6180196	6185169	6190144	6195121
249	6225025	6230016	6235009	6240004	6245001
250	6275025	6280036	6285049	6290064	6295081
251	6325225	6330256	6335289	6340324	6345361
252	6375625	6380676	6385729	6390784	6395841
253	6426225	6431296	6436369	6441444	6446521
254	6477025	6482116	6497209	6492304	6497401
255	6528025	6533136	6538249	6543364	6548481
256	6579225	6584356	6589489	6594624	6599761
257	6630625	6635776	6640929	6646084	6651241
258	6682225	6687396	6692569	6697744	6702921
259	6734025	6739216	6744409	6749604	6754801
260	6786025	6791236	6796449	6801664	6806881
261	6838225	6843456	6848689	6853924	6859161
262	6890625	6895876	6901129	6906384	6911641
263	6943225	6948496	6953769	6959044	6964321
264	6996025	7001316	7006609	7011904	7017201
265	7049025	7054336	7059649	7064964	7070281
266	7102225	7107556	7112889	7118224	7123561
267	7155625	7160976	7166329	7171684	7177041
268	7209225	7214596	7219969	7225344	7230721
269	7263025	7268416	7273809	7279204	7284601
270	7317025	7322436	7327849	7333264	7338681
271	7371225	7376656	7382089	7387524	7392961
272	7425625	7431076	7436529	7441984	7447441
273	7480225	7485696	7491169	7496644	7502121
274	7535025	7540516	7546009	7551504	7557001
275	7590025	7595536	7601049	7606564	7612081
276	7645225	7650756	7656289	7661824	7667361
277	7700625	7706176	7711729	7717284	7722841
278	7756225	7761796	7767369	7772944	7778521
279	7812025	7817616	7823209	7828804	7834401
280	7868025	7873636	7879249	7884864	7890481

Root.	0	1	2	3	4
281	7896100	7901721	7907344	7912969	7918596
282	7952400	7958041	7963684	7969329	7974976
283	8008900	8014561	8020224	8025889	8031556
284	8065600	8071281	8076964	8082649	8088336
285	8122500	8128201	8133904	8139609	8145316
286	8179600	8185321	8191044	8196769	8202496
287	8236900	8242641	8248384	8254129	8259876
288	8294400	8300161	8305924	8311689	8317456
289	8352100	8357881	8363664	8369449	8375236
290	8410000	8415801	8421604	8427409	8433216
291	8468100	8473921	8479744	8485569	8491396
292	8526400	8532241	8538084	8543929	8549776
293	8584900	8590716	8596624	8602489	8608356
294	8643600	8649481	8655364	8661249	8667136
295	8702500	8708401	8714304	8720209	8726116
296	8761600	8767521	8773444	8779369	8785296
297	8820900	8826841	8832784	8838729	8844676
298	8880400	8886361	8892324	8898289	8904256
299	8940100	8946081	8952064	8958049	8964036
300	9000000	9006001	9012004	9018009	9024016
301	9060100	9066121	9072144	9078169	9084196
302	9120400	9126441	9132484	9138529	9144576
303	9180900	9186961	9193024	9199089	9205156
304	9241600	9247681	9253764	9259849	9265936
305	9302500	9308601	9314704	9320809	9326916
306	9363600	9369721	9375844	9381969	9388096
307	9424900	9431041	9437184	9443329	9449476
308	9486400	9492561	9498724	9504889	9511056
309	9548100	9554281	9560464	9566649	9572836
310	9610000	9616201	9622404	9628609	9634816
311	9672100	9678321	9684544	9690769	9696996
312	9734400	9740641	9746884	9753129	9759376
313	9796900	9803161	9809424	9815689	9821956
314	9859600	9865881	9872164	9878449	9884736
315	9922500	9928801	9935104	9941409	9947716

Square Numbers and their Roots.

81

Root	5	6	7	8	9
281	7924125	7929856	7935489	7941124	7946761
282	7980625	7986276	7991929	7997584	8003241
283	8037225	8042896	8048569	8054244	8059921
284	8094025	8099716	8105409	8111104	8116801
285	8151025	8156736	8162449	8168164	8173881
286	8208225	8213956	8219689	8225424	8231161
287	8265625	8271376	8277129	8282884	8288641
288	8323225	8328996	8334769	8340544	8346321
289	8381025	8386816	8392609	8398404	8404201
290	8439025	8444836	8450649	8456464	8462281
291	8497225	8503056	8508889	8514724	8520561
292	8555625	8561476	8567329	8573184	8579041
293	8614225	8620096	8625969	8631844	8637721
294	8673025	8678916	8684809	8690704	8696601
295	8732025	8737936	8743849	8749764	8755681
296	8791225	8797156	8803089	8809024	8814961
297	8850625	8856576	8862529	8868484	8874441
298	8910225	8916196	8922169	8928144	8934121
299	8970025	8976016	8982009	8988004	8994001
300	9030025	9036036	9042049	9048064	9054081
301	9090225	9096256	9102289	9108324	9114361
302	9150625	9156676	9162729	9168784	9174841
303	9211225	9217296	9223369	9229444	9235521
304	9272025	9278116	9284209	9290304	9296401
305	9333025	9339136	9345249	9351364	9357481
306	9394225	9400356	9406489	9412624	9418761
307	9455625	9461776	9467929	9474084	9480241
308	9517225	9523396	9529569	9535744	9541921
309	9579025	9585216	9591409	9597604	9603801
310	9641025	9647236	9653449	9659664	9665881
311	9703225	9709456	9715689	9721924	9728161
312	9765625	9771876	9778129	9784384	9790641
313	9828225	9834496	9840769	9847044	9853321
314	9891025	9897316	9903609	9909904	9916201
315	9954025	9960336	9966649	9972964	9979281

Root	0	1	2	3	4
316	9985600	9991921	9998244	10004569	10010896
317	10048900	10055241	10061584	10067929	10074276
318	10112400	10118761	10125124	10131489	10137856
319	10176100	10182481	10188864	10195249	10201636
320	10240000	10246401	10252804	10259209	10265616
321	10304100	10310521	10316944	10323369	10329796
322	10368400	10374841	10381284	10387729	10394176
323	10432900	10439361	10445824	10452289	10458756
324	10497600	10504081	10510564	10517049	10523536
325	10562500	10569001	10575504	10582009	10588516
326	10627600	10634121	10640644	10647169	10653696
327	10692900	10699441	10705984	10712529	10719076
328	10758400	10764961	10771524	10778089	10784656
329	10824100	10830681	10837264	10843849	10850436
330	10890000	10896601	10903204	10909809	10916416
331	10956100	10962721	10969344	10975969	10982596
332	11022400	11029041	11035684	11042329	11048976
333	11088900	11095561	11102224	11108889	11115556
334	11155600	11162281	11168964	11175649	11182336
335	11222500	11229201	11235904	11242609	11249316
336	11289600	11296321	11303044	11309769	11316496
337	11356900	11363641	11370384	11377129	11383876
338	11424400	11431161	11437924	11444689	11451456
339	11492100	11498881	11505664	11512449	11519236
340	11560000	11566801	11573604	11580409	11587216
341	11628100	11634921	11641744	11648569	11655396
342	11696400	11703241	11710084	11716929	11723776
343	11764900	11771761	11778624	11785489	11792356
344	11833600	11840481	11847364	11854249	11861136
345	11902500	11909401	11916304	11923209	11930116
346	11971600	11978521	11985444	11992369	11999296
347	12040900	12047841	12054784	12061729	12068676
348	12110400	12117361	12124324	12131289	12138256
349	12180100	12187081	12194064	12201049	12208036
350	12250000	12257001	12264004	12271009	12278016

Square Numbers and their Roots

83

Root	5	6	7	8	9
316	10017225	10023556	10029889	10036224	10042561
317	10040681	10047076	10053479	10059884	10066291
318	10064228	10070656	10077099	10083544	10089991
319	10087865	10094316	10100779	10107244	10113721
320	10111600	10118064	10124539	10131014	10137491
321	10135425	10141904	10148389	10154874	10161361
322	10159336	10165824	10172319	10178814	10185311
323	10183325	10189824	10196329	10202844	10209361
324	10207400	10213904	10220419	10226944	10233471
325	10231561	10238064	10244579	10251094	10257621
326	10255800	10262304	10268819	10275344	10281871
327	10280125	10286624	10293139	10299654	10306171
328	10304536	10311024	10317519	10324014	10330521
329	10329025	10335504	10341999	10348494	10354991
330	10353600	10360064	10366539	10373014	10379491
331	10378261	10384704	10391149	10397594	10404041
332	10403000	10409424	10415859	10422294	10428731
333	10427825	10434240	10440669	10447094	10453521
334	10452736	10459136	10465539	10471944	10478351
335	10477725	10484104	10490489	10496874	10503261
336	10502800	10509164	10515529	10521894	10528261
337	10527961	10534304	10540649	10546994	10553341
338	10553200	10559524	10565849	10572174	10578501
339	10578525	10584836	10591149	10597464	10603781
340	10603936	10610224	10616519	10622814	10629111
341	10629525	10635796	10642069	10648344	10654621
342	10655100	10661356	10667619	10673884	10680151
343	10680761	10686996	10693239	10699484	10705731
344	10706500	10712704	10718909	10725114	10731321
345	10732225	10738404	10744589	10750774	10756961
346	10758036	10764196	10770359	10776524	10782691
347	10783825	10789964	10796109	10802254	10808401
348	10809600	10815724	10821849	10827974	10834101
349	10835461	10841564	10847669	10853774	10859881
350	10861400	10867484	10873569	10879654	10885741

Root	0	1	2	3	4
351	12320100	12327121	12334144	12341169	12348196
352	12390400	12397441	12404484	12411529	12418576
353	12460900	12467961	12475024	12482089	12489156
354	12531600	12538681	12545764	12552849	12559936
355	12602500	12609601	12616704	12623809	12630916
356	12673600	12680721	12687844	12694969	12702096
357	12744900	12752041	12759184	12766329	12773476
358	12816400	12823561	12830724	12837889	12845056
359	12888100	12895281	12902464	12909649	12916836
360	12960000	12967201	12974404	12981609	12988816
361	13032100	13039321	13046544	13053769	13060996
362	13104400	13111641	13118884	13126129	13133376
363	13176900	13184161	13191424	13198689	13205956
364	13249600	13256881	13264164	13271449	13278736
365	13322500	13329801	13337104	13344409	13351716
366	13395600	13402921	13410244	13417569	13424896
367	13468900	13476241	13483584	13490929	13498276
368	13542400	13549761	13557124	13564489	13571856
369	13616100	13623481	13630864	13638249	13645636
370	13690000	13697401	13704804	13712209	13719616
371	13764100	13771521	13778944	13786369	13793796
372	13838400	13845841	13853284	13860729	13868176
373	13912900	13920361	13927824	13935289	13942756
374	13987600	13995081	14002564	14010049	14017536
375	14062500	14070001	14077504	14085009	14092516
376	14137600	14145121	14152644	14160169	14167696
377	14212900	14220441	14227984	14235529	14243076
378	14288400	14295961	14303524	14311089	14318656
379	14364100	14371681	14379264	14386849	14394436
380	14440000	14447601	14455204	14462809	14470416
381	14516100	14523721	14531344	14538969	14546596
382	14592400	14600041	14607684	14615329	14622976
383	14668900	14676561	14684224	14691889	14699556
384	14745600	14753281	14760964	14768649	14776336
385	14822500	14830201	14837904	14845609	14853316

Square Numbers and their Roots.

85

Root	5	6	7	8	9
351	12355325	12362256	12369289	12376324	12383361
352	12425625	12432676	12439729	12446784	12453841
353	12496125	12503296	12510369	12517444	12524521
354	12567025	12574116	12581209	12588304	12595401
355	12638025	12645136	12652249	12659364	12666481
356	12709125	12716356	12723489	12730624	12737761
357	12780625	12787776	12794929	12802084	12809241
358	12852225	12859396	12866569	12873744	12880921
359	12924025	12931216	12938409	12945604	12952801
360	12996025	13003236	13010449	13017664	13024881
361	13068225	13075456	13082689	13089924	13097161
362	13140625	13147876	13155129	13162384	13169641
363	13213225	13220496	13227769	13235044	13242321
364	13286025	13293316	13300609	13307904	13315201
365	13359025	13366336	13373649	13380964	13388281
366	13432225	13439556	13446889	13454224	13461561
367	13505625	13512976	13520329	13527684	13535041
368	13579225	13586596	13593969	13601344	13608721
369	13653025	13660416	13667809	13675204	13682601
370	13727025	13734436	13741849	13749264	13756681
371	13801225	13808656	13816089	13823524	13830961
372	13875625	13883076	13890529	13897984	13905441
373	13950225	13957696	13965169	13972644	13980121
374	14025025	14032516	14040009	14047504	14055001
375	14100025	14107536	14115049	14122564	14130081
376	14175225	14182756	14190289	14197824	14205361
377	14250625	14258176	14265729	14273284	14280841
378	14326225	14333796	14341369	14348944	14356521
379	14402025	14409616	14417209	14424804	14432401
380	14478025	14485626	14493249	14500864	14508481
381	14554225	14561856	14569489	14577124	14584761
382	14630625	14638276	14645929	14653584	14661241
383	14707225	14714896	14722569	14730244	14737921
384	14784025	14791716	14799409	14807104	14814801
385	14861025	14868736	14876449	14884164	14891881

Root	0	1	2	3	4
386	14899600	14907321	14915044	14922769	14930496
387	14976900	14984641	14992384	15000129	15007876
388	15054400	15062161	15069924	15077689	15085456
389	15132100	15139881	15147664	15155449	15163236
390	15210000	15217801	15225604	15233409	15241216
391	15288100	15295921	15303744	15311569	15319396
392	15366400	15374241	15382084	15389929	15397776
393	15444900	15452761	15460624	15468489	15476356
394	15523600	15531481	15539364	15547249	15555136
395	15602500	15610401	15618304	15626209	15634116
396	15681600	15689521	15697444	15705369	15713296
397	15760900	15768841	15776784	15784729	15792676
398	15840400	15848361	15856324	15864289	15872256
399	15920100	15928081	15936064	15944049	15952036
400	16000000	16008001	16016004	16024009	16032016
401	16080100	16088121	16096144	16104169	16112196
402	16160400	16168441	16176484	16184529	16192576
403	16240900	16248961	16257024	16265089	16273156
404	16321600	16329681	16337764	16345849	16353936
405	16402500	16410601	16418704	16426809	16434916
406	16483600	16491721	16499844	16507969	16516096
407	16564900	16573041	16581184	16589329	16597476
408	16646400	16654561	16662724	16670889	16679056
409	16728100	16736281	16744464	16752649	16760836
410	16810000	16818201	16826404	16834609	16842816
411	16892100	16900321	16908544	16916769	16924996
412	16974400	16982641	16990884	16999129	17007376
413	17056900	17065161	17073424	17081689	17089956
414	17139600	17147881	17156164	17164449	17172736
415	17222500	17230801	17239104	17247409	17255716
416	17305600	17313921	17322244	17330569	17338896
417	17388900	17397241	17405584	17413929	17422276
418	17472400	17480761	17489124	17497489	17505856
419	17556100	17564481	17572864	17581249	17589636
420	17640000	17648401	17656804	17665209	17673616

Square Numbers and their Roots.

87

Root	5	6	7	8	9
386	14938225	14941936	14953689	14961424	14969161
387	15011625	15023376	15031129	15038884	15046641
388	15091225	15100996	15108769	15116544	15124321
389	15171025	15178816	15186609	15194404	15202201
390	15249025	15256836	15264649	15272464	15280281
391	15327225	15335056	15342889	15350724	15358561
392	15405625	15413476	15421329	15429184	15437041
393	15484225	15492096	15499969	15507844	15515721
394	15563025	15570916	15578809	15586704	15594601
395	15642025	15649936	15657849	15665764	15673681
396	15721225	15729156	15737089	15745024	15752961
397	15800625	15808576	15816529	15824484	15832441
398	15880225	15888196	15896169	15904144	15912121
399	15960025	15968016	15976009	15984004	15992001
400	16040025	16048036	16056049	16064064	16072081
401	16120225	16128256	16136289	16144324	16152361
402	16200625	16208676	16216729	16224784	16232841
403	16281225	16289296	16297369	16305444	16313521
404	16362025	16370116	16378209	16386304	16394401
405	16443025	16451136	16459249	16467364	16475481
406	16524225	16532356	16540489	16548624	16556761
407	16605625	16613776	16621929	16630084	16638241
408	16687225	16695396	16703569	16711744	16719921
409	16769025	16777216	16785409	16793604	16801801
410	16851025	16859236	16867449	16875664	16883881
411	16933225	16941456	16949689	16957924	16966161
412	17015625	17023876	17032129	17040384	17048641
413	17098225	17106496	17114769	17123044	17131321
414	17181025	17189316	17197609	17205904	17214201
415	17264025	17272336	17280649	17288964	17297281
416	17347225	17355556	17363889	17372224	17380561
417	17430625	17438976	17447329	17455684	17464041
418	17514225	17522596	17530969	17539344	17547721
419	17598025	17606416	17614809	17623204	17631601
420	17682025	17690436	17698849	17707264	17715681

Root	0	1	2	3	4
421	17724100	17732521	17740944	17749369	17757796
422	17808400	17816841	17825284	17833729	17842176
423	17892900	17901361	17909824	17918289	17926756
424	17977600	17986081	17994564	18003049	18011536
425	18062500	18071001	18079504	18088009	18096516
426	18147600	18156121	18164644	18173169	18181696
427	18232900	18241441	18249984	18258529	18267076
428	18318400	18326961	18335524	18344089	18352656
429	18404100	18412681	18421264	18429849	18438436
430	18490000	18498601	18507204	18515809	18524416
431	18576100	18584721	18593344	18601969	18610596
432	18662400	18671041	18679684	18688329	18696976
433	18748900	18757561	18766224	18774889	18783556
434	18835600	18844281	18852964	18861649	18870336
435	18922500	18931201	18939904	18948609	18957316
436	19009600	19018321	19027044	19035769	19044496
437	19096900	19105641	19114384	19123129	19131876
438	19184400	19193161	19201924	19210689	19219456
439	19272100	19280881	19289664	19298449	19307236
440	19360000	19368801	19377604	19386409	19395216
441	19448100	19456921	19465744	19474569	19483396
442	19536400	19545241	19554084	19562929	19571776
443	19624900	19633761	19642624	19651489	19660356
444	19713600	19722481	19731364	19740249	19749136
445	19802500	19811401	19820304	19829209	19838116
446	19891600	19900521	19909444	19918369	19927296
447	19980900	19989841	19998784	20007729	20016676
448	20070400	20079361	20088324	20097289	20106256
449	20160100	20169081	20178064	20187049	20196036
450	20250000	20259001	20268004	20277009	20286016
451	20340100	20349121	20358144	20367169	20376196
452	20430400	20439441	20448484	20457529	20466576
453	20520900	20529961	20539024	20548089	20557156
454	20611600	20620681	20629764	20638849	20647936
455	20702500	20711601	20720704	20729809	20738916

Square Numbers and their Roots.

89

Root	5	6	7	8	9
421	17766225	17774656	17783089	17791524	17799961
422	17850625	17859076	17867529	17875984	17884441
423	17935225	17943696	17952169	17960644	17969121
424	18020025	18028516	18037009	18045504	18054001
425	18105025	18113536	18122049	18130564	18139081
426	18190225	18198756	18207289	18215824	18224361
427	18275625	18284176	18292729	18301284	18309841
428	18361225	18369796	18378369	18386944	18395521
429	18447025	18455616	18464209	18472804	18481401
430	18533025	18541636	18550249	18558864	18567481
431	18619225	18627856	18636489	18645124	18653761
432	18705625	18714276	18722929	18731584	18740241
433	18792225	18800896	18809569	18818244	18826921
434	18879025	18887716	18896409	18905104	18913801
435	18966025	18974736	18983449	18992164	19000881
436	19053225	19061956	19070689	19079424	19088161
437	19140625	19149376	19158129	19166884	19175641
438	19228225	19236996	19245769	19254544	19263321
439	19316025	19324816	19333609	19342404	19351201
440	19404025	19412836	19421649	19430464	19439281
441	19492225	19501056	19509889	19518724	19527561
442	19580625	19589476	19598329	19607184	19616041
443	19669225	19678096	19686969	19695844	19704721
444	19758025	19766916	19775809	19784704	19793601
445	19847025	19855936	19864849	19873764	19882681
446	19936225	19945156	19954089	19963024	19971961
447	20025625	20034576	20043529	20052484	20061441
448	20115225	20124196	20133169	20142144	20151121
449	20205025	20214016	20223009	20232004	20241001
450	20295025	20304036	20313049	20322064	20331081
451	20385225	20394256	20403289	20412324	20421361
452	20475625	20484676	20493729	20502784	20511841
453	20566225	20575296	20584369	20593444	20602521
454	20657025	20666116	20675209	20684304	20693401
455	20748025	20757136	20766249	20775364	20784481

Root	0	1	2	3	4
456	20793600	20801721	20811844	20820969	20830096
457	20884900	20894041	20903184	20912329	20921476
458	20976400	20985561	20994724	21003889	21013056
459	21068100	21077281	21086464	21095649	21104836
460	21160000	21169201	21178404	21187609	21196816
461	21252100	21261321	21270544	21279769	21288996
462	21344400	21353641	21362884	21372129	21381376
463	21436900	21446161	21455424	21464689	21473956
464	21529600	21538881	21548164	21557449	21566736
465	21622500	21631801	21641104	21650409	21659716
466	21715600	21724921	21734244	21743569	21752896
467	21808900	21818241	21827584	21836929	21846276
468	21902400	21911761	21921124	21930489	21939856
469	21996100	22005481	22014864	22024249	22033636
470	22090000	22099401	22108804	22118209	22127616
471	22184100	22193521	22202944	22212369	22221796
472	22278400	22287841	22297284	22306729	22316176
473	22372900	22382361	22391824	22401289	22410756
474	22467600	22477081	22486564	22496049	22505536
475	22562500	22572001	22581504	22591009	22600516
476	22657600	22667121	22676644	22686169	22695696
477	22752900	22762441	22771984	22781529	22791076
478	22848400	22857961	22867524	22877089	22886656
479	22944100	22953681	22963264	22972849	22982436
480	23040000	23049601	23059204	23068809	23078416
481	23136100	23145721	23155344	23164969	23174596
482	23232400	23242041	23251684	23261329	23270976
483	23328900	23338561	23348224	23357889	23367556
484	23425600	23435281	23444964	23454649	23464336
485	23522500	23532201	23541904	23551609	23561316
486	23619600	23629321	23639044	23648769	23658496
487	23716900	23726641	23736384	23746129	23755876
488	23814400	23824161	23833924	23843689	23853456
489	23912100	23921881	23931664	23941449	23951236
490	24010000	24019801	24029604	24039409	24049216

Square Numbers and their Roots

91

Root	5	6	7	8	9
416	20839225	20848356	20857489	20866624	20875761
417	20930625	20939776	20948929	20958084	20967241
418	21022225	21031396	21040569	21049744	21058921
419	21114025	21123216	21132409	21141604	21150801
420	21206025	21215236	21224449	21233664	21242881
461	21298225	21307456	21316689	21325924	21335161
462	21390625	21399876	21409129	21418384	21427641
463	21483225	21492496	21501769	21511044	21520321
464	21576025	21585316	21594609	21603904	21613201
465	21669025	21678336	21687649	21696964	21706281
466	21762225	21771556	21780889	21790224	21799561
467	21855625	21864976	21874329	21883684	21893041
468	21949225	21958596	21967969	21977344	21986721
469	22043025	22052416	22061809	22071204	22080601
470	22137025	22146436	22155849	22165264	22174681
471	22231225	22240656	22250089	22259524	22268961
472	22325625	22335076	22344529	22353984	22363441
473	22420225	22429696	22439169	22448644	22458121
474	22515025	22524516	22534009	22543504	22553001
475	22610025	22619536	22629049	22638564	22648081
476	22705225	22714756	22724289	22733824	22743361
477	22800625	22810176	22819729	22829284	22838841
478	22896225	22905796	22915369	22924944	22934521
479	22992025	23001616	23011209	23020804	23030401
480	23088025	23097636	23107249	23116864	23126481
481	23184225	23193856	23203489	23213124	23222761
482	23280625	23290276	23299929	23309584	23319241
483	23377225	23386896	23396569	23406244	23415921
484	23474025	23483716	23493409	23503104	23512801
485	23571025	23580736	23590449	23600164	23609881
486	23668225	23677956	23687689	23697424	23707161
487	23765625	23775376	23785129	23794884	23804641
488	23863225	23872996	23882769	23892544	23902321
489	23961025	23970816	23980609	23990404	24000201
490	24059025	24068836	24078649	24088464	24098281

Root	0	1	2	3	4
491	24108100	24117921	24127744	24137569	24147396
492	24206400	24216241	24226084	24235929	24245776
493	24304900	24314761	24324624	24334489	24344356
494	24403600	24413481	24423364	24433249	24443136
495	24502500	24512401	24522304	24532209	24542116
496	24601600	24611521	24621444	24631369	24641296
497	24700900	24710841	24720784	24730729	24740676
498	24800400	24810361	24820324	24830289	24840256
499	24900100	24910081	24920064	24930049	24940036
500	25000000	25010001	25020004	25030009	25040016
501	25100100	25110121	25120144	25130169	25140196
502	25200400	25210441	25220484	25230529	25240576
503	25300900	25310961	25321024	25331089	25341156
504	25401600	25411681	25421764	25431849	25441936
505	25502500	25512601	25522704	25532809	25542916
506	25603600	25613721	25623844	25633969	25644096
507	25704900	25715041	25725184	25735329	25745476
508	25806400	25816561	25826724	25836889	25847056
509	25908100	25918281	25928464	25938649	25948836
510	26010000	26020201	26030404	26040609	26050816
511	26112100	26122321	26132544	26142769	26152996
512	26214400	26224641	26234884	26245129	26255376
513	26316900	26327161	26337424	26347689	26357956
514	26419600	26429881	26440164	26450449	26460736
515	26522500	26532801	26543104	26553409	26563716
516	26625600	26635921	26646244	26656569	26666896
517	26728900	26739241	26749584	26759929	26770276
518	26832400	26842761	26853124	26863489	26873856
519	26936100	26946481	26956864	26967249	26977636
520	27040000	27050401	27060804	27071209	27081616
521	27144100	27154521	27164944	27175369	27185796
522	27248400	27258841	27269284	27279729	27290176
523	27352900	27363361	27373824	27384289	27394756
524	27457600	27468081	27478564	27489049	27499536
525	27562500	27573001	27583504	27594009	27604516

Square Numbers and their Roots.

93

Root	5	6	7	8	9
491	24157225	24167056	24176889	24186724	24196561
492	24255625	24265476	24275329	24285184	24295041
493	24354225	24364096	24373969	24383844	24393721
494	24453025	24462916	24472809	24482704	24492601
495	24552025	24561936	24571849	24581764	24591681
496	24651225	24661156	24671089	24681024	24690961
497	24750625	24760576	24770529	24780484	24790441
498	24850225	24860196	24870169	24880144	24890121
499	24950025	24960016	24970009	24980004	24990001
500	25050025	25060036	25070049	25080064	25090081
501	25150225	25160256	25170289	25180324	25190361
502	25250625	25260676	25270729	25280784	25290841
503	25351225	25361296	25371369	25381444	25391521
504	25452025	25462116	25472209	25482304	25492401
505	25553025	25563136	25573249	25583364	25593481
506	25654225	25664356	25674489	25684624	25694761
507	25755625	25765776	25775929	25786084	25796241
508	25857225	25867396	25877569	25887744	25897921
509	25959025	25969216	25979409	25989604	25999801
510	26061025	26071236	26081449	26091664	26101881
511	26163225	26173456	26183689	26193924	26204161
512	26265625	26275876	26286119	26296384	26306641
513	26368225	26378496	26388769	26399044	26409321
514	26471025	26481316	26491609	26501904	26512201
515	26574025	26584336	26594649	26604964	26615281
516	26677225	26687556	26697889	26708224	26718561
517	26780625	26790976	26801329	26811684	26822041
518	26884225	26894596	26904969	26915344	26925721
519	26988025	26998416	27008809	27019204	27029601
520	27092025	27102436	27112849	27123264	27133681
521	27196225	27206656	27217089	27227524	27237961
522	27300625	27311076	27321529	27331984	27342441
523	27405225	27415696	27426169	27436644	27447121
524	27510025	27520516	27531009	27541504	27552001
525	27615025	27625536	27636049	27646564	27657081

Root	0	1	2	3	4
526	27667600	27678121	27688644	27699169	27709696
527	27771900	27783441	27793984	27804529	27815076
528	27878400	27888961	27899524	27910089	27920656
529	27984100	27994681	28005264	28015849	28026436
530	28090000	28100601	28111204	28121809	28132416
531	28196100	28206721	28217344	28227969	28238596
532	28302400	28313041	28323684	28334329	28344976
533	28408900	28419561	28430224	28440889	28451556
534	28515600	28526281	28536964	28547649	28558336
535	28622500	28633201	28643904	28654609	28665316
536	28729600	28740321	28751044	28761769	28772496
537	28836900	28847641	28858384	28869129	28879876
538	28944400	28955161	28965924	28976689	28987456
539	29052100	29062881	29073664	29084449	29095236
540	29160000	29170801	29181604	29192409	29203216
541	29268100	29278921	29289744	29300569	29311396
542	29376400	29387241	29398084	29408929	29419776
543	29484900	29495761	29506624	29517489	29528356
544	29593600	29604481	29615364	29626249	29637136
545	29702500	29713401	29724304	29735209	29746116
546	29811600	29822521	29833444	29844369	29855296
547	29920900	29931841	29942784	29953729	29964676
548	30030400	30041361	30052324	30063289	30074256
549	30140100	30151081	30162064	30173049	30184036
550	30250000	30261001	30272004	30283009	30294016
551	30360100	30371121	30382144	30392169	30404196
552	30470400	30481441	30492484	30503529	30514576
553	30580900	30591961	30603024	30614089	30625156
554	30691600	30702681	30713764	30724849	30735936
555	30802500	30813601	30824704	30835809	30846916
556	30913600	30924721	30935844	30946969	30958096
557	31024900	31036041	31047184	31058329	31069476
558	31136400	31147561	31158724	31169889	31181056
559	31248100	31259281	31270464	31281649	31292836
560	31360000	31371201	31382404	31393609	31404816

Square Numbers and their Roots.

95

Root.	5	6	7	8	9
526	27720225	27730756	27741289	27751824	27762361
527	27825625	27836176	27846729	27857284	27867841
528	27931225	27941796	27952369	27962944	27973521
529	28037025	28047616	28058209	28068804	28079401
530	28143025	28153636	28164249	28174864	28185481
531	28249225	28259856	28270489	28281124	28291761
532	28355625	28366276	28376929	28387584	28398241
533	28462225	28472896	28483569	28494244	28504921
534	28569025	28579716	28590409	28601104	28611801
535	28676025	28686736	28697449	28708164	28718881
536	28783225	28793956	28804689	28815424	28826161
537	28890625	28901376	28912129	28922884	28933641
538	28998225	29008996	29019769	29030544	29041321
539	29106025	29116816	29127609	29138404	29149201
540	29214025	29224836	29235649	29246464	29257281
541	29322225	29333056	29343889	29354724	29365561
542	29430625	29441476	29452329	29463184	29474041
543	29539225	29550096	29560969	29571844	29582721
544	29648025	29658916	29669809	29680704	29691601
545	29757025	29767936	29778849	29789764	29800681
546	29866225	29877156	29888089	29899024	29909961
547	29975625	29986576	29997529	30008484	30019441
548	30085225	30096196	30107169	30118144	30129121
549	30195025	30206016	30217009	30228004	30239001
550	30305025	30316036	30327049	30338064	30349081
551	30415225	30426256	30437289	30448324	30459361
552	30525625	30536676	30547729	30558784	30569841
553	30636225	30647296	30658369	30669444	30680521
554	30747025	30758116	30769209	30780304	30791401
555	30858025	30869136	30880249	30891364	30902481
556	30969225	30980356	30991489	31002624	31013761
557	31080625	31091776	31102929	31114084	31125241
558	31192225	31203396	31214569	31225744	31236921
559	31304025	31315216	31326409	31337604	31348801
560	31416025	31427236	31438449	31449664	31460881

Root	0	1	2	3	4
561	31472100	31483321	31494544	31505769	31516996
562	31584400	31595641	31606884	31618129	31629376
563	31696900	31708161	31719424	31730689	31741956
564	31809600	31820881	31832164	31843449	31854736
565	31922500	31933801	31945104	31956409	31967716
566	32035600	32046921	32058244	32069569	32080896
567	32148900	32160241	32171584	32182929	32194276
568	32262400	32273761	32285124	32296489	32307856
569	32376100	32387481	32398864	32410249	32421636
570	32490000	32501401	32512804	32524209	32535616
571	32604100	32615521	32626944	32638369	32649796
572	32718400	32729841	32741284	32752729	32764176
573	32832900	32844361	32855824	32867289	32878756
574	32947600	32959081	32970564	32982049	32993536
575	33062500	33074001	33085504	33097009	33108516
576	33177600	33189121	33200644	33212169	33223696
577	33292900	33304441	33315984	33327529	33339076
578	33408400	33419961	33431524	33443089	33454656
579	33524100	33535681	33547264	33558849	33570436
580	33640000	33651601	33663204	33674809	33686416
581	33756100	33767721	33779344	33790969	33802596
582	33872400	33884041	33895684	33907329	33918976
583	33988900	34000561	34012224	34023889	34035556
584	34105600	34117281	34128964	34140649	34152336
585	34222500	34234201	34245904	34257609	34269316
586	34339600	34351321	34363044	34374769	34386496
587	34456900	34468641	34480384	34492129	34503876
588	34574400	34586161	34597924	34609689	34621456
589	34692100	34703881	34715664	34727449	34739236
590	34810000	34821801	34833604	34845409	34857216
591	34928100	34939921	34951744	34963569	34975396
592	35046400	35058241	35070084	35081929	35093776
593	35164900	35176761	35188624	35200489	35212356
594	35283600	35295481	35307364	35319249	35331136
595	35402500	35414401	35426304	35438209	35450116

Square Numbers and their Roots.

97

Root	5	6	7	8	9
561	31528225	31539456	31550689	31561924	31573161
562	31640625	31651876	31663129	31674384	31685641
563	31753225	31764496	31775769	31787044	31798321
564	31866025	31877316	31888609	31899904	31911201
565	31979025	31990336	32001649	32012964	32024281
566	32092225	32103556	32114889	32126224	32137561
567	32205625	32216976	32228329	32239684	32251041
568	32319225	32330596	32341969	32353344	32364721
569	32433025	32444416	32455809	32467204	32478601
570	32547025	32558436	32569849	32581264	32592681
571	32661225	32672656	32684089	32695524	32706961
572	32775625	32787076	32798529	32809984	32821441
573	32890225	32901696	32913169	32924644	32936121
574	33005025	33016516	33028009	33039504	33051001
575	33120025	33131536	33143049	33154564	33166081
576	33235225	33246756	33258289	33269824	33281361
577	33350625	33362176	33373729	33385284	33396841
578	33466225	33477796	33489369	33500944	33512521
579	33582025	33593616	33605209	33616804	33628401
580	33698025	33709636	33721249	33732864	33744481
581	33814225	33825856	33837489	33849124	33860761
582	33930625	33942276	33953929	33965584	33977241
583	34047225	34058896	34070569	34082244	34093921
584	34164025	34175716	34187409	34199104	34210801
585	34281025	34292736	34304449	34316164	34327881
586	34398225	34409956	34421689	34433424	34445161
587	34515625	34527376	34539129	34550884	34562641
588	34633225	34644996	34656769	34668544	34680321
589	34751025	34762816	34774609	34786404	34798201
590	34869025	34880836	34892649	34904464	34916281
591	34987225	34999056	35010889	35022724	35034561
592	35105625	35117476	35129329	35141184	35153041
593	35224225	35236096	35247969	35259844	35271721
594	35343025	35354916	35366809	35378704	35390601
595	35462025	35473936	35485849	35497764	35509681

Root	0	1	2	3	4
596	35521600	35533521	35545444	35557369	35569296
597	35640900	35652841	35664784	35676729	35688676
598	35760400	35772361	35784224	35796189	35808156
599	35880100	35892081	35904064	35916049	35928036
600	36000000	36012001	36024004	36036009	36048016
601	36120100	36132121	36144144	36156169	36168196
602	36240400	36252441	36264484	36276529	36288576
603	36360900	36372961	36385024	36397089	36409156
604	36481600	36493681	36505764	36517849	36529936
605	36602500	36614601	36626704	36638809	36650916
606	36723600	36735721	36747844	36759969	36772096
607	36844900	36857041	36869184	36881329	36893476
608	36966400	36978561	36990724	37002889	37015056
609	37088300	37100281	37112464	37124649	37136836
610	37210000	37222201	37234404	37246609	37258816
611	37332100	37344321	37356544	37368769	37380996
612	37454400	37466641	37478884	37491129	37503376
613	37576900	37589161	37601424	37613689	37625956
614	37699600	37711881	37724164	37736449	37748736
615	37822500	37834801	37847104	37859409	37871716
616	37945600	37957921	37970244	37982569	37994896
617	38068900	38081241	38093584	38105929	38118276
618	38192400	38204761	38217124	38229489	38241856
619	38316100	38328481	38340864	38353249	38365636
620	38440000	38452401	38464804	38477209	38489616
621	38564100	38576521	38588944	38601369	38613796
622	38688400	38700841	38713284	38725729	38738176
623	38812900	38825361	38837824	38850289	38862756
624	38937600	38950081	38962564	38975049	38987536
625	39062500	39075001	39087504	39100009	39112516
626	39187600	39200121	39212644	39225169	39237696
627	39312900	39325441	39337984	39350529	39363076
628	39438400	39450961	39463524	39476089	39488656
629	39564100	39576681	39589264	39601849	39614436
630	39690000	39702601	39715204	39727809	39740416

Square Numbers and their Roots.

99

Root	5	6	7	8	9
596	35581225	35593156	35605089	35617024	35628961
597	35700625	35712576	35724529	35736484	35748441
598	35820225	35832196	35844169	35856144	35868121
599	35940025	35952016	35964009	35976004	35988001
600	36060025	36072036	36084049	36096064	36108081
601	36180225	36192256	36204289	36216324	36228361
602	36300625	36312676	36324729	36336784	36348841
603	36421225	36433296	36445369	36457444	36469521
604	36542025	36554116	36566209	36578304	36590401
605	36663025	36675136	36687249	36699364	36711481
606	36784225	36796356	36808489	36820624	36832761
607	36905625	36917776	36929929	36942084	36954241
608	37027225	37039396	37051569	37063744	37075921
609	37149025	37161216	37173409	37185604	37197801
610	37271025	37283236	37295449	37307664	37319881
611	37393225	37405456	37417689	37429924	37442161
612	37515625	37527876	37540129	37552384	37564641
613	37638225	37650496	37662769	37675044	37687321
614	37761025	37773316	37785609	37797904	37810201
615	37884025	37896336	37908649	37920964	37933281
616	38007225	38019556	38031889	38044224	38056561
617	38131625	38144296	38156929	38169684	38182441
618	38254225	38266996	38279869	38292744	38305621
619	38378025	38390416	38402809	38415204	38427601
620	38502025	38514436	38526849	38539264	38551681
621	38626225	38638656	38651089	38663524	38675961
622	38750625	38763076	38775529	38787984	38800441
623	38875225	38887696	38900169	38912644	38925121
624	39000025	39012516	39025009	39037504	39050001
625	39125025	39137536	39150049	39162564	39175081
626	39250225	39262756	39275289	39287824	39300361
627	39375625	39388176	39400729	39413284	39425841
628	39501225	39513796	39526369	39538944	39551521
629	39627025	39639616	39652209	39664804	39677401
630	39753025	39765636	39778249	39790864	39803481

100 Square Numbers and their Roots.

Root	0	1	2	3	4
631	39816100	39828721	39841344	39853969	39866596
632	39942400	39955041	39967684	39980329	39992976
633	40068900	40081561	40094224	40106889	40119556
634	40195600	40208281	40220964	40233649	40246336
635	40322500	40335201	40347904	40360609	40373316
636	40449600	40462321	40475044	40487769	40500496
637	40576900	40589641	40602384	40615129	40627876
638	40704400	40717161	40729924	40742689	40755456
639	40832100	40844881	40857664	40870449	40883236
640	40960000	40972801	40985604	40998409	41011216
641	41088100	41100921	41113744	41126569	41139396
642	41216400	41229241	41242084	41254929	41267776
643	41344900	41357761	41370624	41383489	41396356
644	41473600	41486481	41499364	41512249	41525136
645	41602500	41615401	41628304	41641209	41654116
646	41731600	41744521	41757444	41770369	41783296
647	41860900	41873841	41886784	41899729	41912676
648	41990400	42003361	42016324	42029289	42042256
649	42120100	42133081	42146064	42159049	42172036
650	42250000	42263001	42276004	42289009	42302016
651	42380100	42393121	42406144	42419169	42432196
652	42510400	42523441	42536484	42549529	42562576
653	42640900	42653961	42667024	42680089	42693156
654	42771600	42784681	42797764	42810849	42823936
655	42902500	42915601	42928704	42941809	42954916
656	43033600	43046721	43059844	43072969	43086096
657	43164900	43178041	43191184	43204329	43217476
658	43296400	43309561	43322724	43335889	43349056
659	43428100	43441281	43454464	43467649	43480836
660	43560000	43573201	43586404	43599609	43612816
661	43692100	43705321	43718544	43731769	43744996
662	43824400	43837641	43850884	43864129	43877376
663	43956900	43970161	43983424	43996689	44009956
664	44089600	44102881	44116164	44129449	44142736
665	44222500	44235801	44249104	44262409	44275716

Square Numbers and their Roots.

101

Root	5	6	7	8	9
631	39879225	39891856	39904489	39917124	39929761
632	40005625	40018276	40030929	40043584	40056241
633	40132225	40144896	40157569	40170244	40182921
634	40259025	40271716	40284409	40297104	40309801
635	40386025	40398736	40411449	40424164	40436881
636	40513225	40525956	40538689	40551424	40564161
637	40640625	40653376	40666129	40678884	40691641
638	40768225	40780996	40793769	40806544	40819321
639	40896025	40908816	40921609	40934404	40947201
640	41024025	41036836	41049649	41062464	41075281
641	41152225	41165056	41177889	41190724	41203561
642	41280625	41293476	41306329	41319184	41332041
643	41409225	41422096	41434969	41447844	41460721
644	41538025	41550916	41563809	41576704	41589601
645	41667025	41679936	41692849	41705764	41718681
646	41796225	41809156	41822089	41835024	41847961
647	41925625	41938576	41951529	41964484	41977441
648	42055225	42068196	42081169	42094144	42107121
649	42185025	42198016	42211009	42224004	42237001
650	42315025	42328036	42341049	42354064	42367081
651	42445225	42458256	42471289	42484324	42497361
652	42575625	42588676	42601729	42614784	42627841
653	42706225	42719296	42732369	42745444	42758521
654	42837025	42850116	42863209	42876304	42889401
655	42968025	42981136	42994249	43007364	43020481
656	43099225	43112356	43125489	43138624	43151761
657	43230625	43243776	43256929	43270084	43283241
658	43362225	43375396	43388569	43401744	43414921
659	43494025	43507216	43520409	43533604	43546801
660	43626025	43639236	43652449	43665664	43678881
661	43758225	43771456	43784689	43797924	43811161
662	43890625	43903876	43917129	43930384	43943641
663	44023225	44036496	44049769	44063044	44076321
664	44156025	44169316	44182609	44195904	44209601
665	44289025	44302336	44315649	44328964	44342281

Root	0	1	2	3	4
666	44355600	44368921	44382244	44395569	44408896
667	44488900	44502241	44515584	44528929	44542276
668	44622400	44635761	44649124	44662489	44675856
669	44756100	44769481	44782864	44796249	44809636
670	44890000	44903401	44916804	44930209	44943616
671	45024100	45037521	45050944	45064369	45077796
672	45158400	45171841	45185284	45198729	45212176
673	45292900	45306361	45319824	45333289	45346756
674	45427600	45441081	45454564	45468049	45481536
675	45562500	45576001	45589504	45603009	45616516
676	45697600	45711121	45724644	45738169	45751696
677	45832900	45846441	45859984	45873529	45887076
678	45968400	45981961	45995524	46009089	46022656
679	46104100	46117681	46131264	46144849	46158436
680	46240000	46253601	46267204	46280809	46294416
681	46376100	46389721	46403344	46416969	46430596
682	46512400	46526041	46539684	46553329	46566976
683	46648900	46662561	46676224	46689889	46703556
684	46785600	46799281	46812964	46826649	46840336
685	46922500	46936201	46949904	46963609	46977316
686	47059600	47073321	47087044	47100769	47114496
687	47196900	47210641	47224384	47238129	47251876
688	47334400	47348161	47361924	47375689	47389456
689	47472100	47485881	47499664	47513449	47527236
690	47610000	47623801	47637604	47651409	47665216
691	47748100	47761921	47775744	47789569	47803396
692	47886400	47900241	47914084	47927929	47941776
693	48024900	48038761	48052624	48066489	48080356
694	48163600	48177481	48191364	48205249	48219136
695	48302500	48316401	48330304	48344209	48358116
696	48441600	48455521	48469444	48483369	48497296
697	48580900	48594841	48608784	48622729	48636676
698	48720400	48734361	48748324	48762289	48776256
699	48860100	48874081	48888064	48902049	48916036
700	49000000	49014001	49028004	49042009	49056016

Square Numbers and their Roots.

103

Root	5	6	7	8	9
666	4442225	44435556	44448889	44462224	44475561
667	44555625	44568976	44582329	44595684	44609041
668	44689225	44702596	44715969	44729344	44742721
669	44823025	44836416	44849809	44863204	44876601
670	44957025	44970436	44983849	44997264	45010681
671	45091225	45104656	45118089	45131524	45144961
672	45225625	45239076	45252529	45265984	45279441
673	45360225	45373696	45387169	45400644	45414121
674	45495025	45508516	45522009	45535504	45549001
675	45630025	45643536	45657049	45670564	45684081
676	45765225	45778756	45792289	45805824	45819361
677	45900625	45914176	45927729	45941284	45954841
678	46036225	46049796	46063369	46076944	46090521
679	46172025	46185616	46199209	46212804	46226401
680	46308025	46321636	46335249	46348864	46362481
681	46444225	46457856	46471489	46485124	46498761
682	46580625	46594276	46607929	46621584	46635241
683	46717225	46730896	46744569	46758244	46771921
684	46854025	46867716	46881409	46895104	46908801
685	46991025	47004736	47018449	47032164	47045881
686	47128225	47141956	47155689	47169424	47183161
687	47265625	47279376	47293129	47306884	47320641
688	47403225	47416996	47430769	47444544	47458321
689	47541025	47554816	47568609	47582404	47596201
690	47679025	47692836	47706649	47720464	47734281
691	47817225	47831056	47844889	47858724	47872561
692	47955625	47969476	47983329	47997184	48011041
693	48094225	48108096	48121969	48135844	48149721
694	48233025	48246916	48260809	48274704	48288601
695	48372025	48385936	48399849	48413764	48427681
696	48511225	48525156	48539089	48553024	48566961
697	48650625	48664576	48678529	48692484	48706441
698	48790225	48804196	48818169	48832144	48846121
699	48930025	48944016	48958009	48972004	48986001
700	49070025	49084036	49098049	49112064	49126081

194 *Square Numbers and their Roots.*

Root	0	1	2	3	4
701	49140100	49154121	49168144	49182169	49196196
702	49280400	49294441	49308484	49322529	49336576
703	49420900	49434961	49449024	49463089	49477156
704	49561600	49575681	49589764	49603849	49617936
705	49702500	49716601	49730704	49744809	49758916
706	49843600	49857721	49871844	49885969	49900096
707	49984900	49999041	50013184	50027329	50041476
708	50126400	50140561	50154724	50168889	50183056
709	50268100	50282281	50296464	50310649	50324836
710	50410000	50424201	50438404	50452609	50466816
711	50552100	50566321	50580544	50594769	50608996
712	50694400	50708641	50722884	50737129	50751376
713	50836900	50851161	50865424	50879689	50893956
714	50979600	50993881	51008164	51022449	51036736
715	51122500	51136801	51151104	51165409	51179716
716	51265600	51279921	51294244	51308569	51322896
717	51408900	51423241	51437584	51451929	51466276
718	51552400	51566761	51581124	51595489	51609856
719	51696100	51710481	51724864	51739249	51753636
720	51840000	51854401	51868804	51883209	51897616
721	51984100	51998521	52012944	52027369	52041796
722	52128400	52142841	52157284	52171729	52186176
723	52272900	52287361	52301824	52316289	52330756
724	52417600	52432081	52446564	52461049	52475536
725	52562500	52577001	52591504	52606009	52620516
726	52707600	52722121	52736644	52751169	52765696
727	52852900	52867441	52881984	52896529	52911076
728	52998400	53012961	53027524	53042089	53056656
729	53144100	53158681	53173264	53187849	53202436
730	53290000	53304601	53319204	53333809	53348416
731	53436100	53450721	53465344	53479969	53494596
732	53582400	53597041	53611684	53626329	53640976
733	53728900	53743561	53758224	53772889	53787556
734	53875600	53890281	53904964	53919649	53934336
735	54022500	54037201	54051904	54066609	54081316

Square Numbers and their Roots.

105

Root	5	6	7	8	9
701	49210225	49214256	49238289	49252324	49266361
702	49350625	49364676	49378729	49392784	49406841
703	49491225	49505296	49519369	49533444	49547521
704	49632025	49646116	49660209	49674304	49688401
705	49773025	49787136	49801249	49815364	49829481
706	49914225	49928356	49942489	49956624	49970761
707	50055625	50069776	50083929	50098084	50112241
708	50197225	50211396	50225569	50239744	50253921
709	50339025	50353216	50367409	50381604	50395801
710	50481025	50495236	50509449	50523664	50537881
711	50623225	50637456	50651689	50665924	50680161
712	50765625	50779876	50794129	50808384	50822641
713	50908225	50922496	50936769	50951044	50965321
714	51051025	51065316	51079609	51093904	51108201
715	51194025	51208336	51222649	51236964	51251281
716	51337225	51351556	51365889	51380224	51394561
717	51480625	51494976	51509329	51523684	51538041
718	51624225	51638596	51652969	51667344	51681721
719	51768025	51782416	51796809	51811204	51825601
720	51912025	51926436	51940849	51955264	51969681
721	52056225	52070656	52085089	52099524	52113961
722	52200625	52215076	52229529	52243984	52258441
723	52345225	52359696	52374169	52388644	52403121
724	52490025	52504516	52519009	52533504	52548001
725	52635025	52649536	52664049	52678564	52693081
726	52780225	52794756	52809289	52823824	52838361
727	52925625	52940176	52954729	52969284	52983841
728	53071225	53085796	53100369	53114944	53129521
729	53217025	53231616	53246209	53260804	53275401
730	53363025	53377636	53392249	53406864	53421481
731	53509225	53523856	53538489	53553124	53567761
732	53655625	53670276	53684929	53699584	53714421
733	53802225	53816896	53831569	53846244	53860921
734	53949025	53963716	53978409	53993104	54007801
735	54096025	54110736	54125449	54140164	54154881

106 Square Numbers and their Roots.

Root	0	1	2	3	4
736	54169600	54184321	54199044	54213769	54228496
737	54316900	54331641	54346384	54361129	54375876
738	54464400	54479161	54493924	54508689	54523456
739	54612100	54626881	54641664	54656449	54671236
740	54760000	54774801	54789604	54804409	54819216
741	54908100	54922921	54937744	54952569	54967396
742	55056400	55071241	55086084	55100929	55115776
743	55204900	55219761	55234624	55249489	55264356
744	55353600	55368481	55383364	55398249	55413136
745	55502500	55517401	55532304	55547209	55562116
746	55651600	55666521	55681444	55696369	55711296
747	55800900	55815841	55830784	55845729	55860676
748	55950400	55965361	55980324	55995289	56010256
749	56100100	56115081	56130064	56145049	56160036
750	56250000	56265001	56280004	56295009	56310016
751	56400100	56415121	56430144	56445169	56460196
752	56550400	56565441	56580484	56595529	56610576
753	56700900	56715961	56731024	56746089	56761156
754	56851600	56866681	56881764	56896849	56911936
755	57002500	57017601	57032704	57047809	57062916
756	57153600	57168721	57183844	57198969	57214096
757	57304900	57320041	57335184	57350329	57365476
758	57456400	57471561	57486724	57501889	57517056
759	57608100	57623281	57638464	57653649	57668836
760	57760000	57775201	57790404	57805609	57820816
761	57912100	57927321	57942544	57957769	57972996
762	58064400	58079641	58094884	58110129	58125376
763	58216900	58232161	58247424	58262689	58277956
764	58369600	58384881	58400164	58415449	58430736
765	58522500	58537801	58553104	58568409	58583716
766	58675600	58690921	58706244	58721569	58736896
767	58828900	58844241	58859584	58874929	58890276
768	58982400	58997761	59013124	59028489	59043856
769	59136100	59151481	59166864	59182249	59197636
770	59290000	59305401	59320804	59336209	59351616

Square Numbers and their Roots.

107

Root	5	6	7	8	9
736	54243225	54257956	54272689	54287424	54302161
737	54390625	54405376	54420129	54434884	54449641
738	54538225	54552996	54567769	54582544	54597321
739	54686025	54700816	54715609	54730404	54745201
740	54834025	54848836	54863649	54878464	54893281
741	54982225	54997056	55011889	55026724	55041561
742	55130625	55145476	55160329	55175184	55190041
743	55279225	55294096	55308969	55323844	55338721
744	55428025	55442916	55457809	55472704	55487601
745	55577025	55591936	55606849	55621764	55636681
746	55726225	55741156	55756089	55771024	55785961
747	55875625	55890576	55905529	55920484	55935441
748	56025225	56040196	56055169	56070144	56085121
749	56175025	56190016	56205009	56220004	56235001
750	56325025	56340036	56355049	56370064	56385081
751	56475225	56490256	56505289	56520324	56535361
752	56625625	56640676	56655729	56670784	56685841
753	56776225	56791256	56806369	56821444	56836521
754	56927025	56942116	56957209	56972304	56987401
755	57078025	57093136	57108249	57123364	57138481
756	57229225	57244356	57259489	57274624	57289761
757	57380625	57395776	57410929	57426084	57441241
758	57532225	57547396	57562569	57577744	57592921
759	57684025	57699216	57714409	57729604	57744801
760	57836025	57851236	57866449	57881664	57896881
761	57988225	58003456	58018689	58033924	58049161
762	58140625	58155876	58171129	58186384	58201641
763	58293225	58308496	58323769	58339044	58354321
764	58446025	58461316	58476609	58491904	58507201
765	58599025	58614336	58629649	58644964	58660281
766	58752225	58767556	58782889	58798224	58813561
767	58905625	58920976	58936329	58951684	58967041
768	59059225	59074596	59089969	59105344	59120721
769	59213025	59228416	59243809	59259204	59274601
770	59367025	59382436	59397849	59413264	59428681

108 Square Numbers and their Roots.

Root	0	1	2	3	4
771	59444100	59459521	59474944	59490369	59505796
772	59598400	59613841	59629284	59644729	59660176
773	59752900	59768361	59783824	59799289	59814756
774	59907600	59923081	59938564	59954049	59969536
775	60062500	60078001	60093504	60109009	60124516
776	60217600	60233121	60248644	60264169	60279696
777	60372900	60388441	60403984	60419529	60435076
778	60528400	60543961	60559524	60575089	60590656
779	60684100	60699681	60715264	60730849	60746436
780	60840000	60855601	60871204	60886809	60902416
781	60996100	61011721	61027344	61042969	61058596
782	61152400	61168041	61183684	61199329	61214976
783	61308900	61324561	61340224	61355889	61371556
784	61465600	61481281	61496964	61512649	61528336
785	61622500	61638201	61653904	61669609	61685316
786	61779600	61795321	61811044	61826769	61842496
787	61936900	61952641	61968384	61984129	61999876
788	62094400	62110161	62125924	62141689	62157456
789	62252100	62267881	62283664	62299449	62315236
790	62410000	62425801	62441604	62457409	62473216
791	62568100	62583921	62599744	62615569	62631396
792	62726400	62742241	62758084	62773929	62789776
793	62884900	62900761	62916624	62932489	62948356
794	63043600	63059481	63075364	63091249	63107136
795	63202500	63218401	63234304	63250209	63266116
796	63361600	63377521	63393444	63409369	63425296
797	63520900	63536841	63552784	63568729	63584676
798	63680400	63696361	63712324	63728289	63744256
799	63840100	63856081	63872064	63888049	63904036
800	64000000	64016001	64032004	64048009	64064016
801	64160100	64176121	64192144	64208169	64224196
802	64320400	64336441	64352484	64368529	64384576
803	64480900	64496961	64513024	64529089	64545156
804	64641600	64657681	64673764	64689849	64705936
805	64802500	64818601	64834704	64850809	64866916

Square Numbers and their Roots. 109

Root	5	6	7	8	9
771	59521225	59536656	59552089	59567524	59582961
772	59675625	59691076	59706529	59721984	59737441
773	59830225	59845696	59861169	59876644	59892121
774	59985025	60000516	60016009	60031504	60047001
775	60140025	60155536	60171049	60186564	60202081
776	60295225	60310756	60326289	60341824	60357361
777	60450625	60466176	60481729	60497284	60512841
778	60606225	60621796	60637369	60652944	60668521
779	60762025	60777616	60793209	60808804	60824401
780	60918025	60933636	60949249	60964864	60980481
781	61074225	61089856	61105489	61121124	61136761
782	61230625	61246276	61261929	61277584	61293241
783	61387225	61402896	61418569	61434244	61449921
784	61544025	61559716	61575409	61591104	61606801
785	61701025	61716736	61732449	61748164	61763881
786	61858225	61873956	61889689	61905424	61921161
787	62015625	62031376	62047129	62062884	62078641
788	62173225	62188996	62204769	62220544	62236321
789	62331025	62346816	62362609	62378404	62394201
790	62489025	62504836	62520649	62536464	62552281
791	62647225	62663056	62678889	62694724	62710561
792	62805625	62821476	62837329	62853184	62869041
793	62964225	62980096	62995969	63011844	63027721
794	63123025	63138916	63154809	63170704	63186601
795	63282025	63297936	63313849	63329764	63345681
796	63441225	63457156	63473089	63489024	63504961
797	63600625	63616576	63632529	63648484	63664441
798	63760225	63776196	63792169	63808144	63824121
799	63920025	63936016	63952009	63968004	63984001
800	64080025	64096036	64112049	64128064	64144081
801	64240225	64256256	64272289	64288324	64304361
802	64400625	64416676	64432729	64448784	64464841
803	64561225	64577296	64593369	64609444	64625521
804	64722025	64738116	64754209	64770304	64786401
805	64883025	64899136	64915249	64931364	64947481

Root	0	1	2	3	4
806	64963600	64979721	64995844	65011969	65028096
807	65124900	65141041	65157184	65173329	65189476
808	65286400	65302561	65318724	65334889	65351056
809	65448100	65464281	65480464	65496649	65512836
810	65610000	65626201	65642404	65658609	65674816
811	65772100	65788321	65804544	65820769	65836996
812	65934400	65950641	65966884	65983129	65999376
813	66096900	66113161	66129424	66145689	66161956
814	66259600	66275881	66292164	66308449	66324736
815	66422500	66438801	66455104	66471409	66487716
816	66585600	66601921	66618244	66634569	66650896
817	66748900	66765241	66781584	66797929	66814276
818	66912400	66928761	66945124	66961489	66977856
819	67076100	67092481	67108864	67125249	67141636
820	67240000	67256401	67272804	67289209	67305616
821	67404100	67420521	67436944	67453369	67469796
822	67568400	67584841	67601284	67617729	67634176
823	67732900	67749361	67765824	67782289	67798756
824	67897600	67914081	67930564	67947049	67963536
825	68062500	68079001	68095504	68112009	68128516
826	68227600	68244121	68260644	68277169	68293696
827	68392900	68409441	68425984	68442529	68459076
828	68558400	68574961	68591524	68608089	68624656
829	68724100	68740681	68757264	68773849	68790436
830	68890000	68906601	68923204	68939809	68956416
831	69056100	69072721	69089344	69105969	69122596
832	69222400	69239041	69255684	69272329	69288976
833	69388900	69405561	69422224	69438889	69455556
834	69555600	69572281	69588964	69605649	69622336
835	69722500	69739201	69755904	69772609	69789316
836	69889600	69906321	69923044	69939769	69956496
837	70056900	70073641	70090384	70107129	70123876
838	70224400	70241161	70257924	70274689	70291456
839	70392100	70408881	70425664	70442449	70459236
840	70560000	70576801	70593604	70610409	70627216

Square Numbers and their Roots.

111

Root	5	6	7	8	9
806	65044225	65060356	65076489	65092624	65108761
807	65205625	65221776	65237929	65254084	65270241
808	65367225	65383396	65399569	65415744	65431921
809	65529025	65545216	65561409	65577604	65593801
810	65691025	65707236	65723449	65739664	65755881
811	65853225	65869456	65885689	65901924	65918161
812	66015625	66031876	66048129	66064384	66080641
813	66178225	66194496	66210769	66227044	66243321
814	66341025	66357316	66373609	66389904	66406201
815	66504025	66520336	66536649	66552964	66569281
816	66667225	66683556	66699889	66716224	66732561
817	66830625	66846976	66863329	66879684	66896041
818	66994225	67010596	67026969	67043344	67059721
819	67158025	67174416	67190809	67207204	67223601
820	67322025	67338436	67354849	67371264	67387681
821	67486225	67502656	67519089	67535524	67551961
822	67650625	67667076	67683529	67699984	67716441
823	67815225	67831696	67848169	67864644	67881121
824	67980025	67996516	68013009	68029504	68046001
825	68145025	68161536	68178049	68194564	68211081
826	68310225	68326756	68343289	68359824	68376361
827	68475625	68492176	68508729	68525284	68541841
828	68641225	68657796	68674369	68690944	68707521
829	68807025	68823616	68840209	68856804	68873401
830	68973025	68989636	69006249	69022864	69039481
831	69139225	69155856	69172489	69189124	69205761
832	69305625	69322276	69338929	69355584	69372241
833	69472225	69488896	69505569	69522244	69538921
834	69639025	69655716	69672409	69689104	69705801
835	69806025	69822736	69839449	69856164	69872881
836	69973225	69989956	70006689	70023424	70040161
837	70140625	70157376	70174129	70190884	70207641
838	70308225	70324996	70341769	70358544	70375321
839	70476025	70492816	70509609	70526404	70543201
840	70644025	70660836	70677649	70694464	70711281

110

S.

Rox

o

806 649630

807 651240

808 652804

809 654481

810 656110

811 657721

812 659341

813 660951

814 662561

815 664171

816 665781

817 667401

818 669011

819 670621

820 672231

821 673841

822 675451

823 677061

824 678671

825 680281

826 681891

827 683501

828 685111

829 686721

830 688331

831 689941

832 691551

833 693161

834 694771

835 696381

836 697991

837 699601

838 701211

839 702821

840 704431

841 706041

842 707651

843 709261

844 710871

845 712481

846 714091

847 715701

848 717311

849 718921

850 720531

Square Numbers and their Roots.

113

Root	5	6	7	8	9
841	70813225	70829056	70845889	70862724	70879561
842	70989625	70997476	71014329	71031184	71048041
843	71149225	71166096	71182969	71199844	71216721
844	71318025	71334916	71351809	71368704	71385601
845	71487025	71503936	71520849	71537764	71554681
846	71656225	71673156	71690089	71707024	71723961
847	71825625	71842576	71859529	71876484	71893441
848	71995225	72012196	72029169	72046144	72063121
849	72165025	72182016	72199009	72216004	72233001
850	72335025	72352036	72369049	72386064	72403081
851	72505225	72522256	72539289	72556324	72573361
852	72675625	72692676	72709729	72726784	72743841
853	72846225	72863296	72880369	72897444	72914521
854	73017025	73034116	73051209	73068304	73085401
855	73188025	73205136	73222249	73239364	73256481
856	73359225	73376356	73393489	73410624	73427761
857	73530625	73547776	73564929	73582084	73599241
858	73702225	73719396	73736569	73753744	73770921
859	73874025	73891216	73908409	73925604	73942801
860	74046025	74063236	74080449	74097664	74114881
861	74218225	74235456	74252689	74269924	74287161
862	74390625	74407876	74425129	74442384	74459641
863	74563225	74580496	74597769	74615044	74632321
864	74736025	74753316	74770609	74787904	74805201
865	74909025	74926336	74943649	74960964	74978281
866	75082225	75099556	75116889	75134224	75151561
867	75252225	75272976	75290329	75307684	75325041
868	75425625	75446596	75463969	75481344	75498721
869	75600025	75620416	75637809	75655204	75672601
870	75777025	75794436	75811849	75829264	75846681
871	75951225	75968656	75986089	76003524	76020961
872	76125625	76143076	76160529	76177984	76195441
873	76300225	76317696	76335169	76352644	76370121
874	76475025	76492516	76510009	76527504	76545001
875	76650025	76667536	76685049	76702564	76720081

Root	0	1	2	3	4
841	70728100	70744921	70761744	70778569	70795396
842	70896400	70913241	70930084	70946929	70963776
843	71064900	71081761	71098624	71115489	71132356
844	71233600	71250481	71267364	71284249	71301136
845	71402500	71419401	71436304	71453209	71470116
846	71571600	71588521	71605444	71622369	71639296
847	71740900	71757841	71774784	71791729	71808676
848	71910400	71927361	71944324	71961289	71978256
849	72080100	72097081	72114064	72131049	72148036
850	72250000	72267001	72284004	72301009	72318016
851	72420100	72437121	72454144	72471169	72488196
852	72590400	72607441	72624484	72641529	72658576
853	72760900	72777961	72795024	72812089	72829156
854	72931600	72948681	72965764	72982849	72999936
855	73102500	73119601	73136704	73153809	73170916
856	73273600	73290721	73307844	73324969	73342096
857	73444900	73462041	73479184	73496329	73513476
858	73616400	73633561	73650724	73667889	73685056
859	73788100	73805281	73822464	73839649	73856836
860	73960000	73977201	73994404	74011609	74028816
861	74132100	74149321	74166544	74183769	74200996
862	74304400	74321641	74338884	74356129	74373376
863	74476900	74494161	74511424	74528689	74545956
864	74649600	74666881	74684164	74701449	74718736
865	74822500	74839801	74857194	74874409	74891716
866	74995600	75012921	75030244	75047569	75064896
867	75168900	75186141	75203584	75220929	75238276
868	75342400	75359761	75377124	75394489	75411856
869	75516100	75533481	75550864	75568249	75585636
870	75690000	75707401	75724804	75742209	75759616
871	75864100	75881521	75898944	75916369	75933796
872	76038400	76055841	76073284	76090729	76108176
873	76212900	76230361	76247824	76265289	76282756
874	76387600	76405081	76422564	76440049	76457536
875	76562500	76580001	76597504	76615009	76632516

Square Numbers and their Roots.

113

Root	5	6	7	8	9
841	70813225	70829056	70845889	70862724	70879561
842	70984025	70997476	71014329	71031184	71048041
843	71149225	71166096	71182969	71199844	71216721
844	71318025	71334916	71351809	71368704	71385601
845	71487025	71503936	71520849	71537764	71554681
846	71656225	71673156	71690089	71707024	71723961
847	71825625	71842576	71859529	71876484	71893441
848	71995225	72012196	72029169	72046144	72063121
849	72165025	72182016	72199009	72216004	72233001
850	72335025	72352036	72369049	72386064	72403081
851	72505225	72522256	72539289	72556324	72573361
852	72675625	72692676	72709729	72726784	72743841
853	72846225	72863296	72880369	72897444	72914521
854	73017025	73034116	73051209	73068304	73085401
855	73188025	73205136	73222249	73239364	73256481
856	73359225	73376356	73393489	73410624	73427761
857	73530625	73547776	73564929	73582084	73599241
858	73702225	73719396	73736569	73753744	73770921
859	73874025	73891216	73908409	73925604	73942801
860	74046025	74063236	74080449	74097664	74114881
861	74218225	74235456	74252689	74269924	74287161
862	74390625	74407876	74425129	74442384	74459641
863	74563225	74580496	74597769	74615044	74632321
864	74736025	74753316	74770609	74787904	74805201
865	74909025	74926336	74943649	74960964	74978281
866	75082225	75099556	75116889	75134224	75151561
867	75255225	75272576	75290029	75307684	75325041
868	75428625	75446096	75463569	75481344	75498721
869	75603025	75620416	75637809	75655204	75672601
870	75777025	75794436	75811849	75829264	75846681
871	75951225	75968656	75986089	76003524	76020961
872	76125625	76143076	76160529	76177984	76195441
873	76300225	76317696	76335169	76352644	76370121
874	76475025	76492516	76510009	76527504	76545001
875	76650025	76667536	76685049	76702564	76720081

114 Square Numbers and their Roots.

Root	0	1	2	3	4
876	76737600	76755121	76772644	76790169	76807696
877	76912900	76930441	76947984	76965529	76983076
878	77088400	77105961	77123524	77141089	77158656
879	77264100	77281681	77299264	77316849	77334436
880	77440000	77457601	77475204	77492809	77510416
881	77616100	77633721	77651344	77668969	77686596
882	77792400	77810041	77827684	77845329	77862976
883	77968900	77986561	78004224	78021889	78039556
884	78145600	78163281	78180964	78198649	78216336
885	78322500	78340201	78357904	78375609	78393316
886	78499600	78517321	78535044	78552769	78570496
887	78676900	78694641	78712384	78730129	78747876
888	78854400	78872161	78889924	78907689	78925456
889	79032100	79049881	79067664	79085449	79103236
890	79210000	79227801	79245604	79263409	79281216
891	79388100	79405921	79423744	79441569	79459396
892	79566400	79584241	79602084	79619929	79637776
893	79744900	79762761	79780624	79798489	79816356
894	79923600	79941481	79959364	79977249	79995136
895	80102500	80120401	80138304	80156209	80174116
896	80281600	80299521	80317444	80335369	80353296
897	80460900	80478841	80496784	80514729	80532676
898	80640400	80658361	80676324	80694289	80712256
899	80820100	80838081	80856064	80874049	80892036
900	81000000	81018001	81036004	81054009	81072016
901	81180100	81198121	81216144	81234169	81252196
902	81360400	81378441	81396484	81414529	81432576
903	81540900	81558961	81577024	81595089	81613156
904	81721600	81739681	81757764	81775849	81793936
905	81902500	81920601	81938704	81956809	81974916
906	82083600	82101721	82119844	82137969	82156096
907	82264900	82283041	82301184	82319329	82337476
908	82446400	82464561	82482724	82500889	82519056
909	82628100	82646281	82664464	82682649	82700836
910	82810000	82828201	82846404	82864609	82882816

Square Numbers and their Roots.

115

Root.	5	6	7	8	9
876	76825225	78842756	76860289	76877824	76895361
877	77000625	77018176	77035729	77053284	77070841
878	77174225	77193796	77211869	77228944	77246521
879	77354025	77369616	77387209	77404804	77422401
880	77528025	77545636	77563249	77580864	77598481
881	77704225	77721856	77739489	77757124	77774761
882	77880625	77898276	77915929	77933584	77951241
883	78057225	78074896	78092569	78110244	78127921
884	78234025	78251716	78269409	78287104	78304801
885	78411025	78428736	78446449	78464164	78481881
886	78588225	78605956	78623689	78641424	78659161
887	78765625	78783376	78801129	78818884	78836641
888	78943225	78960996	78978769	78996544	79014321
889	79121025	79138816	79156609	79174404	79192201
890	79299025	79316836	79334649	79352464	79370281
891	79477225	79495056	79512889	79530724	79548561
892	79655625	79673476	79691329	79709184	79727041
893	79834225	79852096	79869969	79887844	79905721
894	80013025	80030916	80048809	80066704	80084601
895	80192025	80209936	80227849	80245764	80263681
896	80371225	80389156	80407089	80425024	80442961
897	80550625	80568576	80586529	80604484	80622441
898	80730225	80748196	80766169	80784144	80802121
899	80910025	80928016	80946009	80964004	80982001
900	81090025	81108036	81126049	81144064	81162081
901	81270225	81288256	81306289	81324324	81342361
902	81450625	81468676	81486729	81504784	81522841
903	81631225	81649296	81667369	81685444	81703521
904	81812025	81830116	81848209	81866304	81884401
905	81993025	82011136	82029249	82047364	82065481
906	82174225	82192356	82210489	82228624	82246761
907	82355625	82373776	82391929	82410084	82428241
908	82537225	82555396	82573569	82591744	82609921
909	82719025	82737216	82755409	82773604	82791801
910	82901025	82919236	82937449	82955664	82973881

116 Square Numbers and their Roots.

Root	0	1	2	3	4
911	82992100	83010321	83028544	83046769	83064996
912	83174400	83192641	83210884	83229129	83247376
913	83356900	83375161	83393424	83411689	83429956
914	83539600	83557881	83576164	83594449	83612736
915	83722500	83740801	83759104	83777409	83795716
916	83905600	83923921	83942244	83960569	83978896
917	84088900	84107241	84125584	84143929	84162276
918	84272400	84290761	84309124	84327489	84345856
919	84456100	84474481	84492864	84511249	84529636
920	84640000	84658401	84676804	84695209	84713616
921	84824100	84842521	84860944	84879369	84897796
922	85008400	85026841	85045284	85063729	85082176
923	85192900	85211361	85229824	85248289	85266756
924	85377600	85396081	85414564	85433049	85451536
925	85562500	85581001	85599504	85618009	85636516
926	85747600	85766121	85784644	85803169	85821696
927	85932900	85951441	85969984	85988529	86007076
928	86118400	86136961	86155524	86174089	86192656
929	86304100	86322681	86341264	86359849	86378436
930	86490000	86508601	86527204	86545809	86564416
931	86676100	86694721	86713344	86731969	86750596
932	86862400	86881041	86899684	86918329	86936976
933	87048900	87067561	87086224	87104889	87123556
934	87235600	87254281	87272964	87291649	87310336
935	87422500	87441201	87459904	87478609	87497316
936	87609600	87628321	87647044	87665769	87684496
937	87796900	87815641	87834384	87853129	87871876
938	87984400	88003161	88021924	88040689	88059456
939	88172100	88190881	88209664	88228449	88247236
940	88360000	88378801	88397604	88416409	88435216
941	88548100	88566921	88585744	88604569	88623396
942	88736400	88755241	88774084	88792929	88811776
943	88924900	88943761	88962624	88981489	88990036
944	89113600	89132481	89151364	89170249	89189136
945	89302500	89321401	89340304	89359209	89378116

Square Numbers and their Roots.

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Root	5	6	7	8	9
911	83083225	83101456	83119689	83137924	83156161
912	83261625	83283876	83302129	83320384	83338641
913	83440225	83466496	83484769	83503044	83521321
914	83631025	83649316	83667609	83685904	83704201
915	83811025	83832336	83850649	83868964	83887281
916	83997225	84015556	84033889	84052224	84070561
917	84180625	84198976	84217329	84235684	84254041
918	84364225	84382596	84400969	84419344	84437721
919	84548025	84566416	84584809	84603204	84621601
920	84732025	84750436	84768849	84787264	84805681
921	84916225	84934656	84953089	84971524	84989961
922	85100625	85119076	85137529	85155984	85174441
923	85285225	85303696	85322169	85340644	85359121
924	85470025	85488516	85507009	85525504	85544001
925	85655025	85673536	85692049	85710564	85729081
926	85840225	85858756	85877289	85895824	85914361
927	86026225	86044796	86063329	86081884	86099941
928	86211225	86229796	86248369	86266944	86285521
929	86397025	86415616	86434209	86452804	86471401
930	86583025	86601636	86620249	86638864	86657481
931	86769225	86787856	86806489	86825124	86843761
932	86955625	86974276	86992929	87011584	87030241
933	87142225	87160896	87179569	87198244	87216921
934	87329025	87347716	87366409	87385104	87403801
935	87516025	87534736	87553449	87572164	87590881
936	87703225	87721956	87740689	87759424	87778161
937	87890625	87909376	87928129	87946884	87965641
938	88078225	88096996	88115769	88134544	88153321
939	88266025	88284816	88303609	88322404	88341201
940	88454025	88472836	88491649	88510464	88529281
941	88642225	88661056	88679889	88698724	88717561
942	88830625	88849476	88868329	88887184	88906041
943	89019225	89038096	89056969	89075844	89094721
944	89208025	89226916	89245809	89264704	89283601
945	89397025	89415936	89434849	89453764	89472681

118 Square Numbers and their Roots.

Root	0	1	2	3	4
946	89491600	89519521	89529444	89548369	89567296
947	89680900	89699841	89718784	89737729	89756676
948	89870400	89889361	89908324	89927289	89946256
949	90060100	90079081	90098064	90117049	90136036
950	90250000	90269001	90288004	90307009	90326016
951	90440100	90459121	90478144	90497169	90516196
952	90630400	90649441	90668484	90687529	90706576
953	90820900	90839961	90859024	90878089	90897156
954	91011600	91030681	91049764	91068849	91087936
955	91202500	91221601	91240704	91259809	91278916
956	91393600	91412721	91431844	91450969	91470096
957	91584900	91604041	91623184	91642329	91661476
958	91776400	91795561	91814724	91833889	91853056
959	91968100	91987281	92006464	92025649	92044836
960	92160000	92179201	92198404	92217609	92236816
961	92352100	92371321	92390544	92409769	92428996
962	92544400	92563641	92582884	92602129	92621376
963	92736900	92756161	92775424	92794689	92813956
964	92929600	92948881	92968164	92987449	93006736
965	93122500	93141801	93161104	93180409	93199716
966	93315600	93334921	93354244	93373569	93392896
967	93508900	93528241	93547584	93566929	93586276
968	93702400	93721761	93741124	93760489	93779856
969	93896100	93915481	93934864	93954249	93973636
970	94090000	94109401	94128804	94148209	94167616
971	94284100	94303521	94322944	94342369	94361796
972	94478400	94497841	94517284	94536729	94556176
973	94672900	94692361	94711824	94731289	94750756
974	94867600	94887081	94906564	94926049	94945536
975	95062500	95082001	95101504	95121009	95140516
976	95257600	95277121	95296644	95316169	95335696
977	95452900	95472441	95491984	95511529	95531076
978	95648400	95667961	95687524	95707089	95726656
979	95844100	95863681	95883264	95902849	95922436
980	96040000	96059601	96079204	96098809	96118416

Square Numbers and their Roots. 119

Root	5	6	7	8	9
946	89586225	89605156	89624089	89643024	89661961
947	89775625	89794576	89813529	89832484	89851441
948	89965225	89984196	90003169	90022144	90041121
949	90155025	90174016	90193009	90212004	90231001
950	90345025	90364036	90383049	90402064	90421081
951	90535225	90554256	90573289	90592324	90611361
952	90725625	90744676	90763729	90782784	90801841
953	90916225	90935296	90954369	90973444	90992521
954	91107025	91126116	91145209	91164304	91183401
955	91298025	91317136	91336249	91355364	91374481
956	91489225	91508356	91527489	91546624	91565761
957	91680625	91699776	91718929	91738084	91757241
958	91872225	91891396	91910569	91929744	91948921
959	92064025	92083216	92102409	92121604	92140801
960	92256025	92275236	92294449	92313664	92332881
961	92448225	92467456	92486689	92505924	92525161
962	92640625	92659876	92679129	92698384	92717641
963	92833225	92852496	92871769	92891044	92910321
964	93026025	93045316	93064609	93083904	93103201
965	93219025	93238336	93257649	93276964	93296281
966	93412225	93431556	93450889	93470224	93489561
967	93605625	93624976	93644329	93663684	93683041
968	93799225	93818596	93837969	93857344	93876721
969	93993025	94012416	94031809	94051204	94070601
970	94187025	94206436	94225849	94245264	94264681
971	94381225	94400656	94420089	94439524	94458961
972	94575625	94595076	94614529	94633984	94653441
973	94770225	94789696	94809169	94828644	94848121
974	94965025	94984516	95004009	95023504	95043001
975	95160025	95179536	95199049	95218564	95238081
976	95355225	95374756	95394289	95413824	95433361
977	95550625	95570176	95589729	95609284	95628841
978	95746225	95765796	95785369	95804944	95824521
979	95942025	95961616	95981209	96000804	96020401
980	96138025	96157636	96177249	96196864	96216481

120 Square Numbers and their Roots.

Root	0	1	2	3	4
981	96236100	96255721	96275344	96294969	96314596
982	96432400	96452041	96471684	96491329	96510976
983	96628900	96648561	96668224	96687889	96707556
984	96825600	96845281	96864964	96884649	96904336
985	97022500	97042201	97061904	97081609	97101316
986	97219600	97239321	97259044	97278769	97298496
987	97416900	97436641	97456384	97476129	97495876
988	97614400	97634161	97653924	97673689	97693456
989	97812100	97831881	97851664	97871449	97891236
990	98010000	98029801	98049604	98069409	98089216
991	98208100	98227921	98247744	98267569	98287396
992	98406400	98426241	98446084	98465929	98485776
993	98604900	98624761	98644624	98664489	98684356
994	98803600	98823481	98843364	98863249	98883136
995	99002500	99022401	99042304	99062209	99082016
996	99201600	99221521	99241444	99261369	99281296
997	99400900	99420841	99440784	99460729	99480676
998	99600400	99620361	99640324	99660289	99680256
999	99800100	99820081	99840064	99860049	99880036

Square Numbers and their Roots.

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Root	5	6	7	8	9
981	96334225	96353856	96378489	96393124	96412761
982	96530625	96550276	96569929	96589584	96609241
983	96727225	96746896	96766569	96786244	96805921
984	96924025	96943716	96963409	96983104	97002801
985	97121025	97140736	97160449	97180164	97199881
986	97318225	97337956	97357689	97377424	97397161
987	97515625	97535376	97555129	97574884	97594641
988	97713225	97732996	97752769	97772544	97792321
989	97911025	97930816	97950609	97970404	97990201
990	98109025	98128836	98148649	98168464	98188281
991	98307225	98327056	98346889	98366724	98386561
992	98505625	98525476	98545329	98565184	98585041
993	98704225	98724096	98743969	98763844	98783721
994	98903025	98922916	98942809	98962704	98982601
995	99102025	99121936	99141849	99161764	99181681
996	99301225	99321156	99341089	99361024	99380961
997	99500625	99520576	99540529	99560484	99580441
998	99700225	99720196	99740169	99760144	99780121
999	99900025	99920016	99940009	99960004	99980001

Probl. 4. FIG. 5.

Given the Area of the Oblong ABCD, in Ale-Gallons, and one of the Sides; to find the other.

Theor.

Multiply the Area by 282, and divide the Product by the given Side, the Quotient is the other Side.

S. Or by the Table of Rectilineal Figures.

Divide the Area by the given Side, with the Quotient enter the Table, and against it in the first Column stand the Inches of the other Side, and at the Top the Tenths (if any.)

Probl. 5. FIG. 5.

In the Oblong ABCD, given the Sum of the Sides, and Sum, or Difference of the Diagonal AC, and the Area; to find the Sides.

Theor.

From the Square of A E more 1, subduct 4 times the Sum or Difference of AC and the Area, and extract the Square Root, to or from which add or subduct 1, the Sum or Remainder is AC, and by *Theor. 25.* the Sides are given.

Probl.

Probl. 6. FIG. 5.

In the Oblong ABCD, given the Sum of the Sides, and Sum, or Difference of the Square the Diagonal AC and the Area; to find the Sides.

Theor.

From 4 times the Sum of the Square of AC and the Area, subduct 3 times the Square of AE.

Or from 4 times the Difference of the Square of AC and the Area subduct the Square of AE, the Residue divide by 3, the Square Root of the Remainder, or Quotient is AF, the Difference of the Sides, then—

The Semi-sum $\left\{ \begin{smallmatrix} \text{more} \\ \text{less} \end{smallmatrix} \right\}$ the Semi-differ. $\left\{ \begin{smallmatrix} \text{greater} \\ \text{lesser} \end{smallmatrix} \right\}$ Side $\left\{ \begin{smallmatrix} \text{AB} \\ \text{BC} \end{smallmatrix} \right\}$

Probl. 7. FIG. 5.

In the Oblong ABCD, given the Difference of the Sides, and Sum, or Difference of the Diagonal AC, and the Area; to find the Sides.

Theor.

To double the Sum, or Difference of AC, and the Area, add the Square of AF more 1, and extract the Square Root, from, or to which subduct, or add 1, the Remainder, or Sum, is the Diagonal, and the Sides are given by *Theor. 25.*

Probl.

Probl. 8. FIG. 5.

In the Oblong ABCD, given the Difference of the Sides, and Sum, or Difference of the Square of the Diagonal AC, and the Area; to find the Sides.

Theor.

From 4 times the Sum of the Square of AC and the Area, subtract the Square of AF, the Remainder divide by 3.

Or from 4 times the Difference of the Square of AC and the Area, subtract 3 times the Square of AF, the Square Root of the Quotient, or Remainder is AE the Sum of the Sides, and the Sides are given by *Theor. 6.*

Probl. 9. FIG. 6.

Given AB the Side of a Rhombus, or Rhomboides, and CE the Perpendicular falling from the Obtuse Angle; to find the Area.

1. Defin.

A Rhombus is a Figure contained under four equal Sides, but no right Angles, yet the opposite Angles are equal, viz. two Obtuse, and two Acute, like a Diamond on the Cards, or an ordinary Quarry of Glass.

2. Defin.

A Rhomboides is a Figure consisting of unequal Sides, and Angles, yet those opposite are equal.

Theor.

Theor.

Multiply AB by CE, the Product is the Area in Inches; which divided by 282, or multiplied by .0035461 gives the Area, or Content upon one Inch in Ale-Gallons.

§. Or by the Table of Rectilineal Figures.

Multiply the $\left\{ \begin{array}{l} AB \\ CE \end{array} \right\}$ by $\left\{ \begin{array}{l} CE \\ AB \end{array} \right\}$ the Product is the Area in Ale-Gallons

§. Or by the Tetragonical Table.

A mean Proportional between AB and CE, is the Side of a Square equal to the Rombus, or Romboides, against which in the Table stands the Area.

Note, That a Square, an Oblong, a Rombus and a Romboides are commonly called Parallelograms, of which the two first are Right-angled, and the other Oblique-angled.

Probl. 10. FIG. 6.

Given AB the Side of a Rombus, and one Diagonal; to find the other.

Theor.

Divide the Diagonals AB and BC into two parts making Right angles at the Center, then from the Square of AB, subduct the Square of half the given Diagonal, the Square Root of the Remainder is half the other Diagonal.

Probl.

Probl. 11. FIG. 7.

Given AB and BD the Sides of a Rhomboides, and one Diagonal; to find the other.

Theor.

Seeing the Angle at C is an Acute angle, by *Theor. 47.* find HB the Difference of the Segments of the Base, and the Perpendicular CE, add half HB to AB and the Sum is AF, then to the Square of AF add the Square of $DF=CE$, the Square Root of the Sum is AD.

Or seeing the Angle at B is an Obtuse angle by *Theor. 48.* find BF the distance from the Obtuse angle to the foot of the Perpendicular, and DF the Perpendicular, subduct BF from AB and there remains EB, then to the Square of EB add the Square of $CE=DF$, the Square Root of the Sum is CB.

Probl. 12. FIG. 7.

Given AD and BC the Diagonals of a Rhomboides, and one Side; to find the other.

Theor.

To the Square of AD add the Square of BC, from half the Sum subduct the Square of AB or BD the Square Root of the Remainder is BD or AB according to the intent of the Question.

Probl.

Probl. 13. FIG. 8.

Given the Sides of the Trapezium ABCD, when AB is parallel to CD; to find the Area.

Defin.

A Trapezium is a Figure consisting of four unequal Sides.

Theor.

Multiply half the Sum of AB and CD by Cp the Perpendicular or nearest distance between AB and CD, the Product is the Area in Inches, which divided by 282 or multiplied by .0035461 gives the Content upon one Inch in Ale-Gallons.

Demonstration.

Make EB equal to CD, and draw CE from the Square of AC subduct the Square of Ap the Semi-difference of AB and CD the two parallel Sides, the Square Root of the Remainder is the Perpendicular Cp, then EB multiplied by DF=Cp gives the Area of the Parallelogram EBCD, and half AE multiplied by Cp is equal to the Triangle ACE whose Sum is equal to the Trapezium ABCD.

FIG. 9.

Note, If the Trapezium have two right Angles as G and H, then the operation is more easie, for if half the Sum of GK and HI the two parallel Sides be multiplied by the Perpendicular GH, the Product is the Area of the Trapezium GHIK.

Note,

Note, That all other irregular Rectilineal Figures must with Diagonals be divided into Trapezias, and Triangles, which being severally measured, and their Areas added together, the Sum is the Content of that Figure; and in every such irregular Figure the Triangles are always less by two than the number of given Sides, and the Diagonals by three.

Probl. 14. FIG. 10.

Given the Sides of the Trapezium ABCD when none of them are parallel; to find the Area.

Theor.

Divide it into Triangles by a Diagonal drawn between either pair of opposite Angles as AC, and from the other Angles B, and D, let fall the Perpendiculars BE, and FD, upon the Diagonal, or common Base AC, then

Multiply $\left\{ \begin{array}{l} \text{BE more FD} \\ \text{AC} \end{array} \right\}$ by half $\left\{ \begin{array}{l} \text{AC} \\ \text{BE+FD} \end{array} \right\}$

The Product is the Area in Inches, which divided by 282, or multiplied by .0035461 gives the Area, or Content upon one Inch in Ale-Gallons.

§. Or by the Table of Rectilineal Figures.

Multiply the Number against $\left\{ \begin{array}{l} \text{BE+FD} \\ \text{AC—} \end{array} \right\}$ by half $\left\{ \begin{array}{l} \text{AC} \\ \text{BE+FD} \end{array} \right\}$

The Product is the Area in Ale-Gallons.

§. Or

6. Or by the Tetrugonical Table.

A Geometrical $\{BE+FD\}$ and half $\{AC\}$
mean between $\{AC\}$ and $\{BE+FD\}$
is the Side of a Square equal to the Trapezium, against
which in the Table stands the Area in Ale-Gallons.

Probl. 15. FIG. 8.

Given the Sides of the Trapezium ABCD, when AB is
parallel to CD; to find the Diagonal CB.

Theor.

Find the Perpendicular Cp by *Theor. 14*. then from
AB subduct Ap the Semidifference of AB and CD,
the Remainder is pB, to the Square of which add the
Square of Cp, the Square root of the Sum is CB.

When the Angles are unequal see *Theor. 11*.

Probl. 16. FIG. 11.

Given the Sides of the Trapezium ABCD when none of
them are parallel; and one Diagonal; to find the other.

Theor.

To the Square of AC, add the Square of BC, from
the Sum subduct the Square of AB, divide half the Re-
mainder by AC and the Quotient is CE.

To the Square of AC, add the Square of AD, from
the Sum subduct the Square of CD, half the Remainder
divided by AC quotes AF. K Then

Then by *Theor.* 20. find BE, and DF, add CE to AF, and subtract the Sum from AC the Remainder is EF=DG.

Lastly by *Theor.* 19. you may find BD for the Square of DG more the Square of BG is equal to the Square of BD.

Probl. 17. FIG. 12.

Given AC the Base of a Right-lined plain Triangle and BD the Perpendicular; to find the Area.

Defin.

A Right-lined plain Triangle is a Figure comprehended by three Right lines including three Angles, and of these according to the Length and proportion of their Sides, there are three kinds; *viz.*

Equilateral Isosceles Scalenum	} Having	{ three equal two equal three unequal	} Sides.
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And these again according to the nature and Quantity of their Angles are distinguished into three sorts, *viz.*

Right-angled Acute-angled Obtuse-angled	} Having	{ one Right one Obtuse three Acute	} Angles.
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Hence it is manifest that there are seven sorts of Right-lined Triangles, and that every one contains either three Acute angles, or one Right and two Acute, or one Obtuse and two Acute: for it is impossible that the same Triangle should contain two Right, or two Obtuse

Of Right-angled plain Triangles. 131

Obtuse Angles, or one a Right, and the other an Obtuse angle.

Note, If the Square of one of the Sides be equal to the Sum of the Squares of the other two, then the Angle opposite to that Side is a Right angle, if greater Obtuse, if lesser Acute.

Also in a Right lined Triangle the right angle being always 90 degrees, the other two make also 90 degrees, one being the complement of the other.

Theor.

Multiply AC by half BD (which is always the nearest distance from the Base to the opposite Angle) or Multiply half AC by BD the Product is the Area in Inches, which divided by 282, or multiplied by .0035461 gives the Area, or Content at one Inch deep in Ale-Gallons.

§. Or by the Table of Rectilineal Figures.

Multiply the Number $\left\{ \begin{array}{c} AC \\ BD \end{array} \right\}$ by half $\left\{ \begin{array}{c} BD \\ AC \end{array} \right\}$
answerable to _____
the Product is the Content upon one Inch in Ale-Gallons.

§. Or by the Tetragonical Table.

A Geometrical mean between $\left\{ \begin{array}{c} AC \\ BD \end{array} \right\}$ and half $\left\{ \begin{array}{c} BD \\ AC \end{array} \right\}$
is the Side of a Square whose Area is equal to the Area of the Triangle ABC.

132 Of Right-angled plain Triangles.

Probl. 18. FIG. 12.

In the Triangle ABC, given AB, or BC one of the containing Sides, and the Area; to find the other.

Theor.

Divide the double Area by the given Side, the Quotient is the other Side.

Probl. 19. FIG. 12.

In the Triangle ABC, given AB, and BC, the containing Sides severally; to find the Hypotenuse AC.

Theor.

Square AB, and BC, severally, and add them together, the Square root of the Sum is AC.

Note, That this Problem is the foundation whereon the Dimensions of right-angled plain Triangles do principally depend.

Probl. 20. FIG. 12.

In the Triangle ABC, given AC the Hypotenuse, and AB, or BC, one of the containing Sides; to find the other.

Theor.

From the Square of AC, subduct the Square of the given Side, the Square root of the Remainder is the other Side.

Probl. 21.

Probl. 21. FIG. 12.

In the Triangle ABC any number being given for one of the containing Sides; to form the Triangle.

Theor.

From the Square of the given Number (*if odd*) subduct Unity or 1, half the Remainder is the other Side, to which add Unity or 1, the Sum is the Hypotenuse AC.

Or from the Square of half the given Number (*if even*) subduct 1, the Remainder is the other Side, to which Square add 1, the Sum is the Hypotenuse AC.

Probl. 22. FIG. 12.

In the Triangle ABC, any Number being given for the Hypotenuse AC; to form a Right-angled Triangle whose Sides shall be Proportional.

Theor.

To the Square of AC add a Quarter of the same Square, from the Square Root of the Sum, subduct half AC, the Remainder is BC, which multiplied by AC, the Square Root of the Product is AB.

Which is no more than to divide a Number according to extream and mean Proportion, the greater part being one of the Sides, and a mean Proportional between the Hypotenuse and greater part, the other Side.

Probl. 23. FIG. 11.

In the Triangle ABC, given AB or BC one of the containing Sides, and Sum or Difference of the Hypotenuse AC, and the other Side; to find the Triangle.

Theor.

Divide the Square of the given Side, by the given Sum, or Difference, the Quotient is the Difference, or Sum, according to the intent of the Question, and by *Theor. 6.* the Sides are given.

Probl. 24. FIG. 12.

In the Triangle ABC, given AB, or BC, one of the containing Sides, and Rectangle of the Hypotenuse AC, and the other Side; to find the Triangle.

Theor.

To a Quarter of the Biquadrate of the given Side, add the Square of the Rectangle, the Square Root of the Sum is the Semi-sum of the Squares; to, and from which, add, and subtract half the Square of the given Side, the Square Root of the Sum, and Remainder are the Sides.

Probl.

Probl. 25. FIG. 13.

In the Triangle ABC, given the Hypotenuse AC, and CD, or CE, the Sum, or Difference of the containing Sides; to find the Triangle.

Theor.

From the Square of half CD, subduct the Square of AG, = to half AC; or from the Square of AG, subduct the Square of half CE, the Remainder is half the Rectangle; then divide the Rectangle by AC, the Quotient is BF. And the Sides are given by Theor. 26. or 27.

S. Or thus,

From half the Square of AC, subduct the Square of half CD, or EC, the Square Root of the Remainder is half EC, or CD; and by Theor. 6. the Sides are given.

Probl. 26. FIG. 13.

In the Triangle ABC, given AC the Hypotenuse, and Rectangle of AB, and BC, the containing Sides; to find the Triangle.

Theor.

To, and from the Square of AG = half the Hypotenuse, add and subduct half the Rectangle, the Square Roots of the Sum, and Remainder, are the Semi-sum, and Semi-difference of the Sides; and by Theor. 6. the Sides are given, and the Perpendicular BF is given by the first part of Theor. 25.

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Probl. 27. FIG. 13.

In the Triangle ABC, given BF the Perpendicular falling from the Right angle upon the Hypotenuse AC, and the Hypotenuse; to find the Triangle.

Theor.

Forasmuch as the Angle at B is a Right angle it followeth that $BG = AG$ half the Hypotenuse, therefore from the Square of BG, subduct the Square of BF, the Square Root of the Remainder is FG the Semi-difference of the Segments, and the Sides are given by *Theor. 32.*

Probl. 28. FIG. 13.

In the Triangle ABC, given BF the Perpendicular, and HC the Difference of the Segments of the Hypotenuse; to find the Triangle.

Theor.

To the Square of BF, add the Square of FG the Semi-difference of the Segments, the Square Root of the Sum is $BG = AG$ half the Hypotenuse, and the Sides are given by *Theor. 32.*

Probl. 29. FIG. 13.

In the Triangle ABC, given BF the Perpendicular, and DC, or EC, the Sum, or Difference of the containing Sides; to find the Triangle.

Theor.

Theor.

To the Square of DC, or EC, add the Square of BF,

the Square Root of the Sum $\left\{ \begin{smallmatrix} \text{less} \\ \text{more} \end{smallmatrix} \right\} BF = AC$.

And by *Theor.* 27. the Sides are given,

Probl. 30. FIG. 13.

In the Triangle ABC, given BF the Perpendicular, and Rectangle of AB, and BC, the containing Sides; to find the Triangle.

Theor.

Divide the Rectangle by BF, the Quotient is AC the Hypotenuse, and by *Theor.* 27. the Sides are given.

Probl. 31. FIG. 13.

In the Triangle ABC, given BF the Perpendicular, and Perimeter (that is the Sum of all the Sides,) to find the Triangle.

Theor.

Divide half the Square of the Perimeter, by the Sum of the Perimeter, and Perpendicular, the Quotient is AC the Hypotenuse, and by *Theor.* 27. the Sides are given.

Probl.

Probl. 32. FIG. 13.

In the Triangle ABC, given HC the Difference of the Segments of the Hypotenuse AC, and the Hypotenuse; to find the Triangle.

Theor.

To, and from AG half the Hypotenuse, add, and subduct FG the Semi-difference of the Segments, the Sum, and Remainder, are the Segments FC, and AF; then the Square Root of their Rectangle is BF. And the Sides are given by *Theor. 19.*

S. Or, you may find BF thus—

From AG, subduct AF, the Remainder is FG; and the Square of BG = AG less the Square of FG is = to the Square of BF by *Theor. 20.*

Probl. 33. FIG. 13.

In the Triangle ABC, given HC the Difference of the Segments of the Hypotenuse AC, and AB, or BC one of the containing Sides; to find the Triangle.

Theor.

To the double Square of AB, or BC, add the Square of FG the Semi-difference of the Segments,

The Square Root $\left\{ \begin{array}{l} \text{more} \\ \text{of the Sum} \end{array} \right\}$ less $\left\{ \begin{array}{l} \text{FG, is AC if } \left\{ \begin{array}{l} \text{AB} \\ \text{BC} \end{array} \right\} \text{ were given.}$

And by *Theor. 32.* the Sides are given.

Probl.

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Probl. 34. FIG. 12.

In the Triangle ABC, given HC the Difference of the Segments of the Hypotenuse AC, and DC, or EC, the Sum or Difference of the containing Sides; to find the Triangle.

Theor.

As the Square Root of the double Square of DC, or EC, less the Square of HC, is to DC, or EC;

So is HC, to EC, or DC.

And so is DC, or EC, to AC.

And by Theor. 6. the Sides are given.

Probl. 35. FIG. 13.

In the Triangle ABC, given HC the Difference of the Segments of the Hypotenuse AC, and Rectangle of AB, and BC, the containing Sides; to find the Triangle.

Theor.

To 4 times the Square of the given Rectangle, add a quarter of the Biquadrate of HC, and extract the Square Root, to which add half the Square of HC, the Square Root of this Sum is AC; And by Theor. 32. the Sides are given.

Probl. 36. FIG. 13.

In the Triangle ABC, given the Area, and Perimeter (that is the Sum of all the Sides;) to find the Triangle.

Theor.

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Theor.

Divide the Area by the Semi-perimeter, the Quotient is the Semi-diameter of the Circle Incribed, which subducted from the Semi-perimeter, the Remainder is AC, and the Perimeter less AC, is the Sum of the other Sides, then by Theor. 25. the Sides are given.

Note, That the Square of the Perimeter must always exceed the Quadruple Area.

Probl. 37. FIG. 14.

In the Triangle ABC, given AD, and EC, the Differences between the Hypotenuse AC, and AB, and BC, the containing Sides; to find the Triangle.

Theor.

To the Sum of the given Differences, add the Square Root of their double Rectangle, the Sum is AC, then add that Square Root to the given Differences severally, and the Sums will be the Sides.

Probl. 38. FIG. 13.

In the Triangle ABC, given AB, or BC, one of the containing Sides, and the Alternate or opposite Segment of the Hypotenuse AC; to find the Triangle.

Theor.

Theor.

To the Square of the given Side, add the Square of half the given Segment, to, and from the Square Root of the Sum add, and subduct half the given Segment, the Sum, and Remainder are the Hypotenuse, and other Segment; And by *Theor.* 20. the other Side is given.

Probl. 39. FIG. 13.

In the Triangle ABC, given AB, or BC, one of the containing Sides, and Sum, or Difference of the Hypotenuse AC, and the Area; to find the Triangle.

Theor.

Divide 4 times the Rectangle of the Sum, or Difference of AC and Area, and the given Side, by the Square of the given Side less 4, and Square the Quotient, from a quarter of which, subduct 4 times the Square of the given Sum, or Difference less 4 times the Square of the given Side, divided by the Square of the given Side less 4, and extract the Square Root, which reserve:

Divide double the aforesaid Rectangle, by the Square of the given Side less 4, from, or to the Quotient, subduct, or add the reserved Square Root, the Remainder, or Sum is the Side required.

Probl.

Probl. 40. FIG. 13.

In the Triangle ABC, given DC, the Sum of the containing Sides, and Sum, or Difference of the Hypotenuse AC, and the Area; to find the Triangle.

Theor.

From the Square of DC, more 4, subduct 4 times the Sum, or Difference of AC, and the Area, and extract the Square Root, to, or from which add, or subduct 2, the Sum, or Remainder is AC, and the Sides are given by *Theor. 25.*

Probl. 41. FIG. 13.

In the Triangle ABC, given DC, the Sum of the containing Sides, and Sum or Difference of the Square of the Hypotenuse AC, and the Area; to find the Triangle.

Theor.

From 8 times the Sum of the Square of AC, and the Area, subduct 5 times the Square of DC, and divide the Remainder by 5: or from 8 times the Difference of the Square of AC, and the Area, subduct 3 times the Square of DC, and divide the Remainder by 5, the Square Root of either Quotient is EC, the Difference of the Sides; And by *Theor. 6.* the Sides are given.

Probl.

Probl. 42. FIG. 13.

In the Triangle ABC, given EC the Difference of the containing Sides, and Sum, or Difference of the Hypotenuse AC, and the Area; to find the Triangle.

Theor.

To 4 times the Sum, or Difference of AC, and the Area, add the Square of EC, more 4, and extract the Square Root, from; or to which subduct; or add 2, the Remainder or Sum is AC; and by *Theor. 25.* the Sides are given.

Probl. 43. FIG. 13.

In the Triangle ABC, given EC, the Difference of the containing Sides, and Sum, or Difference of the Square of the Hypotenuse AC, and the Area; to find the Triangle.

Theor.

From 8 times the Sum of the Square of AC, and the Area, subduct 3 times the Square of EC, and divide the Remainder by 5: Or from 8 times the Difference of the Square of AC, and the Area, subduct 9 times the Square of EC, and divide the Remainder by 3; the Square Root of either Quotient is DC, the Sum of the Sides; And by *Theor. 6.* the Sides are given.

Probl.

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Probl. 44. FIG. 13.

In the Triangle ABC, given the Rectangle of AB, and BC, the containing Sides, and Sum, or Difference of the Hypotenuse AC, and Area; to find the Triangle.

Theor.

From the double Sum of AC, and the Area, subduct the given Rectangle: Or from the given Rectangle, subduct the double Difference between AC, and the Area, half the Remainder is AC; And by *Theor. 26.* the Sides are given.

Probl. 45. FIG. 13.

In the Triangle ABC, given the Rectangle of AB, and BC, the containing Sides, and Sum, or Difference of the Square of the Hypotenuse AC, and the Area; to find the Triangle.

Theor.

From, or to the double Sum, or Difference of the Square of AC, and the Area, subduct, or add the given Rectangle, the Square Root of half the Remainder, or Sum will be AC; And the Sides are given by *Theor. 26.*

Probl.

Probl. 46.

In any plain Triangle, given the Sides severally; to find the Area.

Theor.

From the Semi-perimeter or half Sum of all the Sides subduct each Side severally, and multiply the three Differences one into another continually, then multiply the Product by the Semi-perimeter, the Square Root of this last Product will be the Area.

Probl. 47.

In the Triangles ABC, given the Sides severally; to find the Segments of the Base, and the Perpendicular.

Theor.

1. Case. FIG. 15.

When the Triangle hath unequal Acute angles at the Base, and the Perpendicular BD falls within the Triangle:

Divide the Difference of the Squares of AB, and BC, by AC, the Quotient will be AE, the Difference of the Segments of the Base.

Or as AC, is to AF; so is AG, to AE.

And half AC $\left\{ \begin{array}{l} \text{more} \\ \text{less} \end{array} \right\}$ half AE is $\left\{ \begin{array}{l} \text{AD greater} \\ \text{DC lesser} \end{array} \right\}$ Segment.

And by *Theor. 20.* the Perpendicular BD is given.

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2. Case.

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2. Case. FIG. 16.

When the Triangle is Obtuse-angled at the Base and the Perpendicular CD falls without the Triangle:

Divide the Difference of the Squares of AB, and BC, by AB, the Quotient will be AE, the composed line, (that is the Sum of the Base, and double distance from the foot of the Perpendicular to the Obtuse angle)

Or as AB, is to AF; so is AG, to AE.

And half AB $\left\{ \begin{array}{l} \text{more} \\ \text{less} \end{array} \right\}$ half AE is = $\left\{ \begin{array}{l} \text{AD.} \\ \text{DE.} \end{array} \right\}$

And CD is given by Theor. 20. for the Square of AD, or DE subducted from the Square of the respective Side, leaves the Square of CD.

Probl. 48. FIG. 17.

In the Triangle ABC, given the Sides severally; to find the Diameter of the Circle Inscribed, and Circumscribed.

Theor.

From the Semi-perimeter, or half Sum of all the Sides, subduct the three Sides severally, and multiply the three Differences one into another continually; then divide the Product by the Semi-perimeter, the Square Root of the Quotient will be DE, the Semi-diameter of the Circle Inscribed.

And if you divide the Rectangle of AB, and BC, by the Perpendicular CD, the Quotient will be FG, the Semi-diameter of the Circle Circumscribed.

Note,

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Note, That in any Right-angled plain Triangle, the Difference between the Hypotenuse, and Sum of the containing Sides, is the Diameter of the Circle Incribed.

Probl. 49. FIG. 15.

To find an Oblique-angled Triangle as ABC, whose Sides, Segments of the Base, and Perpendicular, may be severally expressed by whole Numbers.

Theor.

Let the Sides be represented by the Letters a, b, c , which may signifie any three Numbers taken at pleasure, then :

Multiply a, b, c , continually double the Product is BD.

From the Square of c multiplied by b , subduct the Square of a multiplied by b , the Remainder is AD.

From the Square of c multiplied by a , subduct the Square of b multiplied by a , the Remainder is DC.

And AC is the Sum of AD, and DC, when BD falls within the Triangle; or AE is the Difference of AD and DC, when BD falls without the Triangle.

To the Square of c multiplied by a add the Square of b multiplied by a the Sum is CB.

To the Square of c multiplied by b , add the Square of a multiplied by b the Sum is AB.

Note, That c must always exceed either of the other Sides.

Probl. 50. FIG. 15.

In the Triangle ABC, given AB, and BC severally, and the Area; to find the Base AC.

Theor.

To the Biquadrate of AF, add the Biquadrate of AG, from a quarter of the Sum, subduct half the Rectangle of the Square of their Sum, and Square of their Difference, more 16 times the Square of the Area, and extract the Square Root, which added to, or subducted from the Square of their Sum, and Square of their Difference, the Square Root of the Sum, or Remainder will be AC, so that two Numbers will be found either of which may be AC.

Probl. 51. FIG. 15.

In the Triangle ABC, given AF, or AG, the Sum, or Difference of the Sides, AC the Base, and BD the Perpendicular; to find the Triangle.

Theor.

As the Difference between the Square of AF, or AG, and the Square of AC,

Is to the said Difference, less or more 4 times the Square of BD;

So is the Square of AC, to the Square of AG, or AF.

And by *Theor. 6.* the Sides are given.

To find where the Perpendicular falls say——

As the first Difference, is to the second;

So

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So is AE , or AG , to a fourth Proportional.
Which is AE the Difference of the Segments, when BD falls within the Triangle, and is always $FIG. 15$, less than AC . Or AE the composed Line 16 , when CD falls without the Triangle, and is ever greater than AB .

But when this fourth Proportional is equal to the Base, then the Perpendicular falls upon the end thereof.

And thus you may know whether the Triangle sought be Acute, Obtuse, or Right-angled at the Base.

Probl. 52. FIG. 15.

In the Triangle ABC , given AF , or AG , the Sum, or Difference of the Sides, AC the Base, and the Area; to find the Triangle.

Theor.

Divide 4 times the Square of the Area, by the Difference between the Square of AC , and the Square of AF , or AG , from, or to a quarter of the Square of AC , subduct, or add the Quotient, the Square Root of the Remainder, or Sum is half AG , or AF ; And by *Theor. 6.* the Sides are given.

Or Divide the double Area by the Base, the Quotient is the Perpendicular BD , and by *Theor. 51.* the Sides are given.

Probl. 53.

In the Triangles ABC , given the Sum or Difference of the Sides, Difference of the Segments of the Base, and Perpendicular; to find the Triangle:

L 3

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Or Sum, or Difference of the Sides, adjoined Line, and Perpendicular; to find the Triangle.

Theor.

1. Case. FIG. 15.

When the Triangle hath unequal Acute angles at the Base, and BD falls within the Triangle.

As the Difference between the Square of AF, or AG, and the Square of AE,

Is to the said Difference, less, or more 4 times the Square of BD;

So is the Square of AF, or AG, to the Square of AC.

And so is the Square of AE, to the Square of AG, or AF.

And AB and BC are given by Theor. 6.

2. Case. FIG. 16.

When the Triangle is Obtuse-angled at the Base, and the Perpendicular CD falls without the Triangle.

As the Difference between the Square of AF, or AG, and the Square of AE,

Is to the said Difference, less or more 4 times the Square of CD;

So is the Square of AF, or AG, to the Square of AB.

And so is the Square of AE, to the Square of AG, or AF.

And by Theor. 6. the Sides are given.

Probl.

Probl. 54. FIG. 15.

In the Triangle ABC, given AF, or AG, the Sum, or Difference of the Sides, Rectangle of AD, and DC, the Segments of the Base, and BD, the Perpendicular; to find the Triangle.

Theor.

To the Square of AF, or AG, add 4 times the Square of BD, and reserve the Sum:

To 16 times the Square of AF, or AG, multiplied by the Square of BD, add 16 times the Rectangle of AD, and DC, and extract the Square Root, which subtracted from, or added to the reserved Sum, gives the Square of AG, or AF, the Difference, or Sum of the Sides,

And by Theor. 6. the Sides are given.

Probl. 55. FIG. 15.

In the Triangle ABC, given the Rectangle of the Sides AB, and BC, Rectangle of AD, and DC, the Segments of the Base, and BD, the Perpendicular; to find the Triangle.

Theor.

To the Square of the Rectangle of AB, and BC, add the Biquadrates of BD, from the Sum subtract the Square of the Rectangle of AD, and DC, and divide the Remainder by the Square of BD; from the Square of half the Quotient subtract the Square of the Rectangle of AB, and BC, to the Square Root of the Remainder add half the said Quotient, the Square Root of the Sum is AB, whence no other *Quæstio* can be unknown.

L 4

Probl.

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Probl. 56. FIG. 15.

In the Triangle ABC, given the Rectangle of the Sides AB, and BC, the Difference of the Segments of the Base AE, and the Perpendicular BD; to find the Triangle.

Theor.

From 16 times the Square of the given Rectangle, subduct the Biquadrate of AE, more 8 times the Square of BD, multiplied by the Square of AE, more 16 times the Biquadrate of BD, and reserve the Difference:

From 8 times the Square of BD, subduct the double Square of AE, to the reserved Difference add the Square of half the Remainder, from the Square Root of the Sum subduct half the aforesaid Remainder, the Square Root of this last Remainder is AC. - Then having the Base (*which is the Sum of the Segments*) and Difference of the Segments, the Segments AD, and DC, are given by Theor. 6.

And the Sides are given by Theor. 19.

Probl. 57. FIG. 15.

In the Triangle ABC, given the Rectangle of the Sides AB, and BC, the Base AC, and the Perpendicular BD; to find the Triangle.

Theor.

From 4 times the Square of the given Rectangle, subduct 4 times the Square of AC, multiplied by the Square of BD, the Square Root of the Remainder is AG,

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AG, the Difference of the Sides; If the double Rect-angle be equal to the Square of AC.

Or to the said Root add the Square of AC, and from the Sum subduct the double Rectangle, the Square Root of the Remainder is AG; If the double Rectangle exceed the Square of AC.

Or to the said Root add the Square of AC, less the double Rectangle, the Square Root of the Sum is AG; If the double Rectangle be less than the Square of AC:

Lastly, To the Square of AG, add the given Rect-angle, the Square Root of the Sum is half AF; And the Sides are given by *Theor. 6.*

Probl. 58. FIG. 15.

In the Triangle ABC, given the Rectangle of the Sides AB, and BC, the Base AC, and the Area; to find the Triangle.

Theor.

Divide the double Area, by the Base AC, the Quotient is the Perpendicular BD, and the Sides are given by *Theor. 57.*

Note, That the Triangle being the most perfect, and justly challenging the prebeminence above all other Geometrical Figures in the Practical part of the Mathematicks, hath encouraged me to collect these Varieties, many of them being extraordinary Cases; and am so far from thinking that I have overburdened this Book with needless curiosities, as I know some will be very apt to conclude, that if opportunity hereafter present it self I shall enlarge my present Conceptions on this Subject: And though some of them may not be of such universal use as others, yet I doubt not but an Ingenious man will find in the meanest of them at one time or other something that will be worth his Observation.

Probl. 59.

Probl. 59. FIG. 18.

GIVEN AB the Diameter of a Circle; to find the Periphery ADBK.

Defn.

A Circle is a plane Figure comprehended under one round Line as ADBK which is the Perimeter, Circumference or Periphery;

In the middle of it is a Point which is called the Center; from whence all right Lines drawn to the Periphery, are equal.

The Diameter is any right Line drawn through the Center from one side of the Periphery to the other, dividing the Circle into two equal parts as AB or DK.

A Chord or Subtense is a right Line reaching from any part of the Periphery to the other dividing the Circle into two unequal parts as FE is the Chord of both the Arches FBE and FAE.

A Right Sine is either the Whole Sine, which is an Arch of 90 degrees equal to the Radius or Semi-diameter, as BC is the Sine of the Quadrant DFB.

Or the Lesser Sine which is half the Chord of the double Arch, as FG, or GE is the Sine of the Arch FB or BE being half the Arch FBE less than a Quadrant, and also the Sine of FA or EA greater than a Quadrant, and LF equal to CG is the Co-Sine of the Arch FB.

A Versed Sine is part of the Radius intercepted between the Periphery, and its right Sine, as GB is the versed Sine of the Arch FB, and GA of FA.

A Tangent is a right Line without the Periphery in the extremity of the Arch, terminated by the Secant as BH

Definitions of a Circle.

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BF is the Tangent of the Arch **BF**; and **DI** is the Co-tangent, *viz.* the Tangent of the Arch **DF**.

A *Secant* is a right Line drawn from the Center through the extremity of the Arch, till it concur with the Tangent, as **CH** is the Secant of the Arch **FB**; and **CI** is the Co-secant, *viz.* the Secant of the Arch **DF**.

A *Quadrant* is a fourth part of the Circle contained under two Semi-diameters drawn from the Center at right Angles.

A *Sector* is any portion of a Circle comprehended by an Arch of the Periphery and two Semi-diameters drawn from the Center, as **DCF**.

A *Segment* is any part of a Circle terminated on one Side by a right Line, and on the other by the Periphery, as **FBE** or **FAE**.

A *Rectilineal Figure* is Inscrib'd in a Circle, or a Circle Conscrib'd about it, when all the Angles touch the Periphery.

A *Rectilineal Figure* is Conscrib'd about a Circle, or a Circle Inscrib'd in it, when all the Sides touch the Periphery.

Theor.

The *Diameter* of a Circle is to its Periphery, as 7 to 22, according to *Archimedes*, which gives the Area somewhat too much; other nearer Proportions there are in whole Numbers, *viz.*

$$\text{As } \left\{ \begin{array}{l} 71 \\ 497 \\ 111 \end{array} \right\} \text{ to } \left\{ \begin{array}{l} 223 \\ 1562 \\ 355 \end{array} \right\}$$

But *Ludolph* of *Ctunen* is the most exact, he hath computed the same to 36 places of Decimals, supposing the *Diameter* to be 1. *viz.* 3.14159265, 35989792, 38462643, 38327950288. Therefore if you multiply
any

any *Diameter* by this Number the Product is the *Periphery*, and in most cases the first 6 places are sufficient,

Illustration. FIG. 19.

Let AB be the Diameter of a Circle and BDA the Semi-periphery equal to BF; then the Right-angled Triangle ABF is as exactly equal to the Circle as the right Line BF is to the Semi-periphery.

Or, Let BFGC be a Right-angled Parallelogram, let BC be equal to the Semi-diameter, and CG equal to the Semi-periphery, and it shall be equal to the Circle, for this Parallelogram is equal to the Triangle.

Every Circle therefore is equal to that Right-angled Triangle one of whose Sides containing the Right-angle is equal to the Diameter, and the other to the Semi-periphery:

Or to that Right-angled Parallelogram whose Breadth and Length are equal, one to the Semi-diameter, and the other to the Semi-periphery.

Probl. 60. FIG. 18.

Given ADBK the Periphery of a Circle; to find the Diameter AB.

Theor.

$$\text{As } \left\{ \begin{array}{l} 22 \\ 223 \\ 355 \\ 1562 \\ 1 \end{array} \right\} \text{ to } \left\{ \begin{array}{l} 7 \\ 71 \\ 113 \\ 497 \\ 0.3183099 \end{array} \right\} \left\{ \begin{array}{l} \text{So is the Periphery} \\ \text{to the Diameter.} \end{array} \right.$$

Note, that .3183099 is the Diameter of a Circle whose Periphery is Unity.

Probl.

Probl. 61. FIG. 18.

Given AB the Diameter of a Circle ; to find the Area.

Theor.

Multiply the Semi-periphery by the Semi-diameter, the Product is the Area according to the *Archimedean* Demonstration ; thus when the *Diameter* is Unity the *Periphery* will be 3.141592654+ according to *Ludolph* whose half multiplied by .5 the Semi-diameter, the Product .7853981635 will be the Area in Inches ; which being divided severally by 282, and 231 gives the Areas in Ale, and Wine Gallons ;

And all *Circles* being in Proportion one to another as the *Squares* of their *Diameters* it will follow, because 1 doth not multiply —

As $\left\{ \begin{array}{l} 1 \\ 1 \\ 1 \end{array} \right\}$ to $\left\{ \begin{array}{l} .785398 \\ .0027851 \\ .0034 \end{array} \right\}$ So is the Square of any Inches. Diameter to its Area in Ale-Gall. Wine-Gall.

And after this manner you may make Tables of *Areas of Circles* in Ale and Wine Gallons, but the *Second Differences* being equal it may be made by *Addition* as directed *Theor. 1.*

The Proportion in whole Numbers are —

As $\left\{ \begin{array}{l} 14 \\ 284 \\ 452 \end{array} \right\}$ to $\left\{ \begin{array}{l} 11 \\ 223 \\ 355 \end{array} \right\}$ So is the Square of any Diameter to its Area in Inches.

Note,

Note, that the Area of a Circle of 8 *Inches Diameter* contains 4 times as much as the Area of a Circle of 4 *Inches Diameter*; the reason is because 4 is contained in 8 twice and the Square of 2 is 4: Wherefore if you can find any number that will divide the given Diameter without a Remainder, the Area of such a number multiplied by the Square of the said number produces the Area of that Diameter.

A T A.

A
CYCLOMETRICAL
TABLE

Exhibiting the
AREAS of CIRCLES
IN

Alc-Gallons

AND
DECIMILLESSIMAL PARTS

*Calculated to every Tenth part, and Quar-
ter of an Inch of the DIAMETER
from 1 to 210 Inches.*

<i>Dia.</i>	.0	.1	.2	.25	.3	.4
1	0.0028	0.0034	0.0040	0.0043	0.0047	0.0055
2	0.0111	0.0123	0.0135	0.0140	0.0147	0.0160
3	0.0251	0.0268	0.0285	0.0294	0.0303	0.0322
4	0.0446	0.0468	0.0491	0.0503	0.0515	0.0539
5	0.0696	0.0724	0.0753	0.0767	0.0782	0.0812
6	0.1003	0.1036	0.1071	0.1087	0.1105	0.1141
7	0.1365	0.1404	0.1444	0.1463	0.1484	0.1525
8	0.1782	0.1827	0.1873	0.1895	0.1919	0.1965
9	0.2256	0.2306	0.2357	0.2383	0.2409	0.2461
10	0.2785	0.2841	0.2898	0.2926	0.2955	0.3012
11	0.3370	0.3432	0.3494	0.3524	0.3556	0.3620
12	0.4011	0.4078	0.4145	0.4179	0.4214	0.4282
13	0.4707	0.4780	0.4853	0.4889	0.4927	0.5001
14	0.5459	0.5537	0.5616	0.5655	0.5695	0.5775
15	0.6266	0.6350	0.6435	0.6477	0.6520	0.6605
16	0.7130	0.7219	0.7309	0.7354	0.7400	0.7491
17	0.8049	0.8144	0.8239	0.8287	0.8336	0.8432
18	0.9024	0.9124	0.9225	0.9276	0.9327	0.9429
19	1.0054	1.0160	1.0267	1.0320	1.0374	1.0482
20	1.1140	1.1252	1.1364	1.1420	1.1477	1.1590
21	1.2282	1.2400	1.2517	1.2576	1.2636	1.2795
22	1.3480	1.3603	1.3726	1.3788	1.3850	1.3975
23	1.4733	1.4861	1.4991	1.5055	1.5120	1.5250
24	1.6042	1.6176	1.6311	1.6378	1.6446	1.6581
25	1.7407	1.7546	1.7686	1.7757	1.7827	1.7968
26	1.8827	1.8972	1.9118	1.9191	1.9264	1.9411
27	2.0303	2.0454	2.0605	2.0681	2.0757	2.0909
28	2.1835	2.1991	2.2148	2.2226	2.2306	2.2464
29	2.3423	2.3585	2.3747	2.3828	2.3910	2.4073
30	2.5066	2.5233	2.5401	2.5485	2.5570	2.5739
31	2.6765	2.6938	2.7111	2.7198	2.7285	2.7460
32	2.8519	2.8698	2.8877	2.8967	2.9057	2.9237
33	3.0330	3.0514	3.0698	3.0792	3.0884	3.1069
34	3.2196	3.2385	3.2576	3.2672	3.2766	3.2958
35	3.4117	3.4313	3.4509	3.4608	3.4705	3.4902

Areas of Circles in Ale-Gallons.

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<i>Dia.</i>	.5	.6	.7	.75	.8	.9
1	0.0063	0.0071	0.0080	0.0085	0.0090	0.0101
2	0.0174	0.0188	0.0203	0.0210	0.0218	0.0234
3	0.0341	0.0361	0.0381	0.0397	0.0402	0.0424
4	0.0564	0.0589	0.0615	0.0628	0.0642	0.0669
5	0.0842	0.0873	0.0905	0.0920	0.0937	0.0969
6	0.1177	0.1213	0.1250	0.1288	0.1288	0.1326
7	0.1567	0.1609	0.1651	0.1672	0.1694	0.1738
8	0.2013	0.2060	0.2108	0.2132	0.2157	0.2206
9	0.2514	0.2567	0.2621	0.2647	0.2675	0.2730
10	0.3071	0.3129	0.3189	0.3218	0.3249	0.3309
11	0.3683	0.3749	0.3814	0.3845	0.3879	0.3945
12	0.4351	0.4422	0.4492	0.4527	0.4563	0.4635
13	0.5076	0.5151	0.5227	0.5265	0.5304	0.5381
14	0.5896	0.5937	0.6018	0.6059	0.6100	0.6181
15	0.6691	0.6778	0.6865	0.6908	0.6953	0.7041
16	0.7582	0.7675	0.7767	0.7813	0.7861	0.7955
17	0.8529	0.8627	0.8725	0.8774	0.8824	0.8924
18	0.9532	0.9635	0.9739	0.9791	0.9844	0.9949
19	1.0590	1.0699	1.0809	1.0863	1.0919	1.1029
20	1.1704	1.1819	1.1934	1.1991	1.2049	1.2166
21	1.2874	1.2994	1.3115	1.3175	1.3236	1.3358
22	1.4100	1.4225	1.4351	1.4414	1.4478	1.4605
23	1.5381	1.5512	1.5644	1.5709	1.5776	1.5909
24	1.6718	1.6854	1.6992	1.7060	1.7129	1.7268
25	1.8110	1.8252	1.8395	1.8467	1.8539	1.8683
26	1.9558	1.9706	1.9855	1.9929	2.0004	2.0153
27	2.1062	2.1216	2.1370	2.1447	2.1524	2.1679
28	2.2622	2.2781	2.2941	2.3020	2.3101	2.3261
29	2.4237	2.4402	2.4567	2.4650	2.4733	2.4899
30	2.5908	2.6079	2.6249	2.6335	2.6421	2.6592
31	2.7635	2.7811	2.7987	2.8076	2.8164	2.8341
32	2.9418	2.9599	2.9781	2.9872	2.9963	3.0146
33	3.1256	3.1443	3.1630	3.1725	3.1818	3.2007
34	3.3150	3.3342	3.3535	3.3633	3.3729	3.3923
35	3.5099	3.5297	3.5496	3.5596	3.5695	3.5895

Dis.	0	.1	.2	.3	.4	.5
36	3.6095	3.6295	3.6497	3.6599	3.6699	3.6901
37	3.8128	3.8334	3.8541	3.8646	3.8747	3.8957
38	4.0217	4.0429	4.0641	4.0749	4.0854	4.1068
39	4.2361	4.2579	4.2797	4.2907	4.3016	4.3236
40	4.4562	4.4785	4.5008	4.5121	4.5233	4.5457
41	4.6818	4.7046	4.7275	4.7390	4.7505	4.7735
42	4.9129	4.9363	4.9598	4.9715	4.9834	5.0069
43	5.1496	5.1736	5.1977	5.2099	5.2218	5.2459
44	5.3920	5.4165	5.4411	5.4534	5.4657	5.4904
45	5.6393	5.6649	5.6901	5.7026	5.7153	5.7405
46	5.8933	5.9189	5.9446	5.9575	5.9704	5.9962
47	6.1523	6.1785	6.2048	6.2179	6.2311	6.2575
48	6.4169	6.4436	6.4705	6.4839	6.4973	6.5243
49	6.6870	6.7143	6.7417	6.7554	6.7692	6.7966
50	6.9628	6.9906	7.0186	7.0325	7.0466	7.0746
51	7.2440	7.2725	7.3010	7.3152	7.3295	7.3581
52	7.5309	7.5599	7.5890	7.6034	7.6181	7.6472
53	7.8233	7.8529	7.8825	7.8973	7.9122	7.9419
54	8.1214	8.1515	8.1816	8.1967	8.2118	8.2421
55	8.4249	8.4556	8.4863	8.5016	8.5171	8.5479
56	8.7341	8.7653	8.7966	8.8122	8.8279	8.8593
57	9.0488	9.0806	9.1124	9.1283	9.1443	9.1762
58	9.3691	9.4014	9.4338	9.4500	9.4662	9.4988
59	9.6949	9.7278	9.7608	9.7772	9.7938	9.8269
60	10.0264	10.0598	10.0933	10.1100	10.1269	10.1605
61	10.3634	10.3974	10.4314	10.4484	10.4655	10.4997
62	10.7059	10.7405	10.7751	10.7924	10.8098	10.8445
63	11.0541	11.0892	11.1244	11.1420	11.1596	11.1949
64	11.4078	11.4434	11.4792	11.4970	11.5150	11.5508
65	11.7670	11.8033	11.8396	11.8577	11.8759	11.9123
66	12.1319	12.1687	12.2055	12.2239	12.2424	12.2794
67	12.5023	12.5397	12.5771	12.5958	12.6145	12.6520
68	12.8783	12.9162	12.9542	12.9731	12.9922	13.0303
69	13.2599	13.2983	13.3368	13.3561	13.3754	13.4140
70	13.6470	13.6860	13.7251	13.7446	13.7642	13.8034

Areas of Circles in Als-Gallons.

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Dia.	5	6	7	7.5	8	9
36	3.7104	3.7308	3.7512	3.7616	3.7717	3.7922
37	3.9161	3.9375	3.9584	3.9699	3.9795	4.0005
38	4.1282	4.1497	4.1712	4.1820	4.1928	4.2144
39	4.3486	4.3696	4.3897	4.4008	4.4117	4.4339
40	4.5683	4.5901	4.6135	4.6250	4.6362	4.6590
41	4.7966	4.8198	4.8430	4.8546	4.8662	4.8895
42	5.0306	5.0543	5.0780	5.0901	5.1019	5.1257
43	5.2701	5.2944	5.3187	5.3308	5.3430	5.3675
44	5.5152	5.5400	5.5649	5.5773	5.5898	5.6148
45	5.7659	5.7912	5.8167	5.8293	5.8421	5.8677
46	6.0224	6.0480	6.0740	6.0870	6.1000	6.1261
47	6.2839	6.3104	6.3369	6.3502	6.3635	6.3902
48	6.5513	6.5783	6.6054	6.6190	6.6325	6.6598
49	6.8248	6.8518	6.8794	6.8932	6.9072	6.9349
50	7.1027	7.1309	7.1591	7.1731	7.1873	7.2157
51	7.3868	7.4155	7.4443	7.4586	7.4731	7.5020
52	7.6764	7.7057	7.7350	7.7497	7.7644	7.7938
53	7.9717	8.0015	8.0314	8.0463	8.0612	8.0912
54	8.2724	8.3028	8.3333	8.3485	8.3638	8.3943
55	8.5788	8.6097	8.6407	8.6562	8.6718	8.7029
56	8.8907	8.9221	8.9538	8.9695	8.9854	9.0171
57	9.2082	9.2403	9.2724	9.2884	9.3046	9.3368
58	9.5313	9.5639	9.5966	9.6129	9.6293	9.6621
59	9.8600	9.8931	9.9263	9.9429	9.9596	9.9930
60	10.1942	10.2279	10.2617	10.2785	10.2955	10.3294
61	10.5339	10.5682	10.6026	10.6197	10.6370	10.6714
62	10.8793	10.9141	10.9490	10.9665	10.9840	10.0190
63	11.2302	11.2656	11.3011	11.3188	11.3366	11.3721
64	11.5867	11.6227	11.6587	11.6767	11.6947	11.7309
65	11.9488	11.9853	12.0219	12.0401	12.0585	12.0952
66	12.3164	12.3535	12.3906	12.4091	12.4278	12.4650
67	12.6896	12.7272	12.7649	12.7837	12.8027	12.8405
68	13.0685	13.1066	13.1448	13.1639	13.1831	13.2215
69	13.4527	13.4915	13.5303	13.5497	13.5691	13.6080
70	13.8426	13.8819	13.9213	13.9410	13.9607	14.0002

Dia.	0	.1	.2	.25	.3	.4
71	14.0397	14.0793	14.1189	14.1387	14.1586	14.1983
72	14.4380	14.4781	14.5183	14.5384	14.5585	14.5988
73	14.8418	14.8825	14.9232	14.9436	14.9640	15.0049
74	15.2512	15.2925	15.3338	15.3544	15.3751	15.4165
75	15.6662	15.7080	15.7499	15.7708	15.7918	15.8337
76	16.0867	16.1291	16.1715	16.1927	16.2140	16.2565
77	16.5129	16.5558	16.5988	16.6202	16.6418	16.6849
78	16.9445	16.9880	17.0316	17.0533	17.0751	17.1188
79	17.3818	17.4258	17.4699	17.4920	17.5141	17.5583
80	17.8246	17.8692	17.9139	17.9362	17.9586	18.0033
81	18.2730	18.3182	18.3634	18.3860	18.4086	18.4540
82	18.7270	18.7727	18.8185	18.8414	18.8643	18.9102
83	19.1866	19.2328	19.2791	19.3024	19.3255	19.3719
84	19.6517	19.6985	19.7454	19.7689	19.7923	19.8393
85	20.1223	20.1697	20.2172	20.2410	20.2646	20.3122
86	20.5986	20.6465	20.6945	20.7187	20.7427	20.7907
87	21.0804	21.1289	21.1775	21.2019	21.2261	21.2747
88	21.5678	21.6169	21.6660	21.6905	21.7150	21.7643
89	22.0608	22.1104	22.1600	22.1849	22.2098	22.2595
90	22.5593	22.6095	22.6597	22.6848	22.7100	22.7603
91	23.0634	23.1141	23.1649	23.1903	23.2157	23.2666
92	23.5731	23.6244	23.6757	23.7014	23.7271	23.7785
93	24.0883	24.1402	24.1920	24.2181	24.2440	24.2960
94	24.6091	24.6615	24.7140	24.7403	24.7665	24.8190
95	25.1355	25.1885	25.2415	25.2681	25.2945	25.3476
96	25.6675	25.7210	25.7745	25.8015	25.8282	25.8818
97	26.2050	26.2591	26.3133	26.3402	26.3674	26.4216
98	26.7481	26.8027	26.8574	26.8847	26.9121	26.9669
99	27.2968	27.3519	27.4072	27.4348	27.4625	27.5178
100	27.8510	27.9067	27.9625	27.9904	28.0184	28.0743
101	28.4108	28.4671	28.5234	28.5517	28.5798	28.6363
102	28.9762	29.0330	29.0899	29.1184	29.1469	29.2039
103	29.5471	29.6045	29.6620	29.6907	29.7195	29.7771
104	30.1236	30.1816	30.2396	30.2686	30.2977	30.3558
105	30.7057	30.7642	30.8228	30.8521	30.8814	30.9401

Areas of Circles in Ale-Gallons.

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Dia.	.5	.6	.7	.75	.8	.9
71	14.2381	14.2780	14.3179	14.3378	14.3579	14.3979
72	14.6392	14.6796	14.7201	14.7403	14.7606	14.8012
73	15.0458	15.0868	15.1278	15.1483	15.1689	15.2100
74	15.4580	15.4993	15.5411	15.5619	15.5827	15.6244
75	15.8758	15.9178	15.9600	15.9810	16.0022	16.0444
76	16.2991	16.3417	16.3844	16.4058	16.4272	16.4700
77	16.7280	16.7712	16.8145	16.8361	16.8578	16.9011
78	17.1625	17.2062	17.2500	17.2719	17.2939	17.3378
79	17.6025	17.6468	17.6912	17.7134	17.7356	17.7801
80	18.0481	18.0930	18.1379	18.1604	18.1829	18.2280
81	18.4993	18.5448	18.5902	18.6130	18.6358	18.6814
82	18.9561	19.0021	19.0481	19.0712	19.0942	19.1403
83	19.4184	19.4650	19.5115	19.5349	19.5582	19.6049
84	19.8863	19.9334	19.9806	20.0043	20.0278	20.0750
85	20.3598	20.4074	20.4551	20.4791	20.5029	20.5507
86	20.8388	20.8870	20.9353	20.9596	20.9836	21.0320
87	21.3234	21.3722	21.4210	21.4456	21.4699	21.5188
88	21.8136	21.8629	21.9123	21.9370	21.9617	22.0112
89	22.3093	22.3592	22.4092	22.4341	22.4592	22.5092
90	22.8107	22.8611	22.9116	22.9369	22.9621	23.0128
91	23.3176	23.3685	23.4196	23.4452	23.4707	23.5219
92	23.8300	23.8816	23.9332	23.9590	23.9848	24.0366
93	24.3480	24.4001	24.4523	24.4785	24.5045	24.5568
94	24.8716	24.9243	24.9770	25.0035	25.0298	25.0826
95	25.4008	25.4540	25.5073	25.5341	25.5606	25.6140
96	25.9355	25.9893	26.0432	26.0702	26.0971	26.1510
97	26.4759	26.5302	26.5846	26.6118	26.6390	26.6935
98	27.0217	27.0766	27.1316	27.1591	27.1866	27.2416
99	27.5732	27.6286	27.6841	27.7119	27.7397	27.7953
100	28.1302	28.1862	28.2423	28.2704	28.2984	28.3546
101	28.6928	28.7494	28.8060	28.8344	28.8627	28.9194
102	29.2610	29.3181	29.3753	29.4038	29.4325	29.4898
103	29.8347	29.8924	29.9501	29.9789	30.0079	30.0657
104	30.4140	30.4722	30.5305	30.5596	30.5889	30.6473
105	30.9989	31.0577	31.1165	31.1459	31.1754	31.2344

Dia.	.9	.1	.2	.25	.3	.4
106	31.2934	31.3525	31.4116	31.4411	31.4708	31.5300
107	31.8866	31.9462	32.0059	32.0357	32.0657	32.1255
108	32.4854	32.5456	32.6058	32.6359	32.6661	32.7265
109	33.0898	33.1505	33.2113	33.2417	33.2722	33.3331
110	33.6997	33.7610	33.8224	33.8530	33.8838	33.9452
111	34.3152	34.3771	34.4390	34.4699	34.5010	34.5630
112	34.9363	34.9987	35.0612	35.0924	35.1237	35.1863
113	35.5629	35.6259	35.6889	35.7204	35.7520	35.8152
114	36.1952	36.2587	36.3222	36.3540	36.3859	36.4496
115	36.8329	36.8970	36.9612	36.9932	37.0254	37.0896
116	37.4763	37.5409	37.6056	37.6380	37.6704	37.7352
117	38.1252	38.1904	38.2557	38.2883	38.3210	38.3864
118	38.7797	38.8455	38.9113	38.9442	38.9772	39.0431
119	39.4398	39.5061	39.5725	39.6056	39.6389	39.7054
120	40.1054	40.1723	40.2392	40.2727	40.3062	40.3733
121	40.7766	40.8441	40.9116	40.9453	40.9791	41.0467
122	41.4534	41.5214	41.5895	41.6234	41.6575	41.7257
123	42.1358	42.2043	42.2729	42.3072	42.3416	42.4103
124	42.8237	42.8928	42.9619	42.9965	43.0312	43.1004
125	43.5172	43.5868	43.6566	43.6914	43.7263	43.7961
126	44.2162	44.2865	44.3567	44.3918	44.4271	44.4974
127	44.9209	44.9916	45.0625	45.0979	45.1336	45.2043
128	45.6311	45.7024	45.7738	45.8095	45.8452	45.9167
129	46.3468	46.4187	46.4907	46.5266	46.5627	46.6347
130	47.0682	47.1406	47.2131	47.2493	47.2857	47.3583
131	47.7952	47.8681	47.9412	47.9777	48.0142	48.0874
132	48.5276	48.6011	48.6747	48.7115	48.7484	48.8221
133	49.2656	49.3397	49.4139	49.4510	49.4881	49.5624
134	50.0093	50.0839	50.1586	50.1960	50.2334	50.3083
135	50.7584	50.8337	50.9090	50.9466	50.9843	51.0597
136	51.5132	51.5890	51.6648	51.7027	51.7407	51.8167
137	52.2735	52.3499	52.4263	52.4644	52.5027	52.5791
138	53.0394	53.1163	53.1933	53.2317	53.2703	53.3474
139	53.8109	53.8884	53.9659	54.0046	54.0434	54.1211
140	54.5880	54.6660	54.7442	54.7830	54.8222	54.9003

Areas of Circles in Ale-Gallons.

167

Dia.	.5	.6	.7	.75	.8	.9
106	31.5893	31.6487	31.7081	31.7377	31.7675	31.8270
107	32.1853	32.2452	32.3052	32.3351	32.3652	32.4253
108	32.7869	32.8474	32.9079	32.9381	32.9685	33.0291
109	33.3940	33.4551	33.5161	33.5467	33.5773	33.6385
110	34.0068	34.0683	34.1300	34.1608	34.1917	34.2534
111	34.6251	34.6872	34.7494	34.7805	34.8116	34.8739
112	35.2489	35.3116	35.3744	35.4057	35.4372	35.5000
113	35.8784	35.9416	36.0049	36.0365	36.0683	36.1317
114	36.5134	36.5772	36.6410	36.6729	36.7049	36.7689
115	37.1539	37.2182	37.2827	37.3149	37.3472	37.4117
116	37.8001	37.8650	37.9300	37.9624	37.9950	38.0601
117	38.4518	38.5173	38.5828	38.6156	38.6484	38.7140
118	39.1091	39.1751	39.2412	39.2742	39.3073	39.3735
119	39.7719	39.8385	39.9052	39.9385	39.9719	40.0386
120	40.4403	40.5075	40.5747	40.6083	40.6420	40.7093
121	41.1143	41.1820	41.2498	41.2837	41.3176	41.3855
122	41.7939	41.8622	41.9305	41.9646	41.9989	42.0673
123	42.4790	42.5479	42.6167	42.6512	42.6857	42.7547
124	43.1697	43.2391	43.3086	43.3433	43.3780	43.4476
125	43.8660	43.9360	44.0059	44.0409	44.0760	44.1461
126	44.5679	44.6384	44.7089	44.7442	44.7795	44.8502
127	45.2755	45.3463	45.4174	45.4530	45.4886	45.5598
128	45.9889	46.0599	46.1315	46.1673	46.2032	46.2750
129	46.7068	46.7790	46.8512	46.8873	46.9235	46.9958
130	47.4309	47.5037	47.5764	47.6128	47.6493	47.7222
131	48.1606	48.2339	48.3073	48.3439	48.3806	48.4541
132	48.8959	48.9697	49.0436	49.0805	49.1176	49.1916
133	49.6367	49.7111	49.7856	49.8228	49.8601	49.9346
134	50.3832	50.4581	50.5331	50.5706	50.6082	50.6833
135	51.1351	51.2106	51.2862	51.3239	51.3618	51.4375
136	51.8922	51.9687	52.0449	52.0829	52.1210	52.1973
137	52.6558	52.7324	52.8091	52.8474	52.8858	52.9626
138	53.4245	53.5017	53.5789	53.6175	53.6562	53.7335
139	54.1987	54.2765	54.3543	54.3931	54.4321	54.5100
140	54.9786	55.0569	55.1352	55.1743	55.2136	55.2922

Dia.	.0	.1	.2	.25	.3	.4
141	55.3706	55.4491	55.5278	55.5670	55.6064	55.6852
142	56.1588	56.2379	56.3171	56.3566	56.3963	56.4756
143	56.9525	57.0322	57.1119	57.1518	57.1917	57.2716
144	57.7518	57.8321	57.9124	57.9525	57.9927	58.0731
145	58.5567	58.6375	58.7184	58.7588	58.7993	58.8802
146	59.3672	59.4485	59.5300	59.5706	59.6114	59.6929
147	60.1832	60.2651	60.3471	60.3881	60.4291	60.5112
148	61.0048	61.0873	61.1698	61.2111	61.2524	61.3350
149	61.8320	61.9150	61.9981	62.0397	62.0812	62.1644
150	62.6648	62.7483	62.8320	62.8738	62.9157	62.9994
151	63.5031	63.5872	63.6714	63.7135	63.7556	63.8400
152	64.3470	64.4316	64.5164	64.5588	64.6012	64.6861
153	65.1964	65.2817	65.3670	65.4097	65.4523	65.5377
154	66.0514	66.1372	66.2231	66.2661	66.3090	66.3950
155	66.9120	66.9984	67.0848	67.1281	67.1713	67.2578
156	67.7782	67.8651	67.9520	67.9956	68.0392	68.1262
157	68.6499	68.7374	68.8249	68.8688	68.9125	69.0002
158	69.5272	69.6153	69.7034	69.7475	69.7915	69.8797
159	70.4101	70.4987	70.5874	70.6318	70.6761	70.7648
160	71.2986	71.3877	71.4769	71.5215	71.5662	71.6555
161	72.1926	72.2823	72.3720	72.4170	72.4619	72.5517
162	73.0922	73.1824	73.2728	73.3179	73.3636	73.4536
163	73.9973	74.0881	74.1790	74.2244	74.2700	74.3609
164	74.9080	74.9994	75.0909	75.1366	75.1824	75.2739
165	75.8243	75.9163	76.0083	76.0542	76.1003	76.1924
166	76.7462	76.8387	76.9313	76.9775	77.0239	77.1165
167	77.6737	77.7667	77.8598	77.9063	77.9530	78.0462
168	78.6067	78.7003	78.7939	78.8407	78.8877	78.9814
169	79.5452	79.6394	79.7336	79.7808	79.8279	79.9222
170	80.4894	80.5841	80.6789	80.7263	80.7737	80.8686
171	81.4391	81.5344	81.6297	81.6774	81.7251	81.8206
172	82.3944	82.4902	82.5861	82.6341	82.6821	82.7781
173	83.3553	83.4517	83.5481	83.5963	83.6446	83.7412
174	84.3217	84.4186	84.5156	84.5641	84.6127	84.7098
175	85.2937	85.3912	85.4888	85.5375	85.5864	85.6840

Areas of Circles in Ale-Gallons. 169

Dia.	.5	.6	.7	.75	.8	.9
141	55.7640	55.8428	55.9217	55.9611	56.0007	56.0797
142	56.5549	56.6343	56.7138	56.7535	56.7933	56.8729
143	57.3515	57.4314	57.5115	57.5514	57.5915	57.6717
144	58.1536	58.2341	58.3147	58.3549	58.3953	58.4760
145	58.9613	59.0423	59.1235	59.1640	59.2047	59.2859
146	59.7745	59.8561	59.9378	59.9786	60.0196	60.1014
147	60.5933	60.6755	60.7578	60.7989	60.8401	60.9224
148	61.4177	61.5005	61.5833	61.6246	61.6661	61.7496
149	62.2477	62.3310	62.4143	62.4560	62.4978	62.5812
150	63.0832	63.1671	63.2510	63.2929	63.3350	63.4190
151	63.9243	64.0087	64.0932	64.1354	64.1777	64.2623
152	64.7710	64.8560	64.9410	64.9835	65.0261	65.1112
153	65.6232	65.7088	65.7943	65.8372	65.8800	65.9657
154	66.4810	66.5671	66.6533	66.6964	66.7395	66.8257
155	67.3444	67.4311	67.5178	67.5612	67.6045	67.6913
156	68.2134	68.3006	68.3878	68.4316	68.4751	68.5625
157	69.0879	69.1756	69.2635	69.3075	69.3513	69.4393
158	69.9680	70.0563	70.1447	70.1890	70.2331	70.3216
159	70.8536	70.9425	71.0314	71.0761	71.1204	71.2095
160	71.7449	71.8343	71.9238	71.9685	72.0133	72.1029
161	72.6417	72.7317	72.8216	72.8667	72.9117	73.0020
162	73.5440	73.6346	73.7252	73.7705	73.8158	73.9066
163	74.4520	74.5431	74.6342	74.6798	74.7255	74.8167
164	75.3655	75.4572	75.5489	75.5947	75.6406	75.7325
165	76.2846	76.3768	76.4691	76.5152	76.5614	76.6538
166	77.2092	77.3020	77.3948	77.4412	77.4877	77.5807
167	78.1395	78.2328	78.3262	78.3728	78.4196	78.5151
168	79.0753	79.1691	79.2631	79.3100	79.3571	79.4511
169	80.0166	80.1111	80.2056	80.2528	80.3001	80.3947
170	80.9636	81.0586	81.1536	81.2011	81.2487	81.3439
171	81.9161	82.0116	82.1072	82.1550	82.2029	82.2986
172	82.8741	82.9702	83.0664	83.1145	83.1626	83.2589
173	83.8378	83.9344	84.0312	84.0795	84.1280	84.2248
174	84.8070	84.9042	85.0014	85.0501	85.0988	85.1962
175	85.7818	85.8796	85.9774	86.0263	86.0753	86.1732

Dia.	.0	.1	.2	.25	.3	.4
176	86.2713	86.3693	86.4674	86.5165	86.5656	86.6638
177	87.2544	87.3530	87.4517	87.5010	87.5504	87.6492
178	88.2431	88.3423	88.4415	88.4911	88.5408	88.6402
179	89.2374	89.3371	89.4369	89.4868	89.5368	89.6367
180	90.2372	90.3375	90.4379	90.4881	90.5383	90.6387
181	91.2427	91.3435	91.4444	91.4949	91.5454	91.6464
182	92.2537	92.3551	92.4565	92.5072	92.5580	92.6596
183	93.2702	93.3722	93.4742	93.5252	93.5763	93.6784
184	94.2923	94.3949	94.4974	94.5487	94.6001	94.7028
185	95.3200	95.4231	95.5263	95.5778	95.6294	95.7327
186	96.3533	96.4570	96.5606	96.6125	96.6644	96.7682
187	97.3923	97.4964	97.6006	97.6528	97.7049	97.8093
188	98.4366	98.5413	98.6461	98.6986	98.7510	98.8559
189	99.4866	99.5919	99.6972	99.7500	99.8026	99.9081
190	100.5421	100.6480	100.7539	100.8069	100.8599	100.9659
191	101.6032	101.7097	101.8161	101.8695	101.9227	102.0292
192	102.6699	102.7769	102.8839	102.9376	102.9910	103.0982
193	103.7422	103.8497	103.9573	104.0112	104.0650	104.1727
194	104.8200	104.9281	105.0363	105.0902	105.1445	105.2527
195	105.9034	106.0121	106.1208	106.1751	106.2295	106.3383
196	106.9924	107.1016	107.2109	107.2655	107.3202	107.4296
197	108.0869	108.1967	108.3065	108.3614	108.4164	108.5263
198	109.1871	109.2974	109.4078	109.4629	109.5182	109.6287
199	110.2927	110.4036	110.5146	110.5700	110.6255	110.7366
200	111.4040	111.5154	111.6269	111.6827	111.7385	111.8501
201	112.5208	112.6328	112.7449	112.8009	112.8570	112.9691
202	113.6432	113.7558	113.8684	113.9247	113.9810	114.0937
203	114.7712	114.8843	114.9974	115.0540	115.1107	115.2239
204	115.9047	116.0184	116.1321	116.1890	116.2459	116.3597
205	117.0438	117.1580	117.2723	117.3295	117.3866	117.5010
206	118.1885	118.3033	118.4181	118.4755	118.5330	118.6480
207	119.3387	119.4541	119.5695	119.6272	119.6849	119.8005
208	120.4946	120.6105	120.7264	120.7844	120.8424	120.9584
209	121.6560	121.7724	121.8889	121.9472	122.0055	122.1221
210	122.8230	122.9399	123.0570	123.1155	123.1741	123.2913

Areas of Circles in Ale-Gallons.

171

Dia.	.5	.6	.7	.75	.8	.9
176	86.7621	86.8605	86.9589	87.0081	87.0573	87.1558
177	87.7481	87.8470	87.9459	87.9954	88.0449	88.1440
178	88.7396	88.8390	88.9385	88.9883	89.0381	89.1377
179	89.7366	89.8366	89.9367	89.9867	90.0368	90.1370
180	90.7393	90.8398	90.9404	90.9908	91.0411	91.1419
181	91.7475	91.8486	91.9498	92.0004	92.0510	92.1523
182	92.7612	92.8629	92.9647	93.0155	93.0665	93.1683
183	93.7806	93.8828	93.9851	94.0363	94.0875	94.1899
184	94.8055	94.9083	95.0112	95.0626	95.1141	95.2170
185	95.8360	95.9393	96.0428	96.0945	96.1462	96.2497
186	96.8720	96.9760	97.0799	97.1319	97.1839	97.2880
187	97.9137	98.0181	98.1227	98.1750	98.2272	98.3319
188	98.9609	99.0659	99.1710	99.2236	99.2761	99.3813
189	100.0136	100.1193	100.2249	100.2778	100.3306	100.4363
190	101.0720	101.1781	101.2843	101.3375	101.3906	101.4969
191	102.1359	102.2426	102.3493	102.4028	102.4561	102.5630
192	103.2054	103.3126	103.4199	103.4737	103.5273	103.6347
193	104.2804	104.3882	104.4961	104.5500	104.6040	104.7120
194	105.3610	105.4693	105.5778	105.6320	105.6863	105.7948
195	106.4472	106.5561	106.6651	106.7196	106.7742	106.8833
196	107.5390	107.6485	107.7580	107.8128	107.8676	107.9772
197	108.6363	108.7463	108.8564	108.9115	108.9666	109.0768
198	109.7392	109.8498	109.9605	110.0158	110.0712	110.1819
199	110.8477	110.9588	111.0700	111.1257	111.1813	111.2926
200	111.9617	112.0734	112.1852	112.2411	112.2970	112.4089
201	113.0813	113.1937	113.3059	113.3621	113.4184	113.5307
202	114.2065	114.3193	114.4322	114.4887	114.5451	114.6581
203	115.3373	115.4506	115.5641	115.6208	115.6776	115.7911
204	116.4736	116.5876	116.7015	116.7585	116.8156	116.9297
205	117.6155	117.7300	117.8445	117.9018	117.9591	118.0738
206	118.7630	118.8780	118.9931	119.0507	119.1083	119.2235
207	119.9160	120.0316	120.1472	120.2051	120.2630	120.3787
208	121.0746	121.1907	121.3070	121.3651	121.4232	121.5396
209	122.2387	122.3555	122.4722	122.5306	122.5891	122.7060
210	123.4085	123.5258	123.6431	123.7018	123.7605	123.8779

Probl. 62. FIG. 18.

Given the Area of a Circle; to find the Diameter AB.

Theor.

As $\left\{ \begin{array}{l} 1 \\ 1 \\ 1 \end{array} \right\}$ to $\left\{ \begin{array}{l} 1.273239 \\ 359.0536 \\ 294.1182 \end{array} \right\}$ so is the $\left\{ \begin{array}{l} \text{Inches} \\ \text{Ale-G.} \\ \text{Wine-G.} \end{array} \right\}$ to the Square of the Diameter.

Note, that these Multipliers are the Squares of the Diameters of a Circle whose Area is Unity :

Viz. $\left\{ \begin{array}{l} 1.128379 \\ 18.9487 \\ 17.1499. \end{array} \right\}$ for $\left\{ \begin{array}{l} \text{Inches} \\ \text{Ale-Gallons.} \\ \text{Wine-Gallons.} \end{array} \right\}$

The Proportions in whole Numbers are—

As $\left\{ \begin{array}{l} 11 \\ 223 \\ 355 \end{array} \right\}$ to $\left\{ \begin{array}{l} 14 \\ 284 \\ 452 \end{array} \right\}$ so is the Area in Inches, to the Square of any Diameter.

**A
CYCLOMETRICAL
TABLE**

**Exhibiting the
AREAS of CIRCLES
IN**

Wine-Gallons

**AND
DECIMILLESSIMAL PARTS,**

*Calculated to every Tenth part, and Quar-
ter of an Inch of the DIAMETER
from 1 to 60 Inches.*

Dia.	1	2	3	4	5	6
1	0.0034	0.0041	0.0049	0.0053	0.0057	0.0067
2	0.0136	0.0149	0.0165	0.0173	0.0180	0.0196
3	0.0306	0.0326	0.0347	0.0359	0.0369	0.0393
4	0.0544	0.0572	0.0600	0.0615	0.0629	0.0659
5	0.0880	0.0885	0.0919	0.0932	0.0955	0.0991
6	0.1224	0.1265	0.1307	0.1328	0.1350	0.1393
7	0.1666	0.1714	0.1763	0.1787	0.1812	0.1862
8	0.2176	0.2231	0.2287	0.2314	0.2343	0.2400
9	0.2754	0.2815	0.2877	0.2910	0.2944	0.3004
10	0.3400	0.3468	0.3537	0.3572	0.3607	0.3678
11	0.4114	0.4189	0.4265	0.4303	0.4341	0.4418
12	0.4896	0.4998	0.5068	0.5102	0.5144	0.5228
13	0.5746	0.5845	0.5925	0.5970	0.6015	0.6106
14	0.6364	0.6470	0.6556	0.6608	0.6653	0.6751
15	0.7050	0.7142	0.7234	0.7297	0.7359	0.7464
16	0.8704	0.8813	0.8923	0.8978	0.9034	0.9145
17	0.9826	0.9942	1.0059	1.0117	1.0176	1.0294
18	1.1016	1.1139	1.1263	1.1325	1.1386	1.1512
19	1.2574	1.2404	1.2535	1.2600	1.2665	1.2797
20	1.3600	1.3736	1.3873	1.3942	1.4011	1.4149
21	1.5095	1.5237	1.5381	1.5453	1.5525	1.5570
22	1.6456	1.6606	1.6757	1.6832	1.6908	1.7060
23	1.7986	1.8142	1.8299	1.8379	1.8457	1.8617
24	1.9584	1.9747	1.9911	1.9994	2.0076	2.0242
25	2.1250	2.1420	2.1591	2.1677	2.1763	2.1936
26	2.2984	2.3161	2.3339	2.3428	2.3517	2.3697
27	2.4786	2.4970	2.5155	2.5247	2.5340	2.5526
28	2.6656	2.6847	2.7038	2.7134	2.7230	2.7423
29	2.8594	2.8791	2.8989	2.9089	2.9188	2.9388
30	3.0600	3.0804	3.1009	3.1112	3.1215	3.1422

Area of Circles in Wine-Gallons.

175

Dia.	.5	.6	.7	.75	.8	.9
1	0.0077	0.0087	0.0098	0.0104	0.0110	0.0118
2	0.0213	0.0230	0.0248	0.0258	0.0267	0.0286
3	0.0416	0.0440	0.0465	0.0478	0.0491	0.0517
4	0.0689	0.0710	0.0731	0.0747	0.0763	0.0786
5	0.1029	0.1057	0.1086	0.1114	0.1144	0.1184
6	0.1437	0.1482	0.1527	0.1549	0.1573	0.1619
7	0.1913	0.1964	0.2016	0.2042	0.2069	0.2112
8	0.2457	0.2515	0.2574	0.2603	0.2633	0.2693
9	0.3069	0.3134	0.3199	0.3233	0.3266	0.3328
10	0.3749	0.3821	0.3893	0.3929	0.3966	0.4049
11	0.4497	0.4576	0.4655	0.4694	0.4735	0.4835
12	0.5313	0.5398	0.5484	0.5537	0.5571	0.5658
13	0.6197	0.6289	0.6382	0.6428	0.6475	0.6569
14	0.7149	0.7248	0.7348	0.7398	0.7448	0.7549
15	0.8169	0.8275	0.8381	0.8434	0.8488	0.8596
16	0.9257	0.9369	0.9482	0.9539	0.9596	0.9711
17	1.0413	1.0532	1.0652	1.0712	1.0773	1.0894
18	1.1637	1.1763	1.1890	1.1953	1.2017	1.2145
19	1.2929	1.3062	1.3195	1.3262	1.3329	1.3464
20	1.4289	1.4428	1.4569	1.4639	1.4710	1.4842
21	1.5716	1.5862	1.6009	1.6084	1.6157	1.6306
22	1.7213	1.7366	1.7520	1.7597	1.7675	1.7829
23	1.8777	1.8937	1.9098	1.9179	1.9259	1.9421
24	2.0409	2.0576	2.0744	2.0828	2.0912	2.1081
25	2.2109	2.2283	2.2457	2.2544	2.2632	2.2808
26	2.3877	2.4057	2.4238	2.4329	2.4420	2.4603
27	2.5713	2.5900	2.6088	2.6182	2.6277	2.6466
28	2.7617	2.7811	2.8006	2.8104	2.8201	2.8397
29	2.9589	2.9789	2.9992	3.0093	3.0194	3.0396
30	3.1629	3.1837	3.2045	3.2149	3.2254	3.2464

Dia.	.0	.1	.2	.25	.3	.4
31	3.2674	3.2885	3.3097	3.3203	3.3309	3.3512
32	3.4816	3.5034	3.5253	3.5362	3.5472	3.5692
33	3.7026	3.7251	3.7476	3.7589	3.7702	3.7929
34	3.9304	3.9536	3.9768	3.9885	4.0001	4.0235
35	4.1650	4.1888	4.2127	4.2247	4.2367	4.2608
36	4.4064	4.4309	4.4555	4.4679	4.4802	4.5949
37	4.6546	4.6798	4.7051	4.7178	4.7304	4.7558
38	4.9096	4.9355	4.9614	4.9744	4.9874	5.0135
39	5.1714	5.1980	5.2246	5.2380	5.2513	5.2781
40	5.4400	5.4673	5.4945	5.5083	5.5220	5.5494
41	5.7154	5.7434	5.7713	5.7854	5.7994	5.8275
42	5.9976	6.0262	6.0549	6.0692	6.0836	6.1124
43	6.2866	6.3159	6.3452	6.3599	6.3746	6.4041
44	6.5824	6.6124	6.6424	6.6574	6.6725	6.7027
45	6.8850	6.9156	6.9463	6.9617	6.9771	7.0079
46	7.1944	7.2257	7.2571	7.2728	7.2886	7.3201
47	7.5106	7.5426	7.5747	7.5908	7.6068	7.6390
48	7.8336	7.8663	7.8990	7.9154	7.9318	7.9647
49	8.1634	8.1968	8.2302	8.2469	8.2637	8.2973
50	8.5000	8.5340	8.5681	8.5852	8.6023	8.6366
51	8.8433	8.8781	8.9129	8.9304	8.9478	8.9827
52	9.1936	9.2290	9.2645	9.2823	9.3000	9.3356
53	9.5506	9.5867	9.6228	9.6409	9.6590	9.6953
54	9.9144	9.9512	9.9880	10.0065	10.0249	10.0619
55	10.2850	10.3224	10.3599	10.3787	10.3975	10.4352
56	10.6624	10.7005	10.7387	10.7579	10.7770	10.8153
57	11.0466	11.0854	11.1243	11.1437	11.1632	11.2022
58	11.4376	11.4771	11.5166	11.5364	11.5562	11.5959
59	11.8354	11.8755	11.9157	11.9359	11.9560	11.9964
60	12.2400	12.2808	12.3217	12.3422	12.3627	12.4038

Areas of Circles in Wine-Gallons.

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Dia.	.5	.6	.7	.75	.8	.9
31	3.3736	3.3951	3.4166	3.4274	3.4382	3.4599
32	3.5913	3.6134	3.6356	3.6467	3.6578	3.6802
33	3.8157	3.8385	3.8614	3.8729	3.8843	3.9073
34	4.0469	4.0704	4.0940	4.1058	4.1176	4.1413
35	4.2849	4.3091	4.3333	4.3454	4.3576	4.3820
36	4.5297	4.5545	4.5794	4.5919	4.6044	4.6295
37	4.7813	4.8068	4.8324	4.8452	4.8581	4.8838
38	5.0397	5.0659	5.0922	5.1054	5.1185	5.1449
39	5.3049	5.3318	5.3588	5.3723	5.3858	5.4128
40	5.5769	5.6044	5.6320	5.6459	5.6597	5.6875
41	5.8557	5.8840	5.9123	5.9265	5.9407	5.9691
42	6.1413	6.1702	6.1992	6.2138	6.2283	6.2574
43	6.4337	6.4633	6.4930	6.5079	6.5227	6.5525
44	6.7329	6.7632	6.7935	6.8089	6.8239	6.8544
45	7.0389	7.0698	7.1009	7.1165	7.1320	7.1632
46	7.3517	7.3834	7.4151	7.4310	7.4469	7.4787
47	7.6713	7.7036	7.7360	7.7522	7.7685	7.8010
48	7.9977	8.0307	8.0638	8.0804	8.0965	8.1301
49	8.3309	8.3649	8.3984	8.4153	8.4322	8.4661
50	8.6709	8.7053	8.7397	8.7570	8.7742	8.8088
51	9.0177	9.0527	9.0879	9.1055	9.1231	9.1583
52	9.3713	9.4070	9.4428	9.4607	9.4787	9.5146
53	9.7317	9.7681	9.8046	9.8229	9.8411	9.8777
54	10.0989	10.1360	10.1731	10.1917	10.2103	10.2476
55	10.4249	10.5107	10.5485	10.5674	10.5864	10.6244
56	10.8537	10.8922	10.9307	10.9500	10.9693	11.0079
57	11.2413	11.2804	11.3196	11.3393	11.3589	11.3982
58	11.6357	11.6755	11.7154	11.7354	11.7553	11.7953
59	12.0369	12.0774	12.1180	12.1383	12.1586	12.1993
60	12.4449	12.4861	12.5273	12.5480	12.5686	12.6100

 Probl. 63. FIG. 18.

Given ADBK the Periphery of a Circle; to find the Area.

Theor.

Multiply the Semi-periphery by the Semi-diameter the Product is the Area; thus when the Periphery is Unity the Diameter will be .3183099 whose half multiplied by .5 the Semi-periphery, the Product .079577475 will be the Area in Inches; which being divided severally by 282, and 231 the Quotients are Areas in Ale, and Wine Gallons.

And the Areas of all Circles being in Proportion one to another as the Square of their Peripheries, it will hold (for 1 doth not multiply) —

As $\left\{ \begin{array}{l} 1 \\ 1 \\ 1 \end{array} \right\}$ to $\left\{ \begin{array}{l} .0795775 \\ .000282197 \\ .00034449 \end{array} \right\}$ So is the Square of any Periphery to its Area in $\left\{ \begin{array}{l} \text{Inches} \\ \text{Ale G.} \\ \text{Wine G.} \end{array} \right\}$

The Proportions in whole Numbers are —

As $\left\{ \begin{array}{l} 88 \\ 892 \\ 1420 \end{array} \right\}$ to $\left\{ \begin{array}{l} 7 \\ 71 \\ 113 \end{array} \right\}$ So is the Square of any Periphery, to its Area in Inches.

 Probl. 64. FIG. 18.

Given the Area of a Circle; to find the Periphery ADBK.

Theor.

Of a Circle and regular Polygons. 179

Theor.

As $\left\{ \begin{array}{l} 1 \\ 1 \\ 1 \end{array} \right\}$ to $\left\{ \begin{array}{l} 12.56637 \\ 3543.71634 \\ 2902.83147 \end{array} \right\}$ So is $\left\{ \begin{array}{l} \text{Inches} \\ \text{Ale-Gal.} \\ \text{Wine-G.} \end{array} \right\}$ To the Square of the Periphery.

Note, that these *Multiplicators* are the *Squares* of the *Peripheries* of a Circle whose *Area* is *Unity*:

Viz. $\left\{ \begin{array}{l} 3.544907 \\ 59.5291 \\ 53.878 \end{array} \right\}$ for $\left\{ \begin{array}{l} \text{Inches} \\ \text{Ale-Gallons} \\ \text{Wine Gallons.} \end{array} \right\}$

The Proportions in whole Numbers are——

As $\left\{ \begin{array}{l} 7 \\ 71 \\ 113 \end{array} \right\}$ to $\left\{ \begin{array}{l} 88 \\ 892 \\ 1420 \end{array} \right\}$ So is the Area in Inches, to the Square of the Periphery.

Probl. 65.

Given the Diameter of a Circle; to find the Side of any regular Polygon Equal to, Inscrib'd in, or Conscrib'd about the same Circle.

Theor.

As 1 is to the proper *Multiplicator*;
So is the *Diameter*, to the *Side*.

180 Of a Circle and regular Polygons.

<i>Polygons.</i>	<i>Sides of the several Polygons equal to a Circle whose Diameter is 1.</i>	<i>Sides of the several Polygons Inscrib'd in a Circle whose Diameter is 1.</i>	<i>Sides of the several Polygons conscrib'd about a Circle whose Diameter is 1.</i>	<i>Polygons.</i>
III	1.346774	.866025	1.732051	III
IV	.886227	.707107	1.000000	IV
V	.675648	.587785	.726543	V
VI	.549818	.500000	.577350	VI
VII	.464897	.433882	.481572	VII
VIII	.403313	.382683	.414214	VIII
IX	.356439	.342020	.363970	IX
X	.319494	.309017	.324920	X
XI	.289585	.281732	.293626	XI
XII	.264856	.258819	.267949	XII

Note, that these *Multiplicators* may be found by multiplying the *Squares* of the *Sides* of the *several Polygons* by .785398, and extracting the *Square Roots* of the *several Products* : for —

As the *Area* of any given *Polygon*,
is to the *Square* of its *Side* ;
So is the *Area* of any other *Polygon*,
to the *Square* of its *Side*.

Or if you multiply the *Sides* of the *several Polygons* in *Theor. 69* by .886227 the *Products* are the *Numbers* in this *Tablet*.

Probl.

Of a Circle and regular Polygons. 181

Probl. 66.

Given the Side of any regular Polygon Equal to, In-
scrib'd in, or Conscrib'd about a Circle; to find the Diamo-
ter of the same Circle.

Theor.

As 1, is to the proper Multiplier;
So is the Side, to the Diameter.

Polygons.	Diameter of the Circle when the Side of the Polygon Equal is 1.	Diameter of the Circle when the Side of the Polygon Inscrib'd is 1.	Diameter of the Circle when the Side of the Polygon Conscrib'd is 1.	Polygons.
III	.742515	1.154701	.577350	III
IV	1.128379	1.414214	1.000000	IV
V	1.480061	1.701302	1.376382	V
VI	1.818784	2.000000	1.732051	VI
VII	2.151014	2.304775	2.076533	VII
VIII	2.479464	2.613126	2.414214	VIII
IX	2.805529	2.923804	2.747477	IX
X	3.129946	3.236068	3.077684	X
XI	3.453211	3.549471	3.404693	XI
XII	3.775637	3.863763	3.732051	XII

Note, that these Multipliers are found by multiply-
ing the Areas of the several Polygons in Theor. 70. by
1.273239, and extracting all the Square Roots.

Or if you multiply the Arithmetical Complements of the
Sides of the several Polygons in Theor. 69. by 1.128379
the Products are the Numbers in this Tablet,

182 Of a Circle and regular Polygons.

Probl. 67.

Given the Periphery of a Circle; to find the Side of any regular Polygon Equal to, Inscrib'd in, or Conscrib'd about the same Circle.

Theor.

As 1, is to the proper Multiplier;
So is the Periphery, to the Side.

Polygons.	Sides of the several Polygons Equal to a Circle whose Periphery is 1.	Sides of the several Polygons Inscrib'd in a Circle whose Periphery is 1.	Sides of the several Polygons conscrib'd about a Circle whose Periphery is 1.	Polygons.
III	.428691.	.275664	.551329	III
IV	.282095.	.225079	.318310	IV
V	.215065.	.187097	.231266	V
VI	.175012.	.159155	.183776	VI
VII	.147981.	.138109	.153289	VII
VIII	.128378.	.121812	.131848	VIII
IX	.113458.	.108868	.115855	IX
X	.101698.	.098263	.103425	X
XI	.092178.	.089678	.093465	XI
XII	.084306.	.082385	.085291	XII

Note, that these Multipliers are found by dividing those in Theor. 65. severally by 3.14159265.

Probl.

Of a Circle and regular Polygons. 183.

Probl. 68.

Given the Side of any regular Polygon Equal to, In-
scrib'd in, or Conscrib'd about a Circle; to find the Periphe-
ry of the same Circle.

Theor.

As 1, is to the proper *Multiplicator*;
So is the Side, to the Periphery.

Polygons.	Periphery when the Side of the Polygon Equal is Unity.	Periphery when the Side of the Polygon Inscri- bed is Unity.	Periphery when the Side of the Polygon Conscri- bed is Unity.	Polygons.
III	2.332683	3.627605	1.813799	III
IV	3.544907	4.442884	3.141593	IV
V	4.649757	5.344821	4.324025	V
VI	5.713894	6.283183	5.441407	VI
VII	6.757624	7.240658	6.523625	VII
VIII	7.789497	8.209372	7.584491	VIII
IX	8.813834	9.185435	8.631479	IX
X	9.833035	10.166424	9.668842	X
XI	10.848576	11.151007	10.699192	XI
XII	11.846155	12.138132	11.724567	XII

Note, that these *Multiplicators* are the Arithmetical
Complements of those in Theor. 67.

184 Of a Circle and regular Polygons.

Probl. 69.

Given the Area of a Circle; to find the Side of any regular Polygon Equal to, Inscrib'd in, or Conscrib'd about the same Circle.

Theor.

As 1, is to the proper Multiplier;
So is the Area to the Square of the Side.

Polygons.	Square of the Sides of the several Polygons Equal to a Circle whose Area is 1 Inch.	Square of the Sides of the several Polygons Inscrib'd in a Circle whose Area is 1 Inch.	Square of the Sides of the several Polygons Conscrib'd about a Circle whose Area is 1 Inch.	Polygons.
III	2.309401	.954930	3.819724	III
IV	1.000000	.636620	1.273239	IV
V	.581234	.439893	.672095	V
VI	.384900	.318310	.424413	VI
VII	.275184	.239692	.295279	VII
VIII	.207107	.186461	.218453	VIII
IX	.161764	.148941	.168671	IX
X	.129968	.121583	.134419	X
XI	.106773	.101060	.109773	XI
XII	.089316	.085291	.091414	XII

Note, that these Multipliers are found by dividing the Squares of the several Numbers in Theor. 65. by .785398.

And

Of a Circle and regular Polygons. 185

And if you multiply these several *Multiplicators* by 282, the Products are the *Numbers* in the next *Tables*.

Polygons.	Square of the Sides of the several Polygons equal to a Circle whose Area is 1 Ale-Gallon.	Square of the Sides of the several Polygons Inscrib'd in a Circle whose Area is 1 Ale-G.	Square of the Sides of the several Polygons Conscrib'd about a Circle whose Area is 1 Ale-G.	Polygons.
III	651.251	269.290	1077.161	III
IV	282.000	179.527	359.054	IV
V	163.908	124.050	189.531	V
VI	108.542	89.763	119.684	VI
VII	77.602	67.593	83.269	VII
VIII	58.404	52.582	61.604	VIII
IX	45.617	42.001	47.565	IX
X	36.651	34.286	37.906	X
XI	30.110	28.499	30.956	XI
XII	25.187	24.052	25.779	XII

Polygons.	Sides of the several Polygons Equal to a Circle whose Area is 1 Inch.	Sides of the several Polygons Inscrib'd in a Circle whose Area is 1 Inch.	Sides of the several Polygons Conscrib'd about a Circle whose Area is 1 Inch.	Polygons.
III	1.519704	.977205	1.954410	III
IV	1.000000	.797885	1.128379	IV
V	.762387	.663244	.819815	V
VI	.620403	.564189	.681470	VI
VII	.524580	.489583	.543396	VII
VIII	.455090	.431811	.467389	VIII
IX	.402199	.385928	.410696	IX
X	.360511	.348688	.366632	X
XI	.326761	.317900	.331320	XI
XII	.298858	.292046	.302348	XII

Note,

186 Of a Circle and regular Polygons.

Note, that the *Multiplicators* in the last *Tablet* are found by multiplying the several *Numbers* in *Theor. 65.* by 1.128379.

Probl. 70.

Given the *Side* of any regular *Polygon* Equal to, *Inscrib'd* in, or *Conscrib'd* about a *Circle*; to find the *Area* of the same *Circle*.

Theor.

As 1, is to the proper *Multiplicator*;
So is the *Square* of the *Side*, to the *Area*.

<i>Polygons.</i>	<i>Area of the Circle in Inches when the Side of the Polygon Equal is 1.</i>	<i>Area of the Circle in Inches when the Side of the Polygon Inscrib'd is 1.</i>	<i>Area of the Circle in Inches when the Side of the Polygon Conscrib'd is 1.</i>	<i>Polygons.</i>
III	0.433013	1.047198	0.261799	III
IV	1.000000	1.570796	0.785398	IV
V	1.720478	2.273280	1.487885	V
VI	2.598075	3.141593	2.356195	VI
VII	3.633931	4.172022	3.386628	VII
VIII	4.828428	5.363052	4.577644	VIII
IX	6.181825	6.714068	5.928701	IX
X	7.694210	8.224834	7.439424	X
XI	9.365659	9.895112	9.103708	XI
XII	11.196130	11.724567	10.939243	XII

Polygons.

Of a Circle and regular Polygons. 187

<i>Polygons.</i>	<i>Area of the Circle in Ale-g. when the Side of the Polygon Equal is 1.</i>	<i>Area of the Circle in Ale-g. when the Side of the Polygon Inscrib'd is 1.</i>	<i>Area of the Circle in Ale-g. when the Side of the Polygon Conscrib'd is 1.</i>	<i>Polygons.</i>
III	.0015355	.0037135	.0009284	III
IV	.0035461	.0055702	.0027851	IV
V	.0061010	.0080613	.0061762	V
IV	.0092130	.0111404	.0083553	VI
VII	.0128863	.0147944	.0120093	VII
VIII	.0171221	.0190179	.0162328	VIII
IX	.0219214	.0238088	.0210238	IX
X	.0272844	.0291661	.0263809	X
XI	.0332115	.0350891	.0323039	XI
XII	.0397027	.0415765	.0387916	XII

*Note, that the Multipliers for Inches are found by multiplying the Squares of the several Numbers in Theor. 66. by .785398, and they being ~~multiplied~~ *Divided* severally by 282, produce the Numbers in this Tablet.*

Probl. 71.

Given the Side of any regular Polygon Equal to a Circle; to find the Side Inscrib'd in, or Conscrib'd about the same Circle.

Theor.

*As 1, is to the proper Multiplier;
So is the Side Equal, to the Side required.*

Polygons.

188 Of a Circle and regular Polygons.

<i>Polygons.</i>	<i>Side of a Polygon Inscrib'd in a Circle the Side Equal being 1.</i>	<i>Side of a Polygon Circscrib'd about a Circle the Side Equal being 1.</i>	<i>Polygons.</i>
III	.643037	1.278649	III
IV	.797885	1.128379	IV
V	.869957	1.075326	V
VI	.909392	1.050075	VI
VII	.933286	1.035868	VII
VIII	.948849	1.027029	VIII
IX	.959547	1.021126	IX
X	.967213	1.016983	X
XI	.972882	1.013954	XI
XII	.977206	1.011678	XII

Note, that these Multipliers are found by dividing the Sides of the several Polygons Inscrib'd in, or Circscrib'd about a Circle whose Diameter is Unity, by the Sides of the several Polygons Equal to a Circle whose Diameter is Unity.

Probl. 72.

Given the Side of any regular Polygon Inscrib'd in a Circle; to find the Side Equal to, or Circscrib'd about the same Circle.

Theor.

*As 1, is to the proper Multiplier;
So is the Side Inscrib'd, to the Side required.*

Polygons.

<i>Polygons.</i>	<i>Side of a Polygon Equal to a Circle the Side Inscrib'd being 1.</i>	<i>Side of a Polygon Circscrib'd about a Circle the Side Inscrib'd being 1.</i>	<i>Polygons.</i>
III	1.555121	2.000000	III
IV	1.253314	1.414214	IV
V	1.149481	1.236068	V
VI	1.099626	1.154701	VI
VII	1.071480	1.109985	VII
VIII	1.053909	1.082392	VIII
IX	1.042158	1.064178	IX
X	1.033904	1.051462	X
XI	1.027874	1.042217	XI
XII	1.023325	1.032576	XII

Note, that these *Multiplicators* are found by dividing the *Sides* of the several *Polygons* Equal to, or *Circscrib'd* about a Circle whose Diameter is Unity by the *Sides* of the several *Polygons* *Inscrib'd* in a Circle whose Diameter is Unity.

Probl. 73.

Given the *Side* of any regular Polygon *Circscrib'd* about a Circle; to find the *Side* Equal to, or *Inscrib'd* in the same Circle.

Theor.

As 1, is to the proper *Multiplicator*;
So is the *Side* *Circscrib'd*, to the *Side* required.

Polygons.

190 Of a Circle and regular Polygons.

<i>Polygons.</i>	<i>Side of a Polygon Equal to a Circle, the Side Conscrib'd being 1.</i>	<i>Side of a Polygon Inscrib'd in a Cir- cle, the Side Con- scrib'd being 1.</i>	<i>Polygons.</i>
III	.5773560	.5000000	III
IV	.8660254	.7071067	IV
V	.9299499	.8090170	V
VI	.9523133	.8660254	VI
VII	.9653774	.9009744	VII
VIII	.9736821	.9238795	VIII
IX	.9793080	.9396926	IX
X	.9833000	.9510565	X
XI	.9862380	.9594930	XI
XII	.9884570	.9659260	XII

Note, that these *Multiplicators* are found by dividing the *Sides* of the several *Polygons* Equal to, or *Inscrib'd* in a Circle whose Diameter is Unity, by the *Sides* of the several *Polygons* Conscrib'd about a Circle whose Diameter is Unity.

Probl. 74.

Given the *Side* of any regular Polygon, and the *Area* ;
to find the *Radius* of the Circle *Inscrib'd*.

Theor.

Divide the *Area* by the *Semi-perimeter* (that is, half the *Sum* of all the *Sides*) the *Quotient* is the *Radius*.

Or

Of a Circle and regular Polygons. 191

Or,

As 1 is to the proper *Multiplicator*;
So is the *Side*, to the *Radius*.

Which multiplied by the *Semi-perimeter* the *Product* will be the *Area* in *Inches*.

Polygons.	Radius when the Side of the Polygon is Unity.
III	0.288675
IV	0.500000
V	0.688191
VI	0.866025
VII	1.038266
VIII	1.207107
IX	1.37739
X	1.538842
XI	1.702847
XII	1.866025

Probl. 75.

Given the *Side* of any regular *Polygon* to augment or diminish it according to any given *Proportion*.

Theor.

Square the given *Side*, and Multiply, or Divide it by the given *Proportion*, the *Square Root* of the *Product*, or *Quotient* is the *Side*.

Here *Note*, the excellency of the foregoing *Stationary Numbers* in that the first *Term* of the *Proportion* is always an *Unit*, whereby the *Theorem* is exceedingly facilitated, and therefore in the *Calculation* of them great care hath been used, as you may prove at your leisure, the *Rise* and *Fabrick* of every single *Multiplicator* and *Divisor* throughout the whole *Book* being plainly laid down: And if *Tables* were made of them to every *Digit*, then any of the laid *Theorems* might be wrought by *Addition* only.

Probl.

Probl. 76. FIG. 20.

Given EF the Chord of a Circle, and DG the Versed Sine of its Segment; to find the Diameter DH.

Theor.

Divide the Square of EG the Semi-chord by DG the Quotient is GH the Remainder of the Diameter, to which add DG, the Sum is DH the Diameter required.

Or to the Square of EG add the Square of DG and divide the Sum by DG, the Quotient is DH.

Probl. 77. FIG. 20.

Given DH the Diameter of a Circle, and DG the Versed Sine of its Segment; to find EF the Chord line.

Theor.

From the Square of $EC=DC$ the Radius, subtract the Square of GC , the Square root of the Remainder is EG the Semi-chord.

Or a Geometrical mean between GH, and DG is $=EG$.

Probl. 78. FIG. 20.

Given DH the Diameter of a Circle, and EF the Chord of its Segment; to find DG the Versed Sine.

Theor.

Theor.

From the Square of $EC=DC$ subduct the Square of EG , the Square Root of the Remainder is GC , which subducted from DC , the Remainder is DG the Versed Sine sought.

Note, that by these three last Theorems you may find the Axe of a Sphere, as also the Diameter at Base, and Altitude of any Frustrum.

Probl. 79. FIG. 20.

Given DH the Diameter of a Circle, and ED the Chord of the half Segments Arch; to find DG the Versed Sine.

Theor.

Divide the Square of ED by DH , the Quotient is DG .

Probl. 80. FIG. 20.

Given DH the Diameter of a Circle, and DG the Versed Sine; to find ED the Chord of the half Segments Arch.

Theor.

A mean Proportional between DH and DG is ED .

Probl. 81. FIG. 20.

Given DH the Diameter of a Circle, and Arch-line of any Portion which has an Angle at the Center; to find the Area.

O

Theor.

Theor.

Multiply half the Arch-line by the Semi-diameter, the Product is the Area.

The reason is evident from the *Archimedean Demonstration*, because the Rectangle of half the Periphery, and Radius gives the Area: And the Arch hath the same Proportion to the Periphery, as the Sector to the Circle.

Probl. 82. FIG. 20.

Given DH the Diameter of a Circle, and DG the Versed Sine; to find the Length of the Arch-line, and Area of the Segment.

Theor.

From 8 times ED the Chord of the half Segments Arch, subduct EF the Chord of the whole Segment, and divide the Remainder by 3, the Quotient is the Length of the Arch-line EDF; then find the Area of the Sector EDFC, and the Area of the Triangle ECF which subducted from the Sector leaves the Segment EDFG.

§. Or by Trigonometry.

1.

As the Log. of EC=DC, is to the Radius;
So is the Log. of EG, to the Sine of the Semi-angle ECG.

2.

As 360 degr. the whole Periphery,
is to the Angle ECF in Degr. and Dec. minutes;
So is the given Periphery ADBH,
to the length of the Arch-line EDF.

And

Of the Segment of a Circle; 193

And so is the Area of the Circle, to the Area of the Sector.

And the Segment is \equiv Sector, less the Triangle.

Note, if you work Trigonometrically for the Segments in the Table, then you must multiply the given Versed Sine by 1.128379.

§. Or by the Table of Segments of a Circle.

As DH the Diameter, is to 1000;

So is DG the Versed Sine, to a fourth Proportional: Which found in the Table under V. S. against it stands a Segment by which if you multiply the Area of the whole Circle, the Product will be the Area of the Segment.

Applicable for finding the Vacuity of a Cylindrical Case part empty; the Ax lying parallel to the Horizon.

The Construction of this Table is comprehended in the three following Precepts, and is the best way of making it I have yet seen, being the same Table published by Adrian Metius to three places of Decimals in his *Pract. Geom. Part.2. Chap.7. pag.216.* which he saith was first calculated by Sybrand Hans; his words in p. 211. run thus:

Generalem & satis exactam mensurandi pragmatiam beneficio Tabellæ, quæ ad omnis generis Dolia se extendit, brevissimo scripto divulgavit Magister Sybrandus Hansonus, Harlinganus, Ludi Magister & Geometra insignis apud Amstelodamenses.

1. Precept.

To the Logar. of the Complement of the Versed Sine DG to the Semi-diameter DC 500, 5000, &c. add this Logar. 9. 301030. (which is the Arithm. Compl. of the supposed Semi-diameter 500, &c.) the Sum is the Co-sine of ECG the Semi-angle at the Center.

O 2

2. Precept.

2. Precept.

To the Logar. of ECF, the Angle at the Center in Degr. and Decimal minutes add this Logar. 9.443697. (*which is the Arithm. Compl. of the Logar. of 360 degrees defective*) the Sum is the Logar. of the Sector EDFC.

3. Precept.

To the Sine and Co-sine of the Semi-angle at the Centre ECG add this Logar. (*which is the Logar. of .31831 the Square of the Semi-diameter of a Circle whose Area is Unity,*) the Sum is the Logar. of the Triangle EFC, which subducted from the Sector EDFC leaves the Segment EDFG.

Note, that the whole operation must be *Decimal Fractions* though wrought in *Whole Numbers* as is common, and though they have an *Index* before them, yet they must be supposed *Defective*.

A TA-

A
TABLE
OF
SEGMENTS
OF A
CIRCLE

Whose AREA is 1.000000.

*And DIAMETER 1.128379 sup-
posed to be intersected at Right-angles
by 1000 parallel Chord-lines.*

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Segments of a Circle whose Area is 1.000000.

V.S.	Segm.	V.S.	Segm.	V.S.	Segm.	V.S.	Segm.
001	000053	036	011469	071	031424	106	056688
002	000151	037	011947	072	032080	107	057473
003	000278	038	012431	073	032740	108	058262
004	000428	039	012921	074	033405	109	059054
005	000599	040	013417	075	034073	110	059849
006	000787	041	013919	076	034746	111	060647
007	000992	042	014426	077	035423	112	061449
008	001211	043	014940	078	036104	113	062253
009	001445	044	015460	079	036789	114	063061
010	001693	045	015985	080	037478	115	063872
011	001951	046	016515	081	038170	116	064686
012	002223	047	017052	082	038867	117	065503
013	002506	048	017593	083	039568	118	066323
014	002800	049	018140	084	040272	119	067146
015	003104	050	018693	085	040980	120	067972
016	003419	051	019250	086	041692	121	068801
017	003743	052	019813	087	042408	122	069633
018	004077	053	020381	088	043127	123	070468
019	004420	054	020954	89	043852	124	071306
020	004771	055	021532	090	044578	125	072146
021	005133	056	022115	091	045309	126	072989
022	005502	057	022703	092	046043	127	073836
023	005880	058	023296	093	046780	128	074686
024	006266	059	023894	094	047523	129	075538
025	006660	060	024496	095	048267	130	076393
026	007061	061	025103	096	049015	131	077251
027	007470	062	025715	097	049767	132	078112
028	007886	063	026331	098	050522	133	078975
029	008310	064	026952	099	051281	134	079841
030	008741	065	027578	100	052044	135	080710
031	009179	066	028208	101	052809	136	081581
032	009623	067	028842	102	053579	137	082455
033	010075	068	029481	103	054351	138	083332
034	010533	069	030124	104	055126	139	084211
035	010998	070	030772	105	055906	140	085094

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Segments of a Circle whose Area is 1.000000.

V.S.	Segm.	V.S.	Segm.	V.S.	Segm.	V.S.	Segm.
141	085979	176	118506	211	153696	246	191102
142	086867	177	119477	212	154736	247	192199
143	087757	178	120450	213	155778	248	193298
144	088650	179	121425	214	156821	249	194399
145	089545	180	122404	215	157866	250	195501
146	090443	181	123381	216	158913	251	196604
147	091343	182	124363	217	159962	252	197709
148	092246	183	125346	218	161013	253	198815
149	093152	184	126332	219	162065	254	199923
150	094060	185	127320	220	163119	255	201032
151	094970	186	128310	221	164175	256	202143
152	095883	187	129301	222	165232	257	203255
153	096799	188	130295	223	166291	258	204368
154	097717	189	131291	224	167352	259	205483
155	098637	190	132289	225	168415	260	206599
156	099560	191	133289	226	169479	261	207717
157	100485	192	134291	227	170545	262	208836
158	101413	193	135295	228	171612	263	209957
159	102343	194	136301	229	172681	264	211078
160	103275	195	137309	230	173752	265	212202
161	104210	196	138319	231	174825	266	213326
162	105147	197	139331	232	175899	267	214452
163	106086	198	140345	233	176975	268	215579
164	107028	199	141360	234	178052	269	216708
165	107972	200	142378	235	179131	270	217838
166	108918	201	143398	236	180211	271	218969
167	109867	202	144419	237	181293	272	220101
168	110818	203	145442	238	182377	273	221235
169	111771	204	146468	239	183462	274	222370
170	112727	205	147495	240	184549	275	223507
171	113684	206	148524	241	185637	276	224644
172	114644	207	149554	242	186727	277	225783
173	115606	208	150587	243	187819	278	226923
174	116570	209	151622	244	188912	279	228065
175	117537	210	152658	245	190006	280	229208

Segments of a Circle whose Area is 1.000000.

V.S.	Segm.	V.S.	Segm.	V.S.	Segm.	V.S.	Segm.
281	230352	316	271124	351	313135	386	356118
282	231497	317	272309	352	314351	387	357398
283	232643	318	273494	353	315567	388	358599
284	233792	319	274681	354	316783	389	359840
285	234940	320	275868	355	318001	390	361082
286	236090	321	277056	356	319220	391	362324
287	237241	322	278245	357	320440	392	363567
288	238394	323	279436	358	321660	393	364810
289	239548	324	280627	359	322881	394	366054
290	240703	325	281819	360	324103	395	367298
291	241859	326	283012	361	325325	396	368543
292	243016	327	284207	362	326549	397	369789
293	244175	328	285402	363	327774	398	371036
294	245334	329	286598	364	328999	399	372283
295	246495	330	287795	365	330224	400	373530
296	247657	331	288992	366	331450	401	374779
297	248820	332	290191	367	332677	402	376026
298	249984	333	291391	368	333905	403	377275
299	251149	334	292592	369	335134	404	378524
300	252315	335	293793	370	336363	405	379774
301	253483	336	294995	371	337593	406	381024
302	254652	337	296199	372	338823	407	382275
303	255821	338	297403	373	340054	408	383526
304	256992	339	298608	374	341286	409	384778
305	258164	340	299814	375	342518	410	386030
306	259337	341	301020	376	343751	411	387283
307	260511	342	302228	377	344985	412	388536
308	261686	343	303436	378	346220	413	389789
309	262862	344	304645	379	347455	414	391043
310	264039	345	305855	380	348691	415	392298
311	265217	346	307066	381	349927	416	393553
312	266397	347	308278	382	351164	417	394808
313	267577	348	309491	383	352401	418	396064
314	268759	349	310705	384	353639	419	397320
315	269941	350	311920	385	354878	420	398576

Segments of a Circle whose Area is 1.000000.

V.S.	Segm.	V.S.	Segm.	V.S.	Segm.	V.S.	Segm.
421	399833	456	444045	491	488539	526	533089
422	401091	457	445317	492	489812	527	534365
423	402349	458	446586	493	491085	528	535632
424	403607	459	447855	494	492358	529	536903
425	404866	460	449124	495	493631	530	538174
426	406125	461	450393	496	494904	531	539445
427	407384	462	451662	497	496177	532	540716
428	408644	463	452932	498	497450	533	541987
429	409904	464	454202	499	498725	534	543258
430	411165	465	455472	500	500000	535	544528
431	412426	466	456742	501	501275	536	545798
432	413687	467	458013	502	502550	537	547068
433	414948	468	459284	503	503823	538	548338
434	416210	469	460555	504	505096	539	549607
435	417472	470	461826	505	506369	540	550876
436	418735	471	463097	506	507642	541	552145
437	419998	472	464368	507	508915	542	553414
438	421261	473	465639	508	510188	543	554683
439	422525	474	466911	509	511461	544	555955
440	423789	475	468183	510	512737	545	557219
441	425053	476	469455	511	514007	546	558487
442	426317	477	470727	512	515280	547	559755
443	427582	478	471999	513	516553	548	561022
444	428847	479	473271	514	517825	549	562289
445	430112	480	474543	515	519097	550	563556
446	431378	481	475815	516	520369	551	564823
447	432644	482	477087	517	521641	552	566090
448	433910	483	478359	518	522913	553	567356
449	435177	484	479631	519	524185	554	568622
450	436444	485	480903	520	525457	555	569888
451	437711	486	482175	521	526729	556	571153
452	438978	487	483447	522	528001	557	572418
453	440245	488	484702	523	529273	558	573683
454	441513	489	485993	524	530545	559	574947
455	442781	490	487266	525	531817	560	576211

Segments of a Circle whose Area is 1.000000.

V.S.	Segm.	V.S.	Segm.	V.S.	Segm.	V.S.	Segm.
561	577425	596	621476	631	664866	666	707408
562	578739	597	622725	632	666095	667	708609
563	580002	598	623974	633	667323	668	709809
564	581265	599	625222	634	668550	669	711008
565	582528	600	626470	635	669776	670	712205
566	583790	601	627717	636	671001	671	713402
567	585052	602	628964	637	672226	672	714598
568	586313	603	630211	638	673451	673	715793
569	587574	604	631457	639	674675	674	716988
570	588835	605	632702	640	675897	675	718181
571	590096	606	633946	641	677119	676	719373
572	591356	607	635190	642	678340	677	720564
573	592616	608	636433	643	679560	678	721755
574	593875	609	637676	644	680780	679	722944
575	595134	610	638918	645	681999	680	724132
576	596393	611	640160	646	683217	681	725319
577	597651	612	641401	647	684433	682	726506
578	598909	613	642642	648	685649	683	727691
579	600167	614	643882	649	686865	684	728876
580	601424	615	645122	650	688080	685	730059
581	602680	616	646361	651	689295	686	731241
582	603936	617	647599	652	690509	687	732423
583	605192	618	648836	653	691722	688	733603
584	606447	619	650073	654	692934	689	734783
585	607702	620	651309	655	694145	690	735961
586	608957	621	652545	656	695355	691	737138
587	610211	622	653780	657	696564	692	738314
588	611464	623	655015	658	697772	693	739485
589	612717	624	656249	659	698980	694	740665
590	613970	625	657482	660	700186	695	741836
591	615222	626	658714	661	701392	696	743008
592	616474	627	659946	662	702596	697	744179
593	617725	628	661177	663	703801	698	745348
594	618976	629	662407	664	705005	699	746517
595	620226	630	663637	665	706207	700	747685

Segments of a Circle whose Area is 1.000000.

V.S.	Segm.	V.S.	Segm.	V.S.	Segm.	V.S.	Segm.
701	748851	736	788921	771	827319	806	863699
702	750016	737	790043	772	828388	807	864705
703	751180	738	791164	773	829455	808	865709
704	752343	739	792283	774	830521	809	866711
705	753505	740	793401	775	831585	810	867711
706	754666	741	794517	776	832648	811	868709
707	755825	742	794632	777	833709	812	869705
708	756984	743	796745	778	834768	813	870699
709	758141	744	797857	779	835825	814	871690
710	759297	745	798968	780	836881	815	872680
711	760452	746	800077	781	837935	816	873668
712	761606	747	801185	782	838987	817	874654
713	762759	748	802291	783	840038	818	875637
714	763910	749	803396	784	841087	819	876619
715	765060	750	804499	785	842138	820	877598
716	766209	751	805601	786	843179	821	878575
717	767357	752	806702	787	844222	822	879550
718	768503	753	807801	788	845264	823	880523
719	769648	754	808898	789	846304	824	881494
720	770792	755	809994	790	847342	825	882463
721	771935	756	811088	791	848378	826	883430
722	773077	757	812181	792	849413	827	884394
723	774217	758	813273	793	850446	828	885356
724	775356	759	814363	794	851476	829	886316
725	776493	760	815451	795	852505	830	887273
726	777630	761	816538	796	853532	831	888229
727	778765	762	817623	797	854558	832	889182
728	779899	763	818707	798	855581	833	890133
729	781031	764	819789	799	856602	834	891082
730	782162	765	820869	800	857622	835	892028
731	783292	766	821948	801	858640	836	892972
732	784421	767	823025	802	859655	837	893914
733	785548	768	824101	803	860669	838	894853
734	786674	769	825175	804	861681	839	895790
735	787798	770	826248	805	862691	840	896725

Segments of a Circle whose Area is 1.000000.

V.S.	Segm.	V.S.	Segm.	V.S.	Segm.	V.S.	Segm.
841	897657	876	928694	911	956148	946	979046
842	898587	877	929532	912	956873	947	979619
843	899515	878	930367	913	957592	948	980187
844	900440	879	931199	914	958308	949	980750
845	901363	880	932029	915	959020	950	981307
846	902283	881	932854	916	959728	951	981860
847	903201	882	933677	917	960432	952	982407
848	904117	883	934497	918	961133	953	982948
849	905030	884	935314	919	961830	954	983485
850	905940	885	936128	920	962522	955	984015
851	906848	886	936939	921	963211	956	984540
852	907754	887	937747	922	963896	957	985060
853	908657	888	938551	923	964577	958	985574
854	909557	889	939353	924	965254	959	986081
855	910455	890	940151	925	965927	960	986583
856	911350	891	940946	926	966595	961	987079
857	912243	892	941738	927	967260	962	987569
858	913133	893	942527	928	967920	963	988053
859	914021	894	943312	929	968576	964	988531
860	914906	895	944095	930	969228	965	989002
861	915788	896	944874	931	969876	966	989467
862	916668	897	945649	932	970519	967	989925
863	917545	898	946422	933	971158	968	990377
864	918419	899	947191	934	971792	969	990821
865	919290	900	947956	935	972422	970	991259
866	920159	901	948719	936	973048	971	991690
867	921025	902	949478	937	973669	972	992114
868	921889	903	950233	938	974285	973	992530
869	922749	904	950985	939	974897	974	992939
870	923607	905	951732	940	975504	975	993341
871	924462	906	952478	941	976106	976	993734
872	925314	907	953219	942	976704	977	994120
873	926164	908	953957	943	977297	978	994498
874	927011	909	954691	944	977885	979	994867
875	927854	910	955422	945	978468	980	995228

Segments of a Circle whose Area is 1.000000.

V.S.	Segm.	V.S.	Segm.	V.S.	Segm.	V.S.	Segm.
981	995580	986	997200	991	998555	996	999572
982	995923	987	997494	992	998789	997	999722
983	996257	988	997777	993	999008	998	999849
984	996581	989	998049	994	999213	999	999947
985	996896	990	998307	995	999401	1000	1.000000

Probl. 83. FIG. 10.

Given DH the Diameter of a Circle, and Area of the Segment EDFG; to find DG the Versed Sine.

Theor.

As the Area of the whole Circle, is to 1000000; so is the Area of the given Segment to a fourth Proportional, which found in the Table under *Segm.* against it stands a Versed Sine, by which if you Multiply the Diameter the Product will be the Versed Sine required.

Applicable for finding the Dry or Wet Inches of a Cylindrical Cask part empty, the Axe lying parallel to the Horizon.

Probl. 84. FIG. 21.

Given AB and CD, the Transverse and Conjugate Diameters of an Ellipsis; to find the Area.

Theor.

Theor.

The *Area* of an *Ellipsis* may be easily found as near the truth as that of a *Circle*, because it hath been proved by *Archimedes* and divers others, to be a mean Proportional between the two *Circles* described severally upon the *Diameters* of the *Ellipsis*. Therefore:

If you multiply the *Diameters* together, and extract the *Square Root*, you will have a *Diameter* whose *Area* is equal to the *Area* of the *Ellipsis*.

S. Or Arithmetically thus:

$$\text{As } \left\{ \begin{array}{l} 1 \\ 1 \\ 1 \end{array} \right\} \text{ to } \left\{ \begin{array}{l} .785398 \\ .0027851 \\ .0034 \end{array} \right\} \left\{ \begin{array}{l} \text{So is the Rectangle} \\ \text{of the Diameters} \\ \text{to the Area in} \end{array} \right\} \left\{ \begin{array}{l} \text{Inches} \\ \text{Ale-G.} \\ \text{Wine-G.} \end{array} \right.$$

Probl. 85. FIG. 21.

Given AB and CD, the Transverse and Conjugate Diameters of an Ellipsis; to describe the same.

Theor.

Draw two Lines equal to the given *Diameters* as *AB* and *CD* crossing one another at *Right-angles* in *K*, and dividing each other into two equal parts, then take between your *Compasses* half the Line *AB* and setting one Foot in *C* cross the Line *AB* in *L* and *M*; with the same extent set one Foot in *D* and cross the two former Arches, in those Points (*which are called the Foci*) drive two Needles, as also another in *C*, then take a Thread and incompass these three Needles in form of a Triangle, pluck the Thread tight, and

and tie the ends together at C, taking out the Needle at C, then hold a Pencil close to the inside of the Thread, and carry it upon the paper round about the Needles, holding the Thread always streight, and you will describe the *Ellipsis* ACBD.

Probl. 86. FIG. 21.

Given AB and CD the Diameters of an Ellipsis, and CE the Versed Sine; to find the Segment HCL.

Theor.

As $CD = NP$ the Lesser Diameter, is to the Segment FCG;

So is AB the Greater Diameter, to the Segm. HCL.

Probl. 87. FIG. 22.

Given AB and CD the Diameters of an Ellipsis, and AE the Versed Sine; to find the Segment HAL.

Theor.

As $AB = LM$ the Transverse Diameter, is to the Segment FAG;

So is CD the Conjugal Diameter, to the Segm. HAL.

Probl.

Probl. 88. FIG. 23.

Given AB the Axe of a Sphere; to find the superficial Content.

Defin.

A Sphere is a solid Figure made by the motion of a Semi-circle turned about its Diameter, the Diameter remaining fixed; hence all right Lines drawn to the Periphery or Superficies are equal.

Theor.

As $\left. \begin{matrix} 1 \\ 1 \\ 1 \end{matrix} \right\}$ to $\left. \begin{matrix} 3.14159265 \\ .0111404 \\ .0136 \end{matrix} \right\}$ So is the Square of Inches
of the Axe to Ale-Gall.
the Content in Wine-Gal.

Note, that either of these *Factors* is the superficial Content of a *Sphere*, whose *Axe* is *Unity*: the two last being found by dividing 3.14159265 severally by 282, and 231.

Probl. 89. FIG. 23.

Given the superficial Content of a Sphere; to find AB the Axe.

Theor.

As $\left. \begin{matrix} 1 \\ 1 \\ 1 \end{matrix} \right\}$ to $\left. \begin{matrix} .3183099 \\ 89.7633918 \\ 73.5295869 \end{matrix} \right\}$ So is the Content of Inches
in A. G. } To the Square of
W. G. } the Axe.

Note,

Note, that either of these *Factors* is the Square of the *Axe* of a Sphere whose *Superficial Content* is *Unity*, the two last being found by multiplying .3183099 severally by 282, and 231.

Note, also that the *Proportion* of the *Axe* to the *Periphery*, is the same with the *Diameter* of a Circle to its *Periphery*, & *Con.*

Probl. 90. FIG. 23.

Given the Periphery of a Sphere; to find the Superficial Content.

Theor.

As $\left\{ \begin{array}{l} 1 \\ 1 \\ 1 \end{array} \right\}$ to $\left\{ \begin{array}{l} .3183099 \\ .00112876 \\ .00137796 \end{array} \right\}$ So is the Square of the Periphery to the Content in $\left\{ \begin{array}{l} \text{Inches.} \\ \text{Ale-Gall.} \\ \text{Wine-Gall.} \end{array} \right\}$

Note, that either of these *Factors* is the *Superficial Content* of a Sphere, whose *Periphery* is *Unity*; the two last being found by dividing .3183099 severally by 282, and 231.

Probl. 91. FIG. 23.

Given the Superficial Content of a Sphere; to find the Periphery.

Theor.

As $\left\{ \begin{array}{l} 1 \\ 1 \\ 1 \end{array} \right\}$ to $\left\{ \begin{array}{l} 3.14159265 \\ 885.9291273 \\ 725.70790215 \end{array} \right\}$ So is the Content in $\left\{ \begin{array}{l} \text{Inches} \\ \text{Ale-G.} \\ \text{W. G.} \end{array} \right\}$ To the Square of the Periphery.

P

Note,

Note; that either of these *Factors* is the Square of the Periphery of a Sphere whose Superficial Content is Unity: The two last found by multiplying 3.14159265 severally by 282, and 231.

Probl. 92. FIG. 23.

Given AB the Axe of a Sphere; to find the Side of a Square Equal to the Superficial Content.

Theor.

As 1, is to 1.772454, so is the *Axe*, to the *Side*.

Note, that this *Factor* is the Square Root of the Superficial Content of a Sphere whose *Axe* is Unity.

Probl. 93. FIG. 23.

Given the Side of a Square to the Superficial Content; to find the Axe.

Theor.

As 1 is to .5641895, so is the *Side* to the *Axe*.

Note, that this *Factor* is the *Axe* of a Sphere whose Superficial Content is Unity.

Probl. 94. FIG. 23.

Given the Periphery of a Sphere; to find the Side of a Square Equal to the Superficial Content.

Theor.

Theor.

As 1, is to .5641895; so is the *Periphery*, to the *Side*.
Note, that this *Factor* is the Square Root of the Superficial Content of a Sphere whose *Periphery* is Unity.

Probl. 95. FIG. 23.

Given the *Side* of a Square Equal to the Superficial Content; to find the *Axe. Periphery*.

Theor.

As 1, is to 1.772434; so is the *Side*, to the *Periphery*.
Note, that this *Factor* is the *Periphery* of a Sphere whose Superficial Content is Unity.

Probl. 96. FIG. 23.

Given the *Axe* of a Sphere; to find the Solid Content.
Note, that a Sphere is two Third parts of a Cylinder of the same Diameter and Altitude;

Now the Diameter and Altitude of a Cylinder being 1, the Solid Content will be .785398, two Thirds of which is .5235988 the Solid Content of a Sphere whose *Axe* is 1, which being divided by 282, and 231 severally gives the Solid Content in Ale, and Wine-Gallons.

And all Solids being in Proportion one to another as the Cubes of their like Sides it will follow, because the Cube of 1, is but 1.

Theor.

As $\left\{ \begin{array}{l} 1 \\ 1 \\ 1 \end{array} \right\}$ to $\left\{ \begin{array}{l} .5235988 \\ .001856733 \\ .002266667 \end{array} \right\}$ So is the *Cube* of the *Axe* to the *Content* in $\left\{ \begin{array}{l} \text{Inches} \\ \text{Ale-Gall.} \\ \text{Wine-Gall.} \end{array} \right\}$

§. Or, Having the *Axe* find the Superficial Content of the Sphere, then Multiply the *Whole* of the one by a *Sixth* part of the other, the Product will be the *Content*.

§. Or, Find the Area of the Circle answerable to the *Axe*, and Multiply the *Whole* of the one by two *Third* parts of the other, the Product will be the *Content*.

Probl. 97. FIG. 23.

Given the Solid Content of a Sphere; to find the Axe AB.

Theor.

As $\left\{ \begin{array}{l} 1 \\ 1 \\ 1 \end{array} \right\}$ to $\left\{ \begin{array}{l} 1.9098592 \\ 538.5802944 \\ 441.1774752 \end{array} \right\}$ So is the *Content* in $\left\{ \begin{array}{l} \text{Inches} \\ \text{A. G.} \\ \text{W. G.} \end{array} \right\}$ to the *Cube* of the *Axe*

Note, that either of these *Factors* is the *Cube* of the *Axe* of a Sphere, whose *Solid Content* is *Unity*: the two last being found by multiplying 1.9098592 severally by 282, and 231.

Probl.

Probl. 98. FIG. 23.

Given the Periphery of a Sphere; to find the Solid Content.

Theor.

As $\left\{ \begin{matrix} 1 \\ 1 \\ 1 \end{matrix} \right\}$ to $\left\{ \begin{matrix} .01688687 \\ .0000598825 \\ .0000731033 \end{matrix} \right\}$ So is the Cube of Inches
the Periphery to Ale-G.
the Content in Wine-G.

Note, that either of these *Factors* is the Solid Content of a Sphere, whose Periphery is Unity: the two last being found by dividing .01688687 severally by 282, and 231.

Probl. 99. FIG. 23.

Given the Solid Content of a Sphere; to find the Periphery.

Theor.

As $\left\{ \begin{matrix} 1 \\ 1 \\ 1 \end{matrix} \right\}$ to $\left\{ \begin{matrix} 59.21761 \\ 16699.36602 \\ 13679.26791 \end{matrix} \right\}$ So is the Inches } to the Cube
the Contents } Ale-Gall. } of the
in } Wine-G. } Periphery.

Note, that either of these *Factors* is the Cube of the Periphery of a Sphere, whose Solid Content is Unity: the two last being found by multiplying 59.21761 severally by 282, and 231.

Probl. 100. FIG. 23.

Given AB the Axe of a Sphere; to find the Side of a Cube Equal to the Solid Content.

Theor.

As 1, is to 805996; so is the *Axe*, to the *Side*.

Note, that this *Factor* is the Cube Root of the Solid Content of a Sphere, whose *Axe* is Unity.

Probl. 101. FIG. 23.

Given the Side of a Cube Equal to the Solid Content; to find AB the Axe.

Theor.

As 1, is to 1.240695; So is the *Side*, to the *Axe*.

Note, that this *Factor* is the *Axe* of a Sphere, whose Solid Content is Unity.

Probl. 102. FIG. 23.

Given the Periphery of a Sphere; to find the Side of a Cube Equal to the Solid Content.

Theor.

As 1, is to .2565565; So is the *Periphery*, to the *Side*.

Note,

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Note, that this *Factor* is the Cube-root of the Solid Content of a Sphere, whose Periphery is Unity.

Probl. 103. FIG. 23.

Given the Side of a Cube Equal to the Solid Content of a Sphere; to find the Periphery.

Theor.

As 1, is to 3.897777; so is the Side, to the Periphery.

Note, that this *Factor* is the Periphery of a Sphere whose Solid Content is Unity.

Probl. 104. FIG. 23.

Given AB the *Axe* of a Sphere, and AG the *Altitude* of the Frustum; to find the Superficial Content.

Theor.

As 1, is to 3.14159265;
So is the *Rectangle* of AB and AG, to the Content.

5. Or, Multiply, the Periphery by AG the *Altitude*, the *Product* will be the Content.

6. Or, As AB the *Axe* is to the Content of the whole Sphere;

So is AG the *Altitude*, to the Content of the Frustum.

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Probl. 105. FIG. 23.

Given AB the Axe of a Sphere, and AG the Altitude of the Frustrum; to find the Solid Content.

Theor.

To GB the Remainder of the Axe add KB the Semi-axe, and multiply the Sum by the Square of AG the Altitude,

The Product $\left\{ \begin{array}{l} 1.04719755 \\ 0.00371347 \\ 0.00453333 \end{array} \right\}$ Produces the $\left\{ \begin{array}{l} \text{Inches} \\ \text{A. Gall.} \\ \text{Content in} \\ \text{W. Gall.} \end{array} \right.$

Note, that these Multipliers are a Third part of those in Theor. 96.

Probl. 106. FIG. 23.

Given AB the Axe of a Sphere, EF the Diameter at the Base, and AG the Frustrum's Altitude; to find the Solid Content.

Theor.

As GB the Remainder of the Axe, is to AG the Altitude; so is AK the Semi-axe, to AH; which added to AG the Sum is GH the Altitude of the Cone EHF, equal to the Frustrum EAF.

S. Or thus without the Axe.

To the Square of EF, the Diameter at the Base, add the Square of AG more a Third part of the same Square

Of a Spheroid.

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Square, and multiply the Sum by half AG, this last Product multiplied by the proper *Factor* in *Theor.* 61. will give the *Content*.

§. *The same by the Cyclometrical Tables.*

To the Area of the Circle EF, add the Area of the Altitude AG, more a Third part of the same Area, the Sum multiplied by half AG produces the *Content*.

Probl. 107. FIG. 24.

Given AB the *Axe*, and CD the *lesser Diameter* of a Spheroid; to find the *Solid Content*.

Defn.

A *Spheroid* is a Solid Figure made by the revolution of half an Ellipsis, the *Axe* remaining fixed, whilst the curv'd part of the Ellipsis is turned about till the motion end at the place where it began.

Theor.

As $\left\{ \begin{array}{l} 1 \\ 1 \\ 1 \end{array} \right\}$ to $\left\{ \begin{array}{l} .5235988 \\ .001856733 \\ .002266667 \end{array} \right\}$ So is the Square of CD $\left\{ \begin{array}{l} \text{Inches} \\ \text{A. G.} \\ \text{W. G.} \end{array} \right\}$ multiplied by AB to the *Content* in

Note, that these *Factors* are the same as in *Theor.* 96.

§. *Or by the Cyclometrical Tables.*

Multiply $\left\{ \begin{array}{l} \text{Area} \\ \text{two thirds of this Area} \end{array} \right\}$ answerable $\left\{ \begin{array}{l} \text{two thirds of AB} \\ \text{to CD by AB.} \end{array} \right\}$

The Product is the *Content* in Ale, or Wine-Gallons.
Probl.

Probl. 108. FIG. 24.

Given AB the Axe of a Spheroid, and the Solid Content in Ale or Wine-Gallons; to find CD the lesser Diameter.

Theor.

Divide the Content by two thirds of AB, the Quotient is the Area answerable to CD, against which in the proper Table stands the Diameter.

Probl. 109. FIG. 24.

Given CD the lesser Diameter of a Spheroid, and Solid Content in Ale, or Wine-Gallons; to find AB the Axe.

Theor.

Divide the Content by the Area correspondent to the lesser Diameter, the Quotient is two third parts of the Axe, the half of which added to its self gives the Axe.

Probl. 110. FIG. 24.

Given AB the Axe, CD the lesser Diameter of a Spheroid, and CG the Frustrums Altitude; to find PV and EF Diameters of the Liquors Surface.

Theor.

Theor.

From the Square of $KV=KC$ subduct the Square of GK , the Square Root of the Remainder is GV by *Theor.* 20. then say:

As KN , is to KB ; so is G , V to GF .

§. Or you may find GF without GR thus :

From the Rectangle of the Squares of KB and CK , subduct the Rectangle of the Squares of KB and GK , Divide the Remainder by the Square of CK , the Square Root of the Quotient is GF , the *Semi-diameter* sought.

Probl. III. *FIG.* 24.

Given CD the Lesser Diameter of a Spheroid, CG the Altitude of the Frustum, and EF the greatest Diameter of the Liquors Surface; to find AB the *Axe*.

Theor.

As the Square Root of the Difference of the Squares of CK the lesser Semi-diameter, and GK the distance from the plane of the Bung to the Surface of the Liquor,

is to CK the said Semi-diameter;
So is GF the given Semi-diameter,
to KB the Semi-axe.

Probl.

Probl. 112. FIG. 22.

Given AB the Axe, CD the lesser Diameter of a Spherocoid, and AE the Altitude of the Frustrum; to find HI the Diameter of the Liquors Surface.

Theor.

From the Rectangle of the Squares of AK and KD the greater and lesser Semi-diameters, subduct the Rectangle of the Squares of EK the distance from the plane of the Bung to the Surface of the Liquor, and KD the lesser Semi-diameter, divide the Remainder by the Square of AK, the Square Root of the Quotient is EI the Semi-diameter required.

Probl. 113. FIG. 22.

Given AB the Axe, CD the lesser Diameter of a Spherocoid, and AE the Altitude; to find the Content of the Frustrum HAI.

Theor.

As the Square of $AB = LM$, is to the Frustrum FAG,
So is the Square of CD, to the Frustrum HAI.

Applicable for finding the Vacuity of a Spherocoidal Cask, the Axe standing Perpendicular to the Horizon.

Probl.

Probl. 114. FIG. 21.

Given AB the Axe, CD the lesser Diameter of a Spherocoid, and CE the Altitude; to find the Content of the Frustrum HCL.

Theor.

As the Cube of $CD=NP$, is to the Frustrum FCG;
So is the Square of CD multiplied by AB, to the Frustrum HCL.

§. Or thus without the Diameters.

To the Rectangle of FG and HI the Conjugates of the Base, add the Square of CE the Altitude, more a third part of the same Square, and multiply the Sum by half CE, this last Product multiplied, or divided by the proper Factor, or Divisor in Theor. 61. gives the Content.

§. Or thus, FIG. 24.

To RD add half CD, and multiply the Sum by the Square of CR reserving the Product:

Divide AB by CD, and by the Quotient multiply the reserved Product, this last Product multiplied, or divided by the proper Factor or Divisor in Theor. 105. gives the Content of the vacant Frustrum cCc. *vid. Smith's Stereom. p. 206.*

Applicable for finding the Vacuity of a Spherocoidal Cask, the Surface of the Liquor not cutting the Heads, and the Axe lying parallel to the Horizon.

Probl.

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Probl. 115. FIG. 24.

Given CD the Bung Diameter, cd the Head Diameter, and the Axe or Length of a Cask taken as the middle Frustum of a Spheroid; to find AB the Spheroids Axe.

Theor.

As the Square Root of the Difference of the Squares of CK, and cO the Semi-diameters at Bung and Head, is to CK the Semi-diameter at Bung;
So is KO the Cask's Semi-length,
to KB the Spheroid's Semi-axe.

Probl. 116. FIG. 31.

Given AB the Bung Diameter, CD the Head Diameter, and EP the Length of a Cask taken as the middle Frustum of a Spheroid, also EI the Dry Inch; to find GH the Diameter of the Liquors Surface.

Theor.

As the Square of EK the Semi-length,
is to the Difference of the Squares of KB and ED, the Semi-diameters at Bung and Head;
So is the Square of IK the distance from the Plane of the Bung to the Surface of the Liquor,
to the Difference of the Squares of KB and IH;
which Subducted from the Square of KB, the Square Root of the Remainder is IH the Semi-diameter required. Compare this with *Probl. 4. Chap. IX. of Mr. Everard's Stereom.*

Probl.

 Probl. 117. FIG. 24.

Given AB the Spheroid's Axe, CD the Bung Diameter, and LO the Length of a Cask taken as the middle Frustum of a Spheroid; to find cd the Head Diameter, & Con.

Theor.

As KB the Spheroid's Semi-axe,
is to KO the Cask's Semi-length;
So is CK, or c O the Semi-diameter at Bung or Head,
to the Square Root of the Differences of their Squares.

Probl. 118. FIG. 25, 36.

Given AB the Base of a Prism, and GP the Altitude;
to find the Solid Content.

Defn.

A Prism is a Solid Figure contained under several Planes, two of which being opposite are called the Bases, and are equal, parallel, and alike situate, but the rest of the Planes are Parallelograms in which a right line may be every where applied from one Base to another (which may be a Triangle, Quadrangle, Pentagon, or any other plain Surface.)

Under this Definition is comprehended that Solid having two Circular Bases commonly called a Cylinder, which is generated by the motion of a right angled Parallelogram, one of the Sides remaining fixed till the

Pa-

Parallelogram be turned about to the place whence first it began to move.

Theor.

Multiply the Area of the Base AB by the Altitude GP, the Product will be the Content.

Note, that by *Theor.* 19, and 20. you may find the Diagonal Altitude, and Base of a Prism.

Probl. 119. FIG. 25, 26.

Given GP the Altitude of a Prism, and Solid Content ; to find AB the Base.

Theor.

Divide the Content by the Altitude the Quotient will be the Content upon one Inch, and the Base is given, if it be Square, or Circular, by the Tetragonical, or Cyclometrical Tables.

But if the Base be a right angled Parallelogram or an Ellipsis, then Multiply the Content in Ale-Gallons by 282, or 359.0536. and divide the Product by the Rectangle of the given Diameter and Altitude, the Quotient will be the other.

Probl. 120. FIG. 25, 26.

Given AB the Base of a Prism, and Solid Content ; to find GP the Altitude.

Theor.

Theor.

Divide the Content by the Area correspondent to the Base, if it be a Square or a Circle, the Quotient will be the *Altitude*.

But if the Base be a right angled Parallelogram, or an *Ellipsis*: then,

Multiply the Content in Ale-Gallons by 282, or 359.0536 and divide the Product by the Rectangle of the Diameters, the Quotient will be the *Altitude*.

Probl. 121. FIG. 25, 26.

Given AH the Side at the Base of a Pyramid, and ZP the *Altitude*; to find the Solid Content.

Defn.

A *Pyramid* is a Solid Figure contained under divers Planes, set upon one right lin'd Plane, which is called the *Base*, from whence it decreaseth equally less and less till it end in a Point at the Top or *Vertex*; also in either of these Planes a right line may be every where applied from the *Base* to the *Vertex*.

Under this *Definition* is comprehended that Solid whose Base is a Circle commonly called a *Cone*, which is made by the motion of a right-angled plain Triangle, one of the containing Sides remaining fixed till the Triangle is turned about to the place from whence it first moved.

226 Of the Frustrum of a Pyramid.

Theor.

Multiply the Area of the Base (*whether it be Triangular, Quadrangular, Pentagonal, &c.*) by one third part of the Altitude, the Product will be the Solid Content. & Cont.

He that can vary the Sides of a right angled plain Triangle must needs understand how to find the Hypotenuse, Altitude, or Base of a Pyramid.

Probl. 122. FIG. 25, 26.

Given AH and CI the Sides at the Base of the Frustrum of a Pyramid, and GP the Altitude; to find the Solid Content.

Theor.

As AK the Semi-difference of AH and CI,
is to CK = GP the Frustrum's Altitude;
So is AP = half AH the Side of the greater Base,
to ZP the Altitude of the Pyramid AZB.

From ZP subduct GP, the Remainder is ZG the Altitude of the Pyramid CZD.

Lastly, from AZB the whole Pyramid, subduct the Lesser Pyramid CZD, the Remainder will be the Content of the Frustrum ABCD.

§. Or thus without ZP.

From the Square of the Sum of AH and CI, subduct their Rectangle, the Remainder multiplied, or divided by the proper Factor or Divisor in Theor. 130. Tablet 1. gives the Area in Inches or Ale-Gallons.

§. Or

Of the Frustum of a Pyramid. 227

§. Or to the Rectangle of the Bases, add a third part of the Square of their Difference, the Sum multiplied by the proper Factor in Theor. 69. produces a mean Area.

§. Or by the Tetragonical, or Cyclometrical Tables.

Multiply the Bases together and extract the Square Root, to the Area answerable thereto, add a third part of the Area of the Difference of the Bases, the Sum is a mean Area, or the Content upon one Inch in Ale-Gallons.

§. Or to the Area correspondent to the Semi-sum of the Diameters, add a third part of the Area of the Semi-difference the Sum is a mean Area, which multiplied by the Altitude the Product will be the Content of the Frustum.

And if the Conjugates in each Base differ, find two Geometrical means, one between the Conjugates above and the other below, so are the Bases reduced either to Squares, or Circles; then finish the operation as before directed.

Probl. 123. FIG. 25, 26.

Given AB and CD the Bases of the Frustum of a Pyramid, and Solid Content; to find GP the Altitude.

Theor.

Divide the Content by the mean Area, the Quotient will be the Altitude.

228 Of the Frustum of a Pyramid.

Probl. 124. FIG. 25, 26.

Given AB and CD the Bases of the Frustum of a Pyramid, and GP the Altitude; to find AC the slant Height.

Theor.

To the Square of $CK=GP$ add the Square of AK the Semi-difference of AB and CD , the Square Root of the Sum will be AC the *slant Height*.

Probl. 125. FIG. 25, 26.

Given AB and CD the Bases of the Frustum of a Pyramid, and GP the Altitude; to find CB the Diagonal.

Theor.

To the Square of KB the Semi-sum of AB and CD , add the Square of $CK=GP$ the Square Root of the Sum is CB the *Diagonal*.

Probl. 126. FIG. 25, 26.

Given AB and CD the Bases of the Frustum of a Pyramid, and CB the Diagonal; to find GP the Altitude.

Theor.

Theor.

From the Square of CB, subduct the Square of KB the Semi-sum of AB and CD, the Square Root of the Remainder is CK=GP the *Altitude*.

Probl. 127. FIG. 25, 26.

Given AB and CD the Bases of the Frustum of a Pyramid, and AC the slant Height; to find GP the Altitude.

Theor.

From the Square of AC, subduct the Square of AK the Semi-difference of AB and CD, the Square Root of the Remainder is CK=GP the *Altitude*.

Probl. 128. FIG. 25, 26.

Given CB the Diagonal of the Frustum of a Pyramid, GP the Altitude, and AC the slant Height; to find the Bases AB and CD.

Theor.

From the Square of CB, subduct the Square of CK=GP, the Square Root of the Remainder is KB the *Semi-sum*.

From the Square of AC subduct the Square of CK, the Square Root of the Remainder is AK the *Semi-difference*, And by *Theor. 6*. AB and CD are given.

Probl. 129. FIG. 25, 26.

Given *CB* the *Diagonal* of the *Frustrum* of a *Pyramid*,
GP the *Altitude*, and *AB* or *CD* one of the *Bases*; to find
the other.

Theor.

From the Square of *CB* subduct the Square of
 $CK = GP$ the Square Root of the Remainder is *KB*
the Semi-sum of the *Bases*, and their Sum less the *given*
Base is the other.

Probl. 130. FIG. 27.

A *Prismoid* is a solid Figure comprehended under
divers Planes, two of which being opposite are called
the *Bases*, and are to be rect-angular Parallelograms,
parallel one another, and alike (square, &c.) the Length
above opposite to the Length below, and both in the
same Plane; also the Sides without any Curvature (i.e.)
in either of these Planes a right Line may be every
where applied from one Base to the other, and the re-
maining Planes are called by *Mr. Dary*, The *Peripe-*
tafma.

Under this *Definition* is comprehended the *Frustrum's*
of *Pyramids* and *Prisms* whose *Bases* are rect-angular Pa-
rallelograms.

If the *Peripetafma* be not constituted by flat Sides,
but shall be clothed about with Curvature from Cir-
cles or Ellipses, the Solid is then called by him a *Cylind-*
roid.

Under this *Definition* is comprehended the *Frustrum's*
of *Cones*, and *Cylinders*.

Theor.

Theor.

To AB add half EF, and multiply the Sum by CD,
To EF add half AB, and multiply the Sum by GH,
the Sum of these two Products multiplied, or divided
by the proper *Factor* or *Divisor* in *Table I.* gives the
Content upon one Inch, which multiplied by the At-
titude produces the *Content* of the Frustrum.

5. Or find two Geometrical means between the
Top and Bottom, as directed *Theor. 122.*

Note, that there is the same proportion between a
Circle and an *Ellipsis*, as between a *Square* and an *Ob-*
long; And between an *Oblong* and an *Ellipsis*, as be-
tween a *Square* and a *Circle*, therefore what is said of the
one hath relation also to the other.

Now to reduce the former Theorems to Practice.

You must take notice that Brewers Tuns, though of
various Forms are generally comprehended under
these three, *viz. Square, Round, and Elliptical*; their
Sides being supposed to be Strait from Top to Bottom,
and their Bases parallel.

I. Of *Square Tuns*, *FIG. 25, 27.*

1. When the Bases are both Square, and Equal, the
Tun is a *Prism* and Gauged by *Theor. 118.*
2. When the Bases are both Rectangular Parallelo-
gramsequal, and alike situate the Tun is a *Prism.*
3. When the Bases are both Square, but unequal,
the Tun is the *Frustrum* of a *Pyramid*, and Gauged by
Theor. 122.

4. When the Bases are both Rectangular Parallelograms proportional, and alike situate, but unequal, the Tun is the *Frustum of a Pyramid*.

5. When the Bases are both Rectangular Parallelograms, but unequal, and Disproportional, or Inverted; or if one Base be a Square, and the other a Rectangular Parallelogram the Tun is called a *Prismocoid*, and Gauged by *Theor. 130.*

II. Of Round Tuns. FIG. 26.

1. When the Bases are both Circular, and equal, the Tun is a *Cylinder* and Gauged by *Theor. 118.*

2. When the Bases are both Circular, but unequal, the Tun is the *Frustum of a Cone*, and Gauged by *Theor. 122.*

III. Of Elliptical Tuns. FIG. 27.

1. When the Bases are both Elliptical, equal, and alike situate, the Tun is a *Prism*, and Gauged by *Theor. 118.*

2. When the Bases are both Elliptical, proportional, and alike situate, but unequal, the Tun is the *Frustum of an Elliptical Cone*, and Gauged by *Theor. 122.*

3. When the Bases are both Elliptical, but unequal, and Disproportional, or Inverted; Or if one Base be a Circle, and the other an Ellipsis, the Tun is called a *Cylindroecid*, and Gauged by *Theor. 130.*

Note, that by *Theor. 130.* you may find the Content of any Tun whose *Sides* are *strait*, and *Bases parallel*, the *Conjugates* in each *Base* cutting one another at *right Angles*, whether they be *equal*, or *unequal*, *proportional*, or *disproportional*, *alike situate*, or *inverted*.

When

When the Conjugates in each Base are proportional, the *Analogy* is this:

As AB the Length below, to EF the Length above;
So CD the Breadth below, to GH the Breadth above.

Lastly, if the Sides are Curv'd, then the Tun may be taken for the Frustrum of a Spherocoid, and Gauged by *Theor.* 135.

How to Inch any regular Polygon Pyramidal Tun, the Conjugates of each Base being Equal.

FIG. 25, 26.

I. From the greater Base AB.

1. From 3 times the Square of AB, subduct 3 times AB less (D), multiplied by (D), the Remainder is the Content upon the *first Inch*.

2. From 3 times the Square of AB, subduct 9 times AB less 7 times (D), multiplied by (D), the Remainder is the Content on the *second Inch*.

The *Excess* call the *first Difference*.

Both the Remainders being first multiplied or divided by the proper *Factor* or *Divisor* taken out of *Table* 1.

3. Six times the Square of (D) multiplied or divided as aforesaid gives the *second Difference*, which varies not.

Then Subduct the *second Difference* from the first, and add the Remainder to the Content on the *second Inch*, the Sum is the Content on the *third Inch*, and so on to every Inch of the Depth.

II. From the Lesser Base CD.

1. To 3 times the Square of CD, add 3 times CD more (D), multiplied by (D) the Sum is the Content on the *first Inch*.

2. To

2. To 3 times the Square of CD add 9 times (D) more 7 times (D), multiplied by (D) the Sum is the Content on the *second Inch*.

The *Excess* call the *first Difference*.

Both the Sums being first multiplied, or divided by the proper *Factor* or *Divisor* taken out of *Tablet 1*.

3. Six times the Square of (D) multiplied, or divided aforesaid gives the *second Difference*, which alters not.

Then add the *second Difference* to the first, and the Sum to the Content on the *second Inch*, and you will have the Content on the *third Inch*, and so proceed by a continual Addition till you have compleated the whole Depth.

To find (D) the common Subducent or Addend say:

As the Depth, is to the Difference of the Diameters,

So is Unity, to the Decrement or Increment at 1 Inch distance in the Depth according to the position of the greater, or lesser Base.

I. Tablet.					
Polig.	Factor for Inches.	Divisor for Inches.	Factor for Ale-Gallons.	Divisor for Ale-Gallons.	Polig.
Cone	0.261799	3.819717	.0009284	1077.161	Cone
III	0.144338	6.928203	.0005118	1953.753	III
IV	0.333333	3.000000	.0011820	846.000	IV
V	0.573493	1.743702	.0020337	491.724	V
VI	0.866025	1.154700	.0037100	325.626	VI
VII	1.211310	0.825552	.0042954	232.806	VII
VIII	1.609476	0.621321	.0057074	175.212	VIII
IX	2.060608	0.485292	.0073071	136.851	IX
X	2.564737	0.389904	.0090948	109.953	X
XI	3.121886	0.320319	.0110705	90.330	XI
XII	3.732050	0.267948	.0132342	75.561	XII

Note,

Note, that these Factors are a third part of the Factors, and the Divisors triple the Divisors in Tablet II.

How to find the Content at any assigned Depth without Incbing the whole Tun, by Aid of three Stationary Numbers.

1. A third part of the Square of D, is the *first Number*.

2. The Rectangle of AB, or CD, and D, is the *second Number*.

3. The Square of AB, or CD is the *third Number*.

All of them being first severally multiplied, or divided by the *Factor*, or *Divisor* in *Tablet II.* proper and peculiar to the given *Figure*: then,

1. *From the greater Base AB.*

Multiply the *first Number* by the assigned *Depth*, and subtract the *Product* from the *second*, the *Remainder* multiplied by the *Depth*, and the *Product* subtracted from the *third*, leaves a *mean Area*, which multiplied by the *Depth*, the *Product* will be the *Content*.

2. *From the lesser Base CB.*

Multiply the *first Number* by the *Depth*, and to the *Product* add the *second*, the *Sum* multiplied by the *Depth*, and the *Product* added to the *third*, will give a *mean Area*, which multiplied by the *Depth* produces the *Content*.

II. *Tablet.*

II. Tablet.					
Polyg.	Factor for Inches.	Divisor for Inches.	Factor for Ale-Gallons.	Divisor for Ale-Gallons.	Polyg.
Cone	0.785398	1.273239	.0027855	359.054	Cone
III	0.433013	2.309401	.0015355	651.250	III
IV	1.000000	1.000000	.0035461	282.000	IV
V	1.720478	0.581234	.0061010	163.908	V
VI	2.596075	0.384900	.0092130	108.542	VI
VII	3.633931	0.275184	.0128863	77.602	VII
VIII	4.828428	0.207107	.0171221	58.404	VIII
IX	6.181825	0.161764	.0219214	45.617	IX
X	7.694210	0.129968	.0272844	36.650	X
XI	9.365659	0.106773	.0332115	30.110	XI
XII	11.196150	0.089316	.0397027	25.187	XII

But seeing that not one Tun of 1000 is exactly regular, and none but such can be Inched by the former *Theorems*, the usual way hath been to take a competent number of mean Diameters in the middle of every Foot, or half Foot of the Depth; but the readiest way is to take them in the middle of every 10 Inches, for then there will be no need of *Multiplication*, then calculate the Areas, or from the proper Table, insert against each respective Diameter its respective Area, removing the Point one place more towards the right hand, and adding them together, the *Sum* will be the *Content*.

Now by help of these Numbers, a Table may be made which shall shew by inspection what Liquor is in the Tun at any number of Dry Inches, by subducting the mean Area of the first 10 Inches, continually from the whole Content till you come at the tenth Inch; and then the mean Area of the second 10 Inches from that Remainder till you come at the twentieth Inch, and so forwards still, subducting the mean

mean Area of the next Segment from the last Remainder till you have compleated the whole Depth.

And if the last result of each respective 10 Inches leave the Remainder in the Tun, then is the operation true, so that every 10 Inches proves its own work.

To take the Dimensions Actually do thus. FIG. 28.

1. With a sliding Rule cross the Tun both ways at the greatest extent you can guess, and with Chalk make 4 short lines, then place the end of the Rule first in one, and then in the other of those Lines; note the Distances from line to line, and adding them together take a fourth part, to which set the Rule, and measure from the first line as far as it will reach, and there make a short line, rubbing out the other if they do not touch, and so go round, and thus the Tun will be exactly *Quartered*.

2. Take the Diameters both ways, and if they differ not above two Inches, you may take an Arithmetical mean, set the Rule to that mean, and see what part of the Tun will bear it; this done, from the ends of the Rule so set with Chalk, draw two lines down the Sides from the Top to the Surface of the Liquor at the Fall, and cross the said lines 5 Inches from the Top; then at every 10 Inches from thence make the like marks, for by this means the Tun is reduced into so many Prisms, each containing 10 Inches in Altitude.

3. Take the Diameters of the several Segments by setting the ends of the Rule in those marks as AB, CD, EF, entring them in a Book beginning always at the Top, and finish the work as before directed.

4. IF

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4 If you measure in the Side line, you must allow for the Difference between the Perpendicular Height, and the Side line in every 10 Inches proportionably thus :

As KP, is to the Difference between IP and KP;

So is 10 Inches, to a fourth Proportional,
to be added to 10 Inches, and the Sum to be esteemed as 10 Inches in the Perpendicular.

And if the last Segment want of 10 Inches (*which oftentimes happens*) measure from E the last Diameter to P the Diameter at Bottom, and subtracting 5 Inches the Remainder is the Altitude of the Segment, in the middle whereof take the Diameter GH.

How to Inch any regular Polygonal Pyramidal Tun, the Conjugates at each Base being unequal. FIG. 27.

First find the Subducend or Addend for the Length, which call D; as also that for the Breadth, and call it *d*: then,

I. If you would gauge it from the greater Base.

1. From 3 times AB, subtract 1.5 (D) and multiply the Remainder by CD reserving the Product :

Then from 1.5 AB subtract (D) and multiply the Remainder by (*d*) the Difference of these Products is the Content on the first Inch.

2. From 3 times AB subtract 4.5 (D) and multiply the Remainder by CD reserving the Product :

Then from 4.5 AB subtract 7 times (D) and multiply the Remainder by (*d*) the Difference of these Products is the Content on the second Inch.

The Excess of these two Contents call the first Difference, both the Products being first multiplied, or divided by the proper Factor or Divisor in Tablet 1.

3. Six

3. Six times the Rectangle of (D) into (*d*) multiplied or divided as aforesaid, gives the second *Difference* which varies not through the whole operation.

Then subduct the second *Difference* from the first, and add the Remainder to the Content on the second *Inch*, the Sum is the Content on the third *Inch*, and after the same manner proceed till you have run through the Depth.

II. From the Lesser Base.

1. To 3 times EF, add 1.5 (D) and multiply the Sum by GH reserving the *Product*:

Then to 1.5 EF add (D) and multiply the Sum by (*d*) the Sum of these *Products* is the Content on the first *Inch*.

2. To 3 times EF add 4.5 D, and multiply the Sum by GH, reserving the *Product*:

Then to 4.5 EF add 7 times D, and multiply the Sum by *d*, the Sum of these two *Products* is the Content on the second *Inch*.

The *Excess* of these two Contents call the first *Difference*.

Both the *Products* being first multiplied, or divided by the proper *Factor* or *Divisor* in *Tablet 1*.

3. Six times the Rectangle of D into *d*, multiplied, or divided as aforesaid gives the second *Difference*, which is common through the whole work.

Then add the second *Difference* to the first, and the Sum to the Content on the second *Inch*, and you will have the Content on the third *Inch*, and so on for every *Inch* of the Tuns Depth.

How to find the Content at any given Depth, without making a Table, by Aid of three Stationary Numbers.

1. A third part of the Rectangle of D into d , is the first Number.

2. The Semi-Sum of the Rectangle of AB and EF, and their alternate Subducends or Addends (d) and (D) is the second Number.

3. The Rectangle of AB and CD, or EF and GH is the third Number.

All of them being first severally multiplied, or divided by the Factor or Divisor proper to the given Polygon in *Table II*.

1. *Then if you Gauge from the greater Base.*

Multiply the first Number by the Depth, and subtract the Product from the second, the Remainder multiplied by the Depth, and the Product subtracted from the third leaves a *mean Area*, which multiplied by the Depth produces the *Content*.

2. *If you Gauge from the lesser Base.*

Multiply the first Number by the Depth, and to the Product add the second, the Sum multiplied by the Depth, and the Product added to the third, gives a *mean Area*, which multiply by the Depth, and you will have the *Content*.

But seeing that most of these Tuns are irregular also in their *Peripetasma's*, the best way is to take the Diameters in the middle of every 10 Inches of the Depth, and to calculate their Areas, or to find the several Geometrical means, and to insert against them from the proper

proper Table their respective Areas, then removing the Point one place more towards the right hand, and adding them together, the Sum will be the Content.

And from these Numbers likewise by *subduction* you may make a Table which shall show what Liquor is in the Tun at any Depth required.

To take the Dimensions Actually do thus :

Quarter the Tun, and from thence draw lines down the Sides to Surface of the Liquor at the Fall, and on those Lines place the Inches at the same distances as before, and take the Dimensions of the several Segments, and if any odd Inches remain, take the Diameter in the middle thereof, then finish the rest of the work according to the former Directions.

Probl. 131.

To find the Drip or Fall of a Tun.

Pour in Liquor by some known measure till you see the Bottom just covered (must fix the Tuns for the convenience of Cleansing, standing lower on the fore-side by three or four Inches from a true Horizontal plane, or level of the Liquor) and when the Liquor hath done moving, take the Depth at the Fall, which subducted out of the whole Depth, the Remainder is the true Depth, and the Liquor put in by measure must be added to the Content found by this Depth.

*Probl. 132.**To Gauge the Worts in any Back or Cooler.*

Divide the Sum of the wet Inches taken in all the places, which in large Backs ought to be in every 5 or 6 Feet Square, by the number of places they were taken at, the Quotient will be the true Dipping place which you are to mark accordingly.

*Probl. 133. FIG. 29.**To Gauge, and Inch a Brewers Copper.*1. *For the upper part, viz. A B V W.*

With a sliding Rule Quarter it as before directed, and find a mean Diameter, to which set the Rule, and see what part of the Copper will bear it, then draw Lines down the Sides as low as the Crown, and mark with Chalk the said Lines 3 Inches from the Top, then at every 6 Inches from thence, make the like marks, and the Copper is reduced into so many Cylinders, each 6 Inches in Depth: Lastly, take the Diameters of each Segment as MN, OP, QR, (*allowing somewhat more in proportion to the Difference between the Perpendicular and slant Height of the Coppers Side, as directed for a Pyramidal Tun.*)

And if the last Segment want of 6 Inches, draw a touch-Line to the top of the Crown, as GH, measure from S the last Diameter to G this touch-Line, and subtracting 3 Inches the Remainder is the Altitude of this Segment in the middle whereof take the Diameter ST, And thus you have all the mean Diameters
above

above the Top of the Crown; then insert against each Diameter its answerable Area, and multiply them severally by 6, placing the Products in another Column, and adding them together.

2. For the Crown.

Lay a Line over the Top, and with a Plummert let fall two Perpendiculars from K and L, to C and D, the distance between them is equal to CD the Diameter at Bottom, for you cannot come to take it actually.

Or, measure the distance from L to B, which doubled and subducted from AB the Diameter at the Top, the Remainder is CD.

Then let fall a Perpendicular from the Top of the Copper to the Top of the Crown, viz. from E to F, which subducted from KC=LD the Depth, the Remainder is FI the Crown's Altitude.

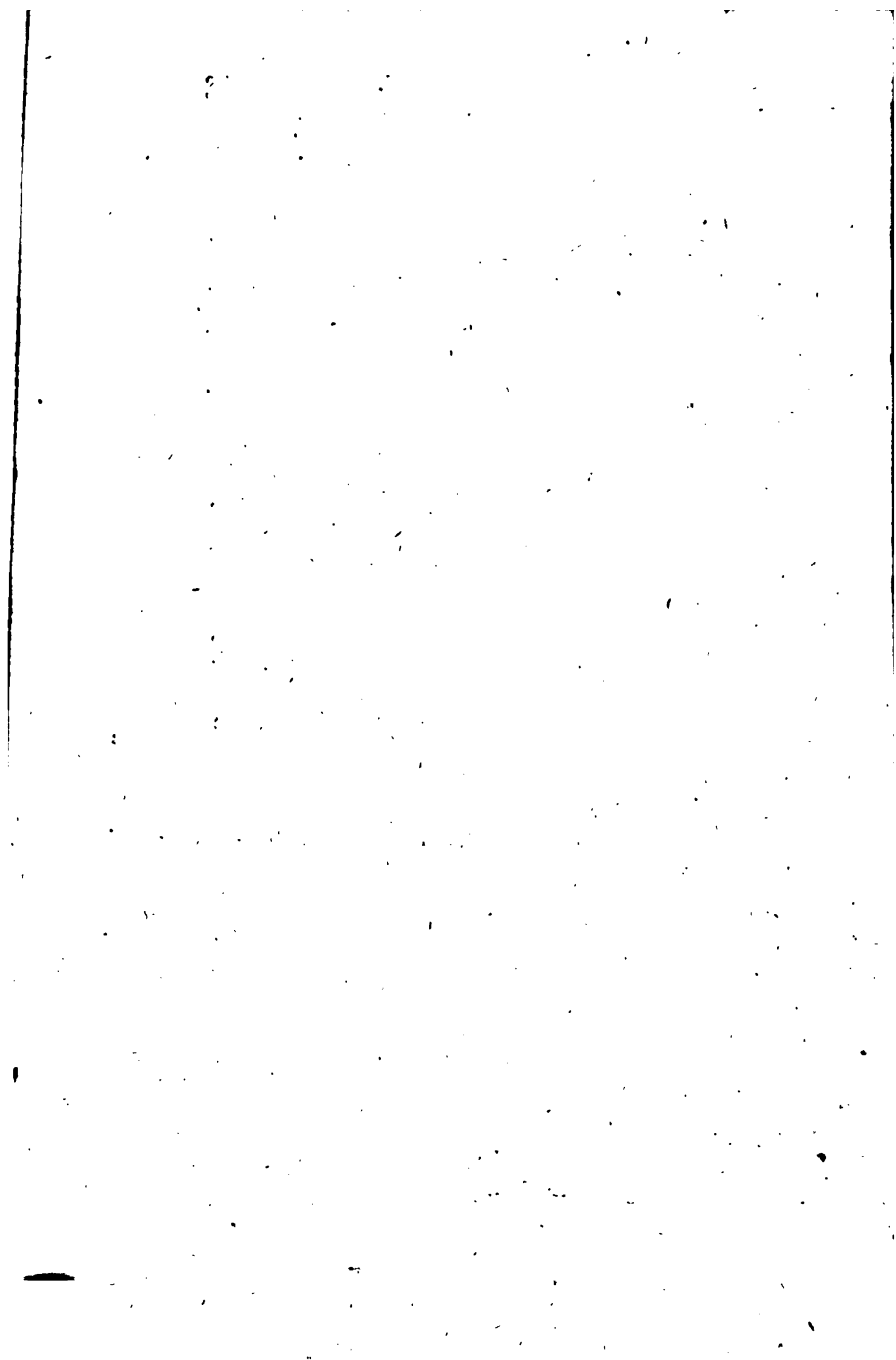
If the Crown have equal Curvature at the Top, and towards the Bottom, it may be Gauged as the *Frustum of a Sphere*.

But if more Curvature at the Top than towards the Bottom, it may be measured as the *Frustum of a Paraboloid*, viz. by multiplying the Area of the Diameter at the Base, by half the Altitude.

3. For the Lower part.

Take an Arithmetical mean between GH and CD, Diameters at the Top and Bottom of the Crown, viz. VW, and multiply the Area answerable thereto by IF the Altitude of that Segment, the Product is the Content of GHDC from which subduct the Content of the Crown CFD, the remainder is the Liquor lying about the Crown when just covered, which added to the upper part, the Sum is the Content of the whole Copper.

And from hence by subduction may be made a Table which shall shew how many Gallons remain in this Copper at any assigned Depth.



A
TABLE
OF
ALLOWANCES
FOR
COMMON BREWERS,

Of { 3 Barrels in 23 for Beer,
and
2 Barrels in 22 for Ale,

At 2 s. 6 d. the Barrel Strong, and 6 d. the
Barrel Small, in Net Mony; from 1 Fir-
kin to 10000 Barrels to the Centesimal
part of a Penny.

Calculated by Mr. RICHARD WALKER.

746 Allowances for Common Brewers.

	Strong Beer.				Ale.				Small Beer.			
Bar.	l.	s.	d.	f. pts	l.	s.	d.	f. pts	l.	s.	d.	f. pts
$\frac{1}{4}$	00.	00.	06.	2. 09	00.	00.	06.	3. 27	00.	00.	01.	1. 22
$\frac{1}{2}$	00.	01.	01.	0. 17	00.	01.	01.	2. 55	00.	00.	02.	2. 43
$\frac{3}{4}$	00.	01.	07.	2. 26	00.	01.	08.	1. 82	00.	00.	03.	3. 65
1	00.	02.	02.	0. 35	00.	02.	03.	1. 09	00.	00.	05.	0. 87
2	00.	04.	04.	0. 69	00.	04.	06.	2. 18	00.	00.	10.	1. 74
3	00.	06.	06.	1. 04	00.	06.	09.	3. 27	00.	01.	03.	2. 61
4	00.	08.	08.	1. 39	00.	09.	01.	0. 36	00.	01.	08.	3. 48
5	00.	10.	10.	1. 74	00.	11.	04.	1. 45	00.	02.	02.	0. 35
6	00.	13.	00.	2. 09	00.	13.	07.	2. 54	00.	02.	07.	1. 22
7	00.	15.	02.	2. 43	00.	15.	10.	3. 64	00.	03.	00.	2. 09
8	00.	17.	04.	2. 78	00.	18.	02.	0. 73	00.	03.	05.	2. 96
9	00.	19.	06.	3. 13	01.	00.	05.	1. 82	00.	03.	10.	3. 83
10	01.	01.	08.	3. 48	01.	02.	08.	2. 91	00.	04.	04.	0. 69
11	01.	03.	10.	3. 83	01.	05.	00.	0. 00	00.	04.	09.	1. 56
12	01.	06.	01.	0. 17	01.	07.	03.	1. 09	00.	05.	02.	1. 43
13	01.	08.	03.	0. 52	01.	09.	06.	2. 18	00.	05.	07.	3. 30
14	01.	10.	05.	0. 87	01.	11.	09.	3. 27	00.	06.	01.	0. 17
15	01.	12.	07.	1. 22	01.	14.	01.	0. 36	00.	06.	06.	1. 04
16	01.	14.	09.	1. 56	01.	16.	04.	1. 45	00.	06.	11.	1. 94
17	01.	16.	11.	1. 91	01.	18.	07.	2. 54	00.	07.	04.	2. 78
18	01.	19.	01.	2. 26	02.	00.	10.	3. 64	00.	07.	09.	3. 65
19	01.	01.	03.	2. 61	02.	03.	01.	0. 73	00.	08.	03.	0. 52
20	02.	03.	05.	2. 96	02.	05.	05.	1. 82	00.	08.	08.	1. 39
21	02.	05.	07.	3. 30	02.	07.	08.	2. 91	00.	09.	01.	2. 26
22	02.	07.	09.	3. 65	02.	10.	00.	0. 00	00.	09.	06.	3. 13
23	02.	10.	00.	0. 00	02.	12.	03.	1. 09	00.	10.	00.	0. 00
24	02.	12.	02.	0. 35	02.	14.	06.	2. 18	00.	10.	05.	0. 87
25	02.	14.	04.	0. 69	02.	16.	09.	3. 27	00.	10.	10.	1. 74
26	02.	16.	06.	1. 04	02.	19.	01.	0. 36	00.	11.	03.	2. 61
27	02.	18.	08.	1. 39	03.	01.	04.	1. 45	00.	11.	08.	3. 48
28	03.	00.	10.	1. 74	03.	03.	07.	2. 54	00.	12.	02.	0. 35
29	03.	03.	00.	2. 09	03.	05.	10.	3. 64	00.	12.	07.	1. 22
30	03.	05.	02.	2. 43	03.	08.	02.	0. 73	00.	13.	00.	2. 09

Allowances for Common Brewers. 247

	<i>Strong Beer.</i>					<i>Ale.</i>					<i>Small Beer.</i>				
Bar.	l.	s.	d.	f.	pts	l.	s.	d.	f.	pts	l.	s.	d.	f.	pts
31	03.	07.	04.	2.	78	03.	10.	05.	1.	82	00.	13.	05.	2.	96
32	03.	09.	06.	3.	13	03.	12.	08.	2.	91	00.	13.	10.	3.	83
33	03.	11.	08.	3.	48	03.	15.	00.	0.	00	00.	14.	04.	0.	69
34	03.	13.	10.	3.	83	03.	17.	03.	1.	09	00.	14.	09.	1.	56
35	03.	16.	01.	0.	17	03.	19.	06.	2.	18	00.	15.	02.	2.	43
36	03.	18.	03.	0.	52	04.	01.	09.	3.	27	00.	15.	07.	3.	30
37	04.	00.	05.	0.	87	04.	04.	01.	0.	36	00.	16.	01.	0.	17
38	04.	02.	07.	1.	22	04.	06.	04.	1.	45	00.	16.	06.	1.	04
39	04.	04.	09.	1.	56	04.	08.	07.	2.	54	00.	16.	11.	1.	91
40	04.	06.	11.	1.	91	04.	10.	10.	3.	64	00.	17.	04.	2.	74
41	04.	09.	01.	2.	26	04.	13.	02.	0.	73	00.	17.	09.	3.	65
42	04.	11.	03.	2.	61	04.	15.	05.	1.	82	00.	18.	00.	3.	52
43	04.	13.	05.	2.	96	04.	17.	08.	2.	91	00.	18.	08.	1.	39
44	04.	15.	07.	3.	30	05.	00.	00.	0.	00	00.	19.	01.	2.	26
45	04.	17.	09.	3.	65	05.	02.	03.	1.	09	00.	19.	06.	3.	13
46	05.	00.	00.	0.	00	05.	04.	06.	2.	18	01.	00.	00.	0.	00
47	05.	02.	02.	0.	35	05.	06.	09.	3.	27	01.	02.	05.	0.	87
48	05.	04.	04.	0.	69	05.	09.	01.	0.	36	01.	00.	10.	1.	74
49	05.	06.	06.	1.	04	05.	11.	04.	1.	45	01.	01.	03.	2.	61
50	05.	08.	08.	1.	39	05.	13.	07.	2.	54	01.	01.	08.	3.	48
51	05.	10.	10.	1.	74	05.	15.	10.	3.	64	01.	02.	02.	0.	35
52	05.	13.	00.	2.	09	05.	18.	02.	0.	73	01.	02.	07.	1.	22
53	05.	15.	02.	2.	43	06.	00.	05.	1.	82	01.	03.	00.	2.	09
54	05.	17.	04.	2.	78	06.	02.	08.	2.	91	01.	03.	05.	2.	96
55	05.	19.	06.	3.	13	06.	05.	00.	0.	00	01.	03.	10.	3.	83
56	06.	01.	08.	3.	48	06.	07.	03.	1.	09	01.	04.	04.	0.	69
57	06.	03.	10.	3.	83	06.	09.	06.	2.	18	01.	04.	09.	1.	56
58	06.	06.	01.	0.	17	06.	11.	09.	3.	27	01.	05.	02.	2.	43
59	06.	08.	03.	0.	52	06.	14.	01.	0.	36	01.	05.	07.	3.	30
60	06.	10.	05.	0.	87	06.	16.	04.	1.	45	01.	06.	01.	0.	17

248 Allowances for Common Brewers.

	Strong Beer.					Ale.					Small Beer.				
Bar.	l.	s.	d.	f.	pts	l.	s.	d.	f.	pts	l.	s.	d.	f.	pts
61	06.	12.	07.	1.	22	06.	18.	07.	2.	54	01.	06.	06.	1.	04
62	06.	14.	09.	1.	56	07.	10.	00.	3.	64	01.	06.	11.	1.	51
63	06.	16.	11.	1.	91	07.	03.	02.	0.	73	01.	07.	04.	2.	78
64	06.	19.	01.	2.	16	07.	05.	05.	1.	82	01.	07.	09.	3.	65
65	06.	01.	03.	2.	61	07.	07.	08.	2.	91	01.	08.	03.	0.	52
66	07.	03.	05.	2.	96	07.	10.	00.	0.	00	01.	08.	08.	1.	39
67	07.	05.	07.	3.	30	07.	12.	03.	1.	09	01.	09.	01.	2.	26
68	07.	07.	09.	3.	65	07.	14.	06.	2.	18	01.	09.	06.	3.	13
69	07.	10.	00.	0.	00	07.	16.	09.	3.	27	01.	10.	00.	0.	00
70	07.	12.	02.	0.	35	07.	19.	01.	0.	36	01.	10.	05.	0.	87
71	07.	14.	04.	0.	69	08.	01.	04.	1.	45	01.	10.	10.	11.	74
72	07.	16.	06.	1.	04	08.	03.	07.	2.	54	01.	11.	03.	2.	81
73	07.	18.	08.	1.	39	08.	05.	10.	3.	64	01.	11.	08.	3.	48
74	08.	00.	10.	1.	74	08.	08.	02.	0.	73	01.	12.	02.	0.	35
75	08.	03.	00.	2.	09	08.	10.	05.	1.	82	01.	12.	07.	1.	22
76	08.	05.	02.	2.	43	08.	12.	08.	2.	91	01.	13.	00.	2.	09
77	08.	07.	04.	2.	78	08.	15.	00.	0.	00	01.	13.	05.	2.	96
78	08.	09.	06.	3.	13	08.	17.	03.	1.	09	01.	13.	10.	3.	83
79	08.	11.	08.	3.	48	08.	19.	06.	2.	18	01.	14.	04.	0.	69
80	08.	13.	10.	3.	83	09.	01.	09.	3.	27	01.	14.	09.	1.	56
81	08.	16.	01.	0.	17	09.	04.	01.	0.	36	01.	15.	02.	2.	43
82	08.	18.	03.	0.	52	09.	06.	04.	1.	45	01.	15.	07.	3.	30
83	09.	00.	05.	0.	87	09.	08.	07.	2.	54	01.	16.	01.	0.	17
84	09.	02.	07.	1.	22	09.	10.	10.	3.	64	01.	16.	06.	1.	04
85	09.	04.	09.	1.	56	09.	13.	02.	0.	73	01.	16.	11.	1.	91
86	09.	06.	11.	1.	91	09.	15.	05.	1.	82	01.	17.	04.	2.	78
87	09.	09.	01.	2.	26	09.	17.	08.	2.	91	01.	17.	09.	3.	65
88	09.	11.	03.	2.	61	10.	00.	00.	0.	00	01.	18.	03.	0.	52
89	09.	13.	05.	2.	96	10.	02.	03.	1.	09	01.	18.	08.	1.	39
90	09.	15.	07.	3.	30	10.	04.	06.	2.	18	01.	19.	01.	2.	26

Allowances for Common Brewers.

249

<i>Strong Beer.</i>					<i>Ala.</i>					<i>Small Beer.</i>				
Bar.	l.	s.	d.	f. pts	l.	s.	d.	f. pts		l.	d.	s.	f. pts	
91	09.	17.	09.	3. 55	10.	06.	09.	3. 27		01.	19.	06.	3. 13	
92	10.	00.	00.	0. 00	10.	09.	01.	0. 36		02.	00.	00.	0. 00	
93	10.	02.	02.	0. 35	10.	11.	04.	1. 45		02.	00.	05.	0. 87	
94	10.	04.	04.	0. 69	10.	13.	07.	2. 54		02.	00.	10.	1. 74	
95	10.	06.	06.	1. 04	10.	15.	10.	3. 64		02.	01.	03.	2. 61	
96	10.	08.	08.	1. 39	10.	18.	02.	0. 73		02.	01.	08.	3. 48	
97	10.	10.	10.	1. 74	11.	00.	05.	1. 82		02.	02.	02.	0. 35	
98	10.	13.	00.	2. 09	11.	02.	08.	2. 91		02.	02.	07.	1. 22	
99	10.	15.	02.	2. 43	11.	05.	00.	0. 00		02.	03.	00.	2. 09	
100	10.	17.	04.	2. 78	11.	07.	03.	1. 09		02.	03.	05.	2. 96	
200	21.	14.	09.	1. 56	22.	14.	06.	2. 18		04.	06.	11.	1. 91	
300	32.	12.	02.	0. 35	34.	01.	09.	3. 27		06.	10.	05.	0. 87	
400	43.	09.	06.	3. 13	45.	09.	01.	0. 36		08.	13.	10.	3. 83	
500	54.	06.	11.	1. 91	56.	16.	04.	1. 45		10.	17.	04.	2. 78	
600	65.	04.	04.	0. 69	68.	03.	07.	2. 54		13.	00.	10.	1. 74	
700	76.	01.	08.	3. 48	79.	10.	10.	3. 63		15.	04.	04.	0. 70	
800	86.	19.	01.	2. 26	90.	18.	02.	0. 72		17.	07.	09.	3. 65	
900	97.	16.	06.	1. 04	102.	05.	05.	1. 82		19.	11.	03.	2. 61	
1000	108.	13.	10.	3. 82	113.	12.	08.	2. 91		21.	14.	09.	1. 56	
2000	217.	07.	09.	3. 65	227.	05.	05.	1. 82		43.	09.	06.	3. 13	
3000	326.	01.	08.	3. 47	340.	18.	02.	0. 73		65.	04.	04.	0. 69	
4000	434.	15.	07.	3. 30	454.	10.	10.	3. 64		86.	19.	01.	2. 26	
5000	543.	08.	06.	3. 12	568.	03.	07.	2. 54		108.	13.	10.	3. 83	
6000	652.	03.	05.	2. 95	681.	16.	04.	1. 45		130.	08.	08.	1. 39	
7000	760.	17.	04.	2. 77	795.	09.	01.	0. 36		152.	03.	05.	2. 95	
8000	866.	11.	03.	2. 60	909.	01.	09.	3. 27		173.	18.	03.	0. 51	
9000	978.	04.	02.	2. 42	1022.	14.	06.	2. 18		195.	13.	00.	2. 08	
10000	1085.	15.	01.	2. 25	1136.	07.	03.	1. 08		217.	07.	09.	3. 65	

Note, that you may find the Net *Excise* of any number of Barrels by *Multiplication* using these *Factors*.

$$\text{Viz. } \left. \begin{array}{l} .10869565 \\ .11363636 \\ .02173913 \end{array} \right\} \text{ for } \left\{ \begin{array}{l} \text{Strong Beer.} \\ \text{Ale.} \\ \text{Small Beer.} \end{array} \right.$$

Which are found by dividing 2.5 the Decimal of 2^d. 10^s. the Duty of 20 Barrels of *Strong Beer* or *Ale*, severally by 23 and 22, and by dividing .5 the Decimal of 10^s. the Duty of 20 Barrels of *Small Beer* by 23, distinguishing as many places as are required by *Decimal Multiplication*.

And the value of the *Fraction* is given by that short Rule in *Pag.* 17.

Probl. 134.

To Gauge any Square, or Round Mash Tun.

Theor.

The *Square* of the $\{ .0042017 \}$ or divi- $\{ 238 \}$ gives the *Area*
Base multiplied by $\{ .0033003 \}$ ded by $\{ 303 \}$ in Gallons.

Note, that the *Corn-Gallon* contains 272.25 *Cube-Inches*, two whereof make a *Peck*, 4 *Pecks* a *Bushel*, and 8 *Bushels* a *Quarter*.

The reason why I lay down these *Divisors* is because I have found that an indifferent sort of *Malt*, and of ordinary grinding when three Worts have passed through it, will not be contracted above an *Eight* part, but if the *Malt* be very fine you may use 227, and 288.

And

And for as much as the *second Differences* are equal, a Table for either may be made by an easie collection.

One for *Round Tuns* I have inserted in the following Table, to every *two Tenths of an Inch of the Diameter* from 21 to 80 Inches, to be used as the *Cyclometrical Tables*; for if you enter with the *mean Diameter* of the Goods usually wet, against it in the proper Column stands the Content in Gallons, &c. upon 1 Inch, which multiplied by any given Depth produces the Content.

Now though by taking the Depth of the Goods, the exact quantity of Malt cannot be discovered by reason of its different goodness and variation in Grinding, some spending it self much more than others, yet by this means you may know how to give an estimate thereto, whereby a considerable Fraud may be discovered.

And the *Divisors* are to be altered by the *Supervisor* according to the *Country Malt* made use of, for his Experience in this case will be the best Director.

A TA.

THE UNIVERSITY OF CHICAGO

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PHYSICS
DEPARTMENT

A
T A B L E
O F
A R E A S
F O R
R O U N D M A S H T U N S
I N
G A L L O N S and Centesimal parts,
Calculated to every *two Tenths* of an *Inch*
O F T H E
D I A M E T E R
From 21 to 80 *Inches.*

254 Round Mash Tuns in Gallons.

Diam.	0	.2	.4	.6	.8
21	1.46	1.48	1.51	1.54	1.57
22	1.60	1.63	1.66	1.69	1.72
23	1.75	1.78	1.81	1.84	1.87
24	1.90	1.93	1.96	2.00	2.03
25	2.06	2.10	2.13	2.16	2.20
26	2.23	2.27	2.30	2.33	2.37
27	2.41	2.44	2.48	2.51	2.55
28	2.59	2.62	2.66	2.70	2.74
29	2.78	2.81	2.85	2.89	2.93
30	2.97	3.01	3.05	3.09	3.13
31	3.17	3.21	3.25	3.30	3.34
32	3.38	3.42	3.46	3.51	3.55
33	3.59	3.64	3.68	3.73	3.77
34	3.82	3.86	3.91	3.95	4.00
35	4.04	4.09	4.14	4.18	4.23
36	4.28	4.32	4.37	4.42	4.47
37	4.52	4.57	4.62	4.67	4.72
38	4.77	4.82	4.87	4.92	4.97
39	5.02	5.07	5.12	5.18	5.23
40	5.28	5.33	5.39	5.44	5.49
41	5.55	5.60	5.66	5.71	5.77
42	5.82	5.88	5.93	5.99	6.05
43	6.10	6.16	6.22	6.27	6.33
44	6.39	6.45	6.51	6.56	6.62
45	6.68	6.74	6.80	6.86	6.92
46	6.98	7.04	7.11	7.17	7.23
47	7.29	7.35	7.42	7.48	7.54
48	7.60	7.67	7.73	7.80	7.86
49	7.92	7.99	8.05	8.12	8.18
50	8.25	8.32	8.38	8.45	8.52

Round Mash Tuns in Gallons. 255

Diam.	0	.2	.4	.6	.8
51	8.58	8.65	8.72	8.79	8.86
52	8.92	8.99	9.06	9.13	9.20
53	9.27	9.34	9.41	9.48	9.55
54	9.62	9.70	9.77	9.84	9.91
55	9.98	10.06	10.13	10.20	10.28
56	10.35	10.42	10.50	10.57	10.65
57	10.72	10.80	10.87	10.95	11.03
58	11.10	11.18	11.26	11.33	11.41
59	11.49	11.57	11.64	11.72	11.80
60	11.88	11.96	12.04	12.12	12.20
61	12.28	12.36	12.44	12.52	12.60
62	12.69	12.77	12.85	12.93	13.02
63	13.10	13.18	13.27	13.35	13.43
64	13.52	13.60	13.69	13.77	13.86
65	13.94	14.03	14.12	14.20	14.29
66	14.38	14.46	14.55	14.64	14.73
67	14.81	14.90	14.99	15.08	15.17
68	15.26	15.35	15.44	15.53	15.62
69	15.71	15.80	15.90	15.99	16.08
70	16.17	16.26	16.36	16.45	16.54
71	16.64	16.73	16.82	16.92	17.01
72	17.11	17.20	17.30	17.40	17.49
73	17.59	17.68	17.78	17.88	17.97
74	18.07	18.17	18.27	18.37	18.47
75	18.56	18.66	18.76	18.86	18.96
76	19.06	19.16	19.26	19.36	19.47
77	19.57	19.67	19.77	19.87	19.98
78	20.08	20.18	20.29	20.39	20.49
79	20.60	20.70	20.81	20.91	21.02
80	21.12	21.23	21.33	21.44	21.55

Probl. 135. FIG. 30.

Given AB the Bung Diameter, cd the Head Diameter, and LE the Axe or Length; to find the Content, if the Cask be taken as the middle Frustum of a Spheroid intercepted between two Planes parallel, bisecting the Axe at right Angles.

Theor.

To the double Square of the Bung Diameter, add the Square of the Head Diameter,

A third part of the Sum multiplied by $\left. \begin{array}{l} .0027851 \\ .0034 \end{array} \right\}$ or divided by $\left. \begin{array}{l} 359.161 \\ 294.118 \end{array} \right\}$ gives the Area of a mean Circle in $\left. \begin{array}{l} \text{A.G.} \\ \text{W.G.} \end{array} \right\}$

§. Or by the Cyclometrical Tables.

To the double Area answerable to the Bung Diameter, add the Area of the Head Diameter, a third part of the Sum is the Area of a mean Circle, which multiplied by the Length, the Product will be the Content in Ale or Wine Gallons, &c.

Probl. 136. FIG. 30.

Given AB the Diameter at Bung, cd the Diameter at Head, and LE the Length; to find the Content, if the Cask be taken as the middle Frustum of a Parabolick Spindle, intercepted between two Planes parallel, bisecting the Axe at right Angles.

Theor.

Theor.

To the double of the *Bung Diameter*, add the Square of the *Head Diameter*, from a third part of the Sum, subduct half the Square of the Difference of the Diameters, the Remainder multiplied or divided by the proper *Factor* or *Divisor* in *Theor.* 135. gives the *Area of a mean Circle*.

S. Or by the Cycloastrical Tables.

To the double *Area* of the *Bung Diameter*, add the *Area* of the *Head Diameter*, from a third part of the Sum subduct half the *Area* correspondent to the Difference of the *Diameters*, the Remainder is the *Area* of a mean Circle which multiplied by the *Length*, produces the *Content* in Ale or Wine Gallons.

Probl. 137. FIG. 30.

Given AB the *Bung Diameter*, cd the *Head Diameter*, and LO the *Length*; to find the *Content*, if the *Cask* be taken as the middle *Frustum* of an *Hyperbolick Spindle*, intercepted between two *Planes* parallel, bisecting the *Axe* at right Angles.

Theor.

To the Double Square of the *Diameter* at *Bung*, add the Square of the *Diameter* at *Head*, from a third part of the Sum, subduct the Square of the Difference of the *Diameters*, the Remainder multiplied or divided by the proper *Factor*, or *Divisor* in *Theor.* 135. gives the *Area of a mean Circle*.

§. Or by the Cyclometrical Tables.

To the *double Area* of the *Bung Diameter*, add the *Area* of the *Head Diameter*; from a *third part* of the *Sum* subtract the *Area* answerable to the *Difference* of the *Diameters*, the *Remainder* is the *Area* of a *mean Circle*, which multiplied by the *Length* produces the *Content* in Ale or Wine Gallons.

Note, if the *Staves* are *curv'd* as the Lines *Aec*, then the Cask is the middle Frustum of a *Spheroid*, and the *Content* is given by *Theor.* 135.

If they are *strait* as the Lines *Abc*, then it is the middle Frustum of *two Cones* abutting upon one *Base*, and the *Content* is found by *Theor.* 133.

Now the nearer the *Staves* are to the *Curv'd Lines* *Aec*, the greater will be the *Content*, and it is best Gauged by *Theor.* 136.

But the nearer they are to the *strait Lines* *Abc*, the less will be the *Content*, and it is best Gauged by *Theor.* 137.

Upon a diligent observation of these *Rules* in a short time you will be able to discover what *Form* to make use of, when the *Cask* lieth before you that is to be Gauged.

If you like to work by a *mean Diameter*, which reduces the Cask to a *Cylinder*, the following *Tablets* are very ready which are calculated for the three *Forms* aforesaid, for if you enter the proper *Tablet* with the *Inches* of the *Difference* of the *Diameters* in the *first Column*, and the *Tenths* (if any) at the *Top* in the answerable *Square* stands a *number*, which added to the *Head Diameter* gives the *mean Diameter*, whose *Area* multiplied by the *Length* produces the *Content*.

Frustum

Frustum of a Spheroid.

D.D	0	1	2	3	4	5	6	7	8	9
1	0.67	0.74	0.81	0.87	0.94	1.01	1.08	1.15	1.21	1.28
2	1.35	1.42	1.49	1.56	1.62	1.69	1.76	1.83	1.90	1.97
3	2.03	2.11	2.17	2.24	2.31	2.38	2.45	2.52	2.59	2.66
4	2.73	2.80	2.87	2.94	3.01	3.08	3.15	3.22	3.29	3.36
5	3.43	3.50	3.57	3.64	3.71	3.78	3.86	3.93	4.00	4.07
6	4.14	4.21	4.27	4.35	4.42	4.49	4.56	4.63	4.71	4.78
7	4.85	4.92	4.99	5.06	5.14	5.21	5.28	5.35	5.42	5.49
8	5.57	5.64	5.71	5.79	5.86	5.93	6.01	6.08	6.15	6.22
9	6.30	6.37	6.45	6.52	6.59	6.67	6.74	6.81	6.89	6.96
10	7.04	7.11	7.19	7.26	7.34	7.41	7.49	7.56	7.64	7.71

Frustum of a Parabolick Spindle.

D.D	0	1	2	3	4	5	6	7	8	9
1	0.66	0.73	0.79	0.86	0.92	0.99	1.05	1.12	1.18	1.25
2	1.32	1.38	1.44	1.51	1.57	1.64	1.70	1.77	1.83	1.89
3	1.96	2.02	2.09	2.15	2.22	2.28	2.34	2.41	2.47	2.53
4	2.60	2.66	2.72	2.78	2.85	2.91	2.97	3.03	3.10	3.16
5	3.22	3.28	3.35	3.41	3.47	3.53	3.59	3.65	3.72	3.78
6	3.84	3.90	3.96	4.02	4.08	4.14	4.20	4.26	4.32	4.38
7	4.44	4.50	4.56	4.62	4.68	4.74	4.80	4.86	4.92	4.98
8	5.04	5.10	5.16	5.21	5.27	5.33	5.39	5.45	5.51	5.56
9	5.62	5.68	5.74	5.79	5.85	5.91	5.97	6.02	6.08	6.14
10	6.20	6.25	6.31	6.36	6.42	6.47	6.53	6.59	6.65	6.71

Frustum of an Hyperbolick Spindle.

D.D	0	1	2	3	4	5	6	7	8	9
1	0.65	0.72	0.78	0.84	0.91	0.97	1.03	1.10	1.16	1.22
2	1.29	1.35	1.41	1.47	1.53	1.59	1.65	1.71	1.77	1.83
3	1.89	1.95	2.01	2.06	2.12	2.18	2.24	2.30	2.35	2.41
4	2.47	2.53	2.58	2.64	2.69	2.75	2.80	2.86	2.91	2.97
5	3.02	3.07	3.13	3.18	3.23	3.28	3.34	3.39	3.44	3.49
6	3.54	3.59	3.64	3.69	3.74	3.79	3.84	3.89	3.94	3.99
7	4.03	4.08	4.13	4.18	4.22	4.27	4.32	4.36	4.41	4.46
8	4.51	4.54	4.59	4.63	4.67	4.72	4.76	4.80	4.85	4.89
9	4.93	4.97	5.01	5.05	5.09	5.13	5.17	5.21	5.25	5.29
10	5.33	5.36	5.40	5.43	5.47	5.50	5.54	5.57	5.61	5.64

§. To take the Dimensions of any close Cask.

1. For the Head Diameter.

Apply the Rule as close as you can to either *Chine*, and take the *Diameters* cross ways, including the thickness of one *Chine*, and if they differ take an *Arithm. mean*.

2. For the Bung Diameter.

Let the Rule descend perpendicularly from the *Bung-hole* to the opposite *Staff*, the inside of the *Bung-hole* is the *Diameter*: And if the *Bung* may not be opened, extend a strait Rule or Line along the *Length* of the *Cask* just to touch the *Bung*. (allowing for the thickness of the *Hoops*) double the Difference between the Rule and outside of the *Chine* added to the *Head Diameter* gives the *Bung Diameter*; And if the *Heads* differ, take an *Arith. mean*.

3. For the Length.

Set the *Length* of the *Chine* over-hanging the *Heads* on the *Hoops* at both ends (allowing for the thickness of both *Heads*) and make two Marks, or stick up two Knives there, the distance between them is near the *Length* of the inside of the *Cask*.

Probl.

Probl. 138. FIG. 31.

Given AB the Bung Diameter, CD the Head Diameter, and EP the Axe or Length of a Cask taken as the middle Frustum of a Spheroid, also GH the Diameter of the Liquor's Surface, and EI or IP the Dry or Wet Inches; to find the Vacuity, or remaining Liquor, viz. GHDC or GCDH, and the Converse.

Theor.

To the double Area of the Bung Diameter, add the Area of the Diameter of the Liquor's Surface, a third part of the Sum is the Area of a mean Circle.

S. Or thus without the Diameter of the Liquor's Surface.

Divide the Difference of the Areas of AB and CD the Bung and Head Diameters, by 3 times the Square of KP the Semi-length, and multiply the Quotient by the Square of IK the distance from the plane of the Bung to the Surface of the Liquor, the Product subducted from the Area of the Bung Diameter will leave the Area of a mean Circle, which multiplied by IK, produces the Content of the Frustum ABHG, and that subducted from, or added to ABDC the Cask's Semi-content according as it is less, or more than half full, the Remainder or Sum will be the Vacuity, or remaining Liquor, viz. GHDC, or GCDH.

Thus this Problem is solved without the Spheroid's Axe, and the work in Mich. Dary much abbreviated.

And forasmuch as the second Differences are equal, the Content upon every Inch may be found by an easie Collection.

262 Of a standing Cask part empty.

Compare this with *Probl. iv. Chap. ix. of Mr. Everard's Stereom.*

§. Or by the Table of Segments of a Spheroid.

Find the *Head Diameter* in the *first Column* of the *Table of Proportional Diameters*, and against it in some of the other *Columns* you will find the *Bung Diameter*, or a number near it, then say—

As the *Axe or Length*, is to the *Radius 100*;

So is the *Dry, or Wet Inches*, to a fourth *Proportional*. Which found in the *Table* under *Axe*, directly against it, in the *same Column* that you found the *Bung Diameter* in, in the *Table of Proportional Diameters* stands a *Segment*, by which if you multiply the *Content* of the *whole Cask*, the *Product* will be the *Vacuity*, or remaining *Liquor*.

The Converse.

Divide the *Difference* between the *Semi-content*, and *Vacuity*, or remaining *Liquor* by the *Area of a mean Circle*, the *Quotient* is *IK* the *distance* from the *plane* of the *Bung* to the *Surface* of the *Liquor*, which subtracted from, or added to *KE* or *KP* the *Semi-length* gives *EI* or *IP* the *Dry, or Wet Inches*.

§. Or by the Table of Segments of a Spheroid.

As the *Cask's whole Content*, is to 1000;

So is the *Vacuity*, or remaining *Liquor*, to a *Segment*. Which found in the *Table* in the *same Column* that you found the *Bung Diameter* in, in the *Table of Proportional Diameters*, against it under [*Axe*] stands a *Number*, by which if you multiply the *Axe or Length*, the *Product* will be *EI* or *IP* the *Dry or Wet Inches*.

A Table of Head and Bung Diameters in Proportion, useful in finding the Vacuity of a Spheroidal or Parabolical Cask, Standing with its AXE perpendicular to the Horizon, Or Lying with its AXE parallel thereto.

H.D.	1	2	3	4	H.D.	1	2	3	4
10.0	11.00	11.60	12.20	12.80	16.0	17.60	18.56	19.52	20.48
.2	11.22	11.83	12.44	13.06	.2	17.82	18.79	19.76	20.74
.4	11.44	12.06	12.69	13.31	.4	18.04	19.02	20.01	20.99
.6	11.66	12.30	12.93	13.57	.6	18.26	19.26	20.25	21.25
.8	11.88	12.53	13.18	13.82	.8	18.48	19.49	20.50	21.50
11.0	12.10	12.76	13.42	14.08	17.0	18.70	19.72	20.74	21.76
.2	12.32	12.99	13.66	14.34	.2	18.92	19.95	20.98	22.00
.4	12.54	13.22	13.91	14.59	.4	19.14	20.18	21.23	22.27
.6	12.76	13.46	14.15	14.85	.6	19.36	20.42	21.47	22.53
.8	12.98	13.69	14.40	15.10	.8	19.58	20.65	21.72	22.78
12.0	13.20	13.92	14.64	15.36	18.0	19.80	20.88	21.96	23.04
.2	13.42	14.15	14.88	15.62	.2	20.02	21.11	22.20	23.30
.4	13.64	14.38	15.13	15.87	.4	20.24	21.34	22.45	23.55
.6	13.86	14.62	15.37	16.13	.6	20.46	21.58	22.69	23.81
.8	14.08	14.85	15.62	16.38	.8	20.68	21.81	22.94	24.06
13.0	14.30	15.08	15.86	16.64	19.0	20.90	22.04	23.18	24.32
.2	14.52	15.31	16.10	16.90	.2	21.12	22.27	23.42	24.58
.4	14.74	15.54	16.35	17.15	.4	21.34	22.50	23.67	24.83
.6	14.96	15.78	16.59	17.41	.6	21.56	22.74	23.91	25.09
.8	15.18	16.01	16.84	17.66	.8	21.78	22.97	24.16	25.34
14.0	15.40	16.24	17.08	17.92	20.0	22.00	23.20	24.40	25.60
.2	15.62	16.47	17.32	18.18	.2	22.22	23.43	24.64	25.86
.4	15.84	16.70	17.57	18.43	.4	22.44	23.66	24.89	26.11
.6	16.06	16.94	17.81	18.69	.6	22.66	23.90	25.13	26.37
.8	16.28	17.17	18.06	18.94	.8	22.88	24.13	25.38	26.60
15.0	16.50	17.40	18.30	19.20	21.0	23.10	24.36	25.62	26.88
.2	16.72	17.63	18.54	19.45	.2	23.32	24.59	25.86	27.14
.4	16.94	17.86	18.79	19.71	.4	23.54	24.82	26.11	27.39
.6	17.16	18.10	19.03	19.97	.6	23.76	25.06	26.35	27.65
.8	17.38	18.33	19.28	20.22	.8	23.98	25.20	26.60	27.90

Head and Bung Diameters in proportion.

HD	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220	221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260	261	262	263	264	265	266	267	268	269	270	271	272	273	274	275	276	277	278	279	280	281	282	283	284	285	286	287	288	289	290	291	292	293	294	295	296	297	298	299	300	301	302	303	304	305	306	307	308	309	310	311	312	313	314	315	316	317	318	319	320	321	322	323	324	325	326	327	328	329	330	331	332	333	334	335	336	337	338	339	340	341	342	343	344	345	346	347	348	349	350	351	352	353	354	355	356	357	358	359	360	361	362	363	364	365	366	367	368	369	370	371	372	373	374	375	376	377	378	379	380	381	382	383	384	385	386	387	388	389	390	391	392	393	394	395	396	397	398	399	400	401	402	403	404	405	406	407	408	409	410	411	412	413	414	415	416	417	418	419	420	421	422	423	424	425	426	427	428	429	430	431	432	433	434	435	436	437	438	439	440	441	442	443	444	445	446	447	448	449	450	451	452	453	454	455	456	457	458	459	460	461	462	463	464	465	466	467	468	469	470	471	472	473	474	475	476	477	478	479	480	481	482	483	484	485	486	487	488	489	490	491	492	493	494	495	496	497	498	499	500	501	502	503	504	505	506	507	508	509	510	511	512	513	514	515	516	517	518	519	520	521	522	523	524	525	526	527	528	529	530	531	532	533	534	535	536	537	538	539	540	541	542	543	544	545	546	547	548	549	550	551	552	553	554	555	556	557	558	559	560	561	562	563	564	565	566	567	568	569	570	571	572	573	574	575	576	577	578	579	580	581	582	583	584	585	586	587	588	589	590	591	592	593	594	595	596	597	598	599	600	601	602	603	604	605	606	607	608	609	610	611	612	613	614	615	616	617	618	619	620	621	622	623	624	625	626	627	628	629	630	631	632	633	634	635	636	637	638	639	640	641	642	643	644	645	646	647	648	649	650	651	652	653	654	655	656	657	658	659	660	661	662	663	664	665	666	667	668	669	670	671	672	673	674	675	676	677	678	679	680	681	682	683	684	685	686	687	688	689	690	691	692	693	694	695	696	697	698	699	700	701	702	703	704	705	706	707	708	709	710	711	712	713	714	715	716	717	718	719	720	721	722	723	724	725	726	727	728	729	730	731	732	733	734	735	736	737	738	739	740	741	742	743	744	745	746	747	748	749	750	751	752	753	754	755	756	757	758	759	760	761	762	763	764	765	766	767	768	769	770	771	772	773	774	775	776	777	778	779	780	781	782	783	784	785	786	787	788	789	790	791	792	793	794	795	796	797	798	799	800	801	802	803	804	805	806	807	808	809	810	811	812	813	814	815	816	817	818	819	820	821	822	823	824	825	826	827	828	829	830	831	832	833	834	835	836	837	838	839	840	841	842	843	844	845	846	847	848	849	850	851	852	853	854	855	856	857	858	859	860	861	862	863	864	865	866	867	868	869	870	871	872	873	874	875	876	877	878	879	880	881	882	883	884	885	886	887	888	889	890	891	892	893	894	895	896	897	898	899	900	901	902	903	904	905	906	907	908	909	910	911	912	913	914	915	916	917	918	919	920	921	922	923	924	925	926	927	928	929	930	931	932	933	934	935	936	937	938	939	940	941	942	943	944	945	946	947	948	949	950	951	952	953	954	955	956	957	958	959	960	961	962	963	964	965	966	967	968	969	970	971	972	973	974	975	976	977	978	979	980	981	982	983	984	985	986	987	988	989	990	991	992	993	994	995	996	997	998	999	1000	1001	1002	1003	1004	1005	1006	1007	1008	1009	1010	1011	1012	1013	1014	1015	1016	1017	1018	1019	1020	1021	1022	1023	1024	1025	1026	1027	1028	1029	1030	1031	1032	1033	1034	1035	1036	1037	1038	1039	1040	1041	1042	1043	1044	1045	1046	1047	1048	1049	1050	1051	1052	1053	1054	1055	1056	1057	1058	1059	1060	1061	1062	1063	1064	1065	1066	1067	1068	1069	1070	1071	1072	1073	1074	1075	1076	1077	1078	1079	1080	1081	1082	1083	1084	1085	1086	1087	1088	1089	1090	1091	1092	1093	1094	1095	1096	1097	1098	1099	1100	1101	1102	1103	1104	1105	1106	1107	1108	1109	1110	1111	1112	1113	1114	1115	1116	1117	1118	1119	1120	1121	1122	1123	1124	1125	1126	1127	1128	1129	1130	1131	1132	1133	1134	1135	1136	1137	1138	1139	1140	1141	1142	1143	1144	1145	1146	1147	1148	1149	1150	1151	1152	1153	1154	1155	1156	1157	1158	1159	1160	1161	1162	1163	1164	1165	1166	1167	1168	1169	1170	1171	1172	1173	1174	1175	1176	1177	1178	1179	1180	1181	1182	1183	1184	1185	1186	1187	1188	1189	1190	1191	1192	1193	1194	1195	1196	1197	1198	1199	1200	1201	1202	1203	1204	1205	1206	1207	1208	1209	1210	1211	1212	1213	1214	1215	1216	1217	1218	1219	1220	1221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SEGMENTS of the middle Frustum of a SPHEROEID whose AXE is supposed to be divided by planes *parallel* to the BUNG DIAMETER into 100 equal parts, and the Content 1.000. Useful in finding the Vacuity of a Spheroidal Cask, the AXE standing *Perpendicular* to the HORIZON.

Axe	1	2	3	4
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.01	.009	.008	.008	.007
.02	.018	.017	.015	.014
.03	.027	.025	.023	.021
.04	.036	.033	.031	.029
.05	.045	.042	.039	.036

.06	.054	.051	.047	.044
.07	.063	.060	.056	.052
.08	.072	.068	.064	.060
.09	.082	.077	.073	.069
.10	.091	.087	.082	.077

.11	.101	.098	.093	.086
.12	.110	.106	.100	.095
.13	.119	.114	.109	.104
.14	.128	.123	.118	.113
.15	.137	.133	.127	.122

.16	.149	.143	.137	.131
.17	.159	.152	.147	.141
.18	.168	.162	.156	.150
.19	.178	.172	.166	.160
.20	.188	.182	.176	.170

Axe	1	2	3	4
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.21	.198	.192	.186	.180
.22	.208	.202	.196	.190
.23	.218	.212	.206	.200
.24	.228	.222	.216	.210
.25	.239	.233	.226	.220

.26	.249	.243	.237	.231
.27	.259	.253	.247	.241
.28	.269	.263	.257	.251
.29	.279	.274	.268	.262
.30	.290	.284	.278	.273

.31	.300	.295	.289	.284
.32	.310	.305	.300	.295
.33	.321	.316	.311	.306
.34	.332	.327	.322	.317
.35	.342	.337	.333	.328

.36	.352	.348	.344	.340
.37	.363	.359	.355	.351
.38	.373	.369	.365	.362
.39	.383	.380	.376	.373
.40	.394	.391	.388	.385

Seg-

Segments of a Spheroid for a Standing Cask.

Axe	1	2	3	4
.41	.405	.402	.399	.396
.42	.415	.413	.410	.407
.43	.426	.424	.421	.419
.44	.436	.434	.432	.430
.45	.447	.445	.444	.442
.46	.458	.456	.455	.453
.47	.468	.467	.466	.465
.48	.479	.478	.477	.477
.49	.489	.489	.489	.488
.50	.500	.500	.500	.500
.51	.511	.511	.511	.512
.52	.521	.522	.523	.523
.53	.532	.533	.534	.535
.54	.542	.544	.545	.547
.55	.553	.555	.556	.558
.56	.564	.566	.568	.570
.57	.574	.576	.579	.581
.58	.585	.587	.590	.593
.59	.595	.598	.601	.604
.60	.606	.609	.612	.615
.61	.617	.620	.624	.627
.62	.627	.631	.635	.638
.63	.637	.641	.645	.649
.64	.648	.652	.656	.660
.65	.658	.663	.667	.672
.66	.669	.673	.678	.683
.67	.679	.684	.689	.694
.68	.690	.695	.700	.705
.69	.700	.705	.711	.716
.70	.710	.716	.722	.727

Axe	1	2	3	4
.71	.721	.726	.732	.738
.72	.731	.737	.743	.749
.73	.741	.747	.753	.759
.74	.751	.757	.763	.769
.75	.761	.767	.774	.780
.76	.772	.778	.784	.790
.77	.782	.788	.794	.800
.78	.792	.798	.804	.810
.79	.802	.808	.814	.820
.80	.812	.818	.824	.830
.81	.822	.828	.834	.840
.82	.832	.838	.844	.850
.83	.841	.847	.853	.859
.84	.851	.857	.863	.869
.85	.861	.867	.873	.878
.86	.871	.876	.882	.887
.87	.880	.886	.891	.896
.88	.890	.895	.900	.905
.89	.899	.904	.909	.914
.90	.909	.913	.918	.923
.91	.918	.923	.927	.931
.92	.928	.932	.936	.940
.93	.937	.940	.944	.948
.94	.946	.949	.953	.956
.95	.955	.958	.961	.964
.96	.964	.967	.969	.971
.97	.973	.975	.977	.979
.98	.982	.983	.985	.986
.99	.991	.992	.992	.993

Probl. 139. FIG. 24.

Given CD the Bung Diameter, cd the Head Diameter, and LO the Axe or Length of a Cask taken as the middle Frustum of a Spheroid, or Parabolick Spindle, also CG or GD the Dry or Wet Inches; to find the Vacuity, or remaining Liquor, viz. $CcSic$, or $SdDdI$: and the Converse.

Theor.

Find the Head Diameter in the first Column of the Table of Proportional Diameters, and against it in some of the other Columns you will find the Bung Diameter, or one very near it; then say —

As the Bung Diameter, is to the Radius 100;

So is the Dry, or Wet Inches, to a Versed Sine.

Which found in the Table under V.S. against it in the same Column that you found the Bung Diameter in, in the Table of Proportional Diameters, stands a Segment, which multiplied by the Content of the whole Cask, the Product will be the Vacuity, or remaining Liquor.

The Converse.

As the whole Content, is to 1000;

So is the Vacuity, or remaining Liquor, to a Segment.

Which found in the Table in the same Column that you found the Bung Diameter in, in the Table of Proportional Diameters, against it under V.S. stands a Versed Sine, by which if you multiply the Bung Diameter, the Product will be CG or GD the Dry, or Wet Inches.

Note, when the Dry Inches are less than the Semi-difference of the Diameters, then the Liquor cuts not the Heads.

A TABLE

O.F.

SEGMENTS of the middle Frustrum of a SPHEROEID whose BUNG DIAMETER is supposed to be divided by planes *parallel* to the AXE into 100 equal parts, and the Content 1.000, Useful in finding the Vacuity of a Spheroidal Cask, the Surface of the Liquor cutting the Heads, and the Axe lying *parallel* to the HORIZON.

V.S.	1	2	3	4	V.S.	1	2	3	4
.01					.01	.144	.139	.135	.131
.02	.001				.22	.154	.150	.145	.141
.03	.003	.001			.23	.161	.156	.151	.147
.04	.007	.005	.002	.001	.24	.166	.162	.157	.153
.05	.012	.008	.006	.003	.25	.171	.167	.162	.158
.06	.017	.013	.010	.007	.26	.176	.172	.167	.163
.07	.023	.019	.015	.012	.27	.181	.177	.172	.168
.08	.029	.025	.021	.018	.28	.186	.182	.177	.173
.09	.036	.032	.028	.024	.29	.191	.187	.182	.178
.10	.043	.039	.034	.030	.30	.196	.192	.187	.183
.11	.051	.046	.042	.038	.31	.201	.197	.192	.188
.12	.059	.054	.050	.046	.32	.206	.202	.197	.193
.13	.067	.062	.058	.054	.33	.211	.207	.202	.198
.14	.076	.071	.066	.062	.34	.216	.212	.207	.203
.15	.085	.080	.075	.071	.35	.221	.217	.212	.208
.16	.094	.089	.084	.080	.36	.226	.222	.217	.213
.17	.103	.098	.094	.090	.37	.231	.227	.222	.218
.18	.113	.108	.104	.100	.38	.236	.232	.227	.223
.19	.123	.118	.114	.110	.39	.241	.237	.232	.228
.20	.133	.128	.124	.120	.40	.246	.242	.237	.233

269 *Segments of a Spheroid for a Lying Cask.*

V.S.	1	2	3	4
.41	.383	.381	.379	.377
.42	.396	.394	.392	.391
.43	.409	.407	.406	.404
.44	.422	.420	.419	.418
.45	.435	.433	.432	.432
.46	.448	.447	.446	.445
.47	.461	.460	.460	.459
.48	.474	.473	.473	.473
.49	.487	.487	.487	.486
.50	.500	.500	.500	.500
.51	.513	.513	.513	.514
.52	.526	.527	.527	.527
.53	.539	.540	.540	.541
.54	.552	.553	.554	.555
.55	.565	.567	.568	.568
.56	.578	.580	.581	.582
.57	.591	.593	.594	.596
.58	.604	.606	.608	.609
.59	.617	.619	.621	.623
.60	.630	.632	.634	.636
.61	.643	.645	.647	.649
.62	.656	.658	.660	.663
.63	.668	.671	.674	.676
.64	.681	.684	.686	.689
.65	.693	.697	.699	.702
.66	.706	.710	.712	.715
.67	.718	.722	.725	.728
.68	.730	.734	.738	.740
.69	.743	.747	.750	.753
.70	.755	.759	.762	.768

V.S.	1	2	3	4
.71	.767	.771	.774	.778
.72	.778	.782	.786	.790
.73	.790	.794	.798	.802
.74	.801	.806	.810	.814
.75	.813	.817	.821	.825
.76	.824	.828	.833	.836
.77	.835	.839	.844	.848
.78	.846	.850	.855	.859
.79	.856	.861	.865	.869
.80	.867	.872	.876	.880
.81	.877	.882	.886	.890
.82	.887	.892	.896	.900
.83	.897	.902	.906	.910
.84	.906	.911	.915	.919
.85	.915	.920	.925	.929
.86	.924	.929	.934	.938
.87	.933	.938	.942	.946
.88	.941	.946	.950	.954
.89	.949	.954	.958	.962
.90	.957	.961	.966	.970
.91	.964	.968	.972	.976
.92	.971	.975	.979	.982
.93	.977	.981	.985	.988
.94	.983	.987	.990	.993
.95	.988	.992	.994	.997
.96	.993	.995	.998	.999
.97	.997	.999		
.98	.999			
.99				

A TABLE

OF

SEGMENTS of the middle Frustrum of a PARABOLICK SPINDLE whose BUNG DIAMETER is supposed to be divided by planes *parallel* to the AXE into 100 equal parts, and the Content 1,000.

Useful in finding the Vacuity of a Parabolical Cask, the Surface of the Liquor cutting the Heads, and the AXE lying *parallel* to the HORIZON.

V.S.	1	2	3	4
.01				
.02				
.03	.003			
.04	.006	.003	.001	
.05	.010	.007	.004	.002
.06	.015	.011	.007	.005
.07	.021	.017	.013	.009
.08	.028	.023	.019	.015
.09	.034	.029	.024	.020
.10	.041	.036	.031	.027
.11	.049	.043	.038	.034
.12	.057	.051	.046	.041
.13	.065	.059	.054	.049
.14	.074	.068	.063	.058
.15	.083	.077	.071	.066
.16	.092	.086	.081	.076
.17	.102	.096	.090	.085
.18	.111	.105	.100	.095
.19	.121	.115	.110	.105
.20	.132	.126	.120	.115

V.S.	1	2	3	4
.21	.142	.136	.131	.126
.22	.153	.147	.142	.137
.23	.163	.158	.153	.148
.24	.174	.169	.164	.159
.25	.186	.180	.175	.171
.26	.197	.192	.187	.183
.27	.209	.203	.199	.195
.28	.220	.215	.211	.207
.29	.232	.227	.223	.219
.30	.244	.239	.235	.231
.31	.256	.252	.248	.244
.32	.268	.264	.260	.257
.33	.280	.276	.273	.269
.34	.293	.289	.286	.282
.35	.305	.302	.298	.295
.36	.318	.315	.312	.309
.37	.331	.328	.325	.322
.38	.343	.340	.338	.335
.39	.356	.354	.351	.349
.40	.369	.367	.364	.362

Seg-

Segments of a Parabolick Spindle for a Lying Cask.

V.S. | 1 | 2 | 3 | 4

.41 | .382 | .380 | .378 | .376
 .42 | .395 | .393 | .391 | .389
 .43 | .408 | .406 | .405 | .403
 .44 | .421 | .420 | .418 | .416
 .45 | .434 | .433 | .432 | .431

.46 | .447 | .446 | .445 | .445
 .47 | .460 | .460 | .459 | .458
 .48 | .474 | .473 | .473 | .472
 .49 | .487 | .487 | .486 | .486
 .50 | .500 | .500 | .500 | .500

.51 | .513 | .513 | .514 | .514
 .52 | .516 | .517 | .517 | .518
 .53 | .540 | .540 | .541 | .542
 .54 | .553 | .554 | .555 | .555
 .55 | .566 | .567 | .568 | .569

.56 | .579 | .580 | .582 | .584
 .57 | .592 | .594 | .595 | .597
 .58 | .605 | .607 | .609 | .611
 .59 | .618 | .620 | .622 | .624
 .60 | .631 | .633 | .636 | .638

.61 | .644 | .646 | .649 | .651
 .62 | .657 | .659 | .662 | .665
 .63 | .669 | .672 | .675 | .678
 .64 | .682 | .685 | .688 | .691
 .65 | .695 | .698 | .702 | .705

.66 | .707 | .711 | .714 | .718
 .67 | .720 | .724 | .727 | .731
 .68 | .732 | .736 | .740 | .743
 .69 | .744 | .748 | .752 | .756
 .70 | .756 | .761 | .765 | .769

V.S. | 1 | 2 | 3 | 4

.71 | .768 | .773 | .777 | .781
 .72 | .780 | .785 | .789 | .793
 .73 | .791 | .797 | .801 | .805
 .74 | .803 | .808 | .813 | .817
 .75 | .814 | .820 | .825 | .829

.76 | .826 | .831 | .836 | .831
 .77 | .837 | .842 | .847 | .852
 .78 | .847 | .853 | .858 | .863
 .79 | .858 | .864 | .869 | .873
 .80 | .868 | .874 | .880 | .885

.81 | .879 | .885 | .890 | .895
 .82 | .889 | .895 | .900 | .905
 .83 | .898 | .904 | .910 | .915
 .84 | .908 | .914 | .919 | .924
 .85 | .917 | .923 | .929 | .934

.86 | .926 | .932 | .937 | .942
 .87 | .935 | .941 | .946 | .951
 .88 | .943 | .949 | .954 | .959
 .89 | .951 | .957 | .962 | .966
 .90 | .959 | .964 | .969 | .973

.91 | .966 | .971 | .976 | .980
 .92 | .972 | .977 | .981 | .985
 .93 | .979 | .983 | .987 | .991
 .94 | .985 | .989 | .993 | .995
 .95 | .990 | .995 | .996 | .998

.96 | .994 | .997 | .999 |
 .97 | .997 |
 .98 |
 .99 |

272 — Of a Lying Cask part empty.

See the *Construction and Demonstration* as I received it from the learned M. Isaac Newton, Professor of the *Mathematicks* in Cambridge.

Construction. FIG. 32.

Let *AVBC* be a *Paraboloid*, whose *Vertex* is *V*, and *Axe* *VC* *Perpendicular* to its *Base* *AB*, and let its *Segment* *ADE* cut off with the plane *DE* *Perpendicular* also to its *Base* be required.

With the *Center* *C*, and *Radius* *AC* describe the *Circle* *AHBI* cut in *F* and *G* by the plane *DE* produced, and in *AB* taking *BK* in proportion to $2\ DC$; as *DC*, to *AC*; erect the *Perpendicular* *HKI* cutting the *Circle* in *H* and *I*, then 8 times the *Segment* *AFDG* less *AH**KI* multiplied by $\frac{1}{3}$ of *VC* produces the *Content* of the *Segment* of the *Paraboloid* *ADE*.

Demonstration. FIG. 32.

For in *AD* take the infinitely little part *Dd*, and draw the *Perpendicular* *fdg* cutting the *Circle* in *f* and *g*, and the *Parabola* in *e*: and take *Bk* to $2\ dC$ as *dC* to *AC*: And if from the equal *Rectangles* *Bk* \times *AC* and $2\ dC^2$, you take the equal *Rectangles* *BK* \times *AC* and $2\ dC^2$ respectively, the *Remainders* *Kk* \times *AC* and $2\ dC^2 - 2\ DC^2$, that is $2\ dD^2 + 4\ d\ DC$ will be equal.

Whence *Kk* will be to *dD* as $4\ DC + 2\ Dd$ to *AC*, that is (if the infinitely little quantity $2\ Dd$ be neglected) as $4\ DC$ to *AC*: *Kk* therefore is equal to $\frac{4\ Dd \times DC}{AC}$.

And

And Because AC is to DC, as 2 DC to BK, and therefore BK equal to $\frac{2 DC^2}{AC}$ and AK is equal to $2 AC - BK$, that is to $\frac{2 AC^2 - 2 DC^2}{AC}$ or $\frac{2 DF^2}{AC}$; therefore the Rectangle AKB that is, KH^2 will be equal to $\frac{4 DF^2 \times DC^2}{AC^2}$ and KH equal to $\frac{2 DF \times DC}{AC}$ and the Area HI i h which is the Content under HI or 2 KM, and K k is equal to $\frac{16 d D \times DC^2}{AC^2}$ which subducted from 8 times the Area FG g f, or $16 d D \times DF$ leaves 16 times $d D \times DF \times \frac{AC^2 - DC^2}{AC^2}$ that is $\frac{16 d D \times DF^2}{AC^2}$ and this drawn into $\frac{1}{12}$ of VC produces $\frac{4 d D \times VC \times DF^2}{3 AC^2}$ that is $\frac{4}{3} d D \times DE \times DF$.

For by the nature of a Parabola DE is to VC, as ADB to ACB, or as DF^2 to AC^2 , and therefore DE is equal to $\frac{VC \times DF^2}{AC^2}$; so then 8 times FG g f - HI i h drawn into $\frac{1}{12}$ of VC is equal to $\frac{4}{3} d D \times DE \times DF$ that is to $\frac{4}{3} d D \times DE \times DF$, that is, to the infinitely thin Parabolick Segment DE ed of the Paraboloid A B V E; For the Height of this Segment is DE or de, the Thickness D d, and the Length of the Base FG, and two thirds of the Content of these three are the Parabolick Segment.

And by the same Argument, if the whole Segment ADE of the Paraboloid be by innumerable parallel planes FG, fg, ϕy &c. divided into an infinite number of parts ED d e, e d $\delta \epsilon$, &c. and the Segment AFG of the Circle be by the same planes divided into the same number of parts FG g f, fgy ϕ , &c. and

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the Segment AHI also divided into the same number of correspondent parts H I i h, h i i x, the Excess whereby any part of the Circular Segment AHI, if it be multiplied by $\frac{1}{2}$ of VC it becomes equal to the correspondent part of the Segment of the Paraboloid. And therefore the Sum of all the Excesses multiplied by $\frac{1}{2}$ of VC that is 8 AFG—AHI in $\frac{1}{2}$ VC is equal to the Sum of all the parts of the Segment ADE of the Paraboloid that is to the whole Segment, which was to be demonstrated.

Now if you divide the Content of the Segment of the Spheroid cBd in FIG. 24. by the Area of its Base cd, the Quotient will be Semi-axe of a Paraboloid whose Content will be equal to that Segment, with which work according to this Rule and you will have the Content of CIF the second Segment of the Spheroid, for—

As the Square of the Axe of the Paraboloid,
is to the Square of the said Axe prolong'd;
So is the Content of the Segment of the Paraboloid,
to the Content of the Segment of the Spheroid.

Which doubled, and subducted from the Frustrum CEF, the Remainder will be the vacant Frustrum CcSIc.

These Directions are necessary to be understood by every Gager, who designs to be truly serviceable to his King and Country; I shall conclude with that saying of Horace, Lib. 1. Epist. ult. used by several of our learned Countrymen:

*Siquid novisti rectius istis,
Candidus imperti, si non, his utere mecum.*

A TABLE

A
TABLE
OF THE
CONTENTS
OF
CYLINDERS
IN
ALE-GALLONS
AND
CENTESSIMAL PARTS.

Calculated to every Tenth part of an *Inch*
of the DIAMETER, from 12 to 40 *Inches*.
And from 1 to 35 *Inches* in DEPTH.

12 Inches Diameter.

Depth	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	0.40	0.41	0.41	0.42	0.43	0.44	0.44	0.45	0.46	0.46
2	0.80	0.82	0.82	0.84	0.86	0.88	0.88	0.90	0.92	0.92
3	1.20	1.22	1.22	1.26	1.29	1.32	1.32	1.35	1.38	1.38
4	1.60	1.64	1.64	1.68	1.72	1.76	1.76	1.80	1.84	1.84
5	2.00	2.05	2.05	2.10	2.15	2.20	2.20	2.25	2.30	2.30
6	2.40	2.46	2.46	2.52	2.58	2.64	2.64	2.70	2.76	2.76
7	2.80	2.87	2.87	2.94	3.01	3.08	3.08	3.15	3.22	3.22
8	3.20	3.28	3.28	3.36	3.44	3.52	3.52	3.60	3.68	3.68
9	3.60	3.69	3.69	3.78	3.87	3.96	3.96	4.05	4.14	4.14
10	4.00	4.10	4.10	4.20	4.30	4.40	4.40	4.50	4.60	4.60
11	4.40	4.51	4.51	4.62	4.73	4.84	4.84	4.95	5.06	5.06
12	4.80	4.92	4.92	5.04	5.16	5.28	5.28	5.40	5.52	5.52
13	5.20	5.33	5.33	5.46	5.59	5.72	5.72	5.85	5.98	5.98
14	5.60	5.74	5.74	5.88	6.02	6.16	6.16	6.30	6.44	6.44
15	6.00	6.15	6.15	6.30	6.45	6.60	6.60	6.75	6.90	6.90
16	6.40	6.56	6.56	6.72	6.88	7.04	7.04	7.20	7.36	7.36
17	6.80	6.97	6.97	7.14	7.31	7.48	7.48	7.65	7.82	7.82
18	7.20	7.38	7.38	7.56	7.74	7.92	7.92	8.10	8.28	8.28
19	7.60	7.79	7.79	7.98	8.17	8.36	8.36	8.55	8.74	8.74
20	8.00	8.20	8.20	8.40	8.60	8.80	8.80	9.00	9.20	9.20
21	8.40	8.61	8.61	8.82	9.03	9.24	9.24	9.45	9.66	9.66
22	8.80	9.02	9.02	9.24	9.46	9.68	9.68	9.90	10.12	10.12
23	9.20	9.43	9.43	9.66	9.89	10.12	10.12	10.35	10.58	10.58
24	9.60	9.84	9.84	10.08	10.32	10.56	10.56	10.80	11.04	11.04
25	10.00	10.25	10.25	10.50	10.75	11.00	11.00	11.25	11.50	11.50
26	10.40	10.66	10.66	10.92	11.18	11.44	11.44	11.70	11.96	11.96
27	10.80	11.07	11.07	11.34	11.61	11.88	11.88	12.15	12.42	12.42
28	11.20	11.48	11.48	11.76	12.04	12.32	12.32	12.60	12.88	12.88
29	11.60	11.89	11.89	12.18	12.47	12.76	12.76	13.05	13.34	13.34
30	12.00	12.30	12.30	12.60	12.90	13.20	13.20	13.50	13.80	13.80
31	12.40	12.71	12.71	13.02	13.33	13.64	13.64	13.95	14.26	14.26
32	12.80	13.12	13.12	13.44	13.76	14.08	14.08	14.40	14.72	14.72
33	13.20	13.53	13.53	13.86	14.19	14.52	14.52	14.85	15.18	15.18
34	13.60	13.94	13.94	14.28	14.62	14.96	14.96	15.30	15.64	15.64
35	14.00	14.35	14.35	14.70	15.05	15.40	15.40	15.75	16.10	16.10

Cylinders in Ale Gallons:

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13 Inches Diameter.

Depth	0	1	2	3	4	5	6	7	8	9
1	0.47	0.48	0.49	0.49	0.50	0.51	0.52	0.52	0.53	0.54
2	0.94	0.96	0.98	0.98	1.00	1.02	1.04	1.04	1.06	1.08
3	1.41	1.44	1.47	1.47	1.50	1.53	1.56	1.56	1.59	1.62
4	1.88	1.92	1.96	1.96	2.00	2.04	2.08	2.08	2.12	2.16
5	2.35	2.40	2.45	2.45	2.50	2.55	2.60	2.60	2.65	2.70
6	2.82	2.88	2.94	2.94	3.00	3.06	3.12	3.12	3.18	3.24
7	3.29	3.36	3.43	3.43	3.50	3.57	3.64	3.64	3.71	3.78
8	3.76	3.84	3.92	3.92	4.00	4.08	4.16	4.16	4.24	4.32
9	4.23	4.32	4.41	4.41	4.50	4.59	4.68	4.68	4.77	4.86
10	4.70	4.80	4.90	4.90	5.00	5.10	5.20	5.20	5.30	5.40
11	5.17	5.28	5.39	5.39	5.50	5.61	5.72	5.72	5.83	5.94
12	5.64	5.76	5.88	5.88	6.00	6.12	6.24	6.24	6.36	6.48
13	6.11	6.24	6.37	6.37	6.50	6.63	6.76	6.76	6.89	7.02
14	6.58	6.72	6.86	6.86	7.00	7.14	7.28	7.28	7.42	7.56
15	7.05	7.20	7.35	7.35	7.50	7.65	7.80	7.80	7.95	8.10
16	7.52	7.68	7.84	7.84	8.00	8.16	8.32	8.32	8.48	8.64
17	7.99	8.16	8.33	8.33	8.50	8.67	8.84	8.84	9.01	9.18
18	8.46	8.64	8.82	8.82	9.00	9.18	9.36	9.36	9.54	9.72
19	8.93	9.12	9.31	9.31	9.50	9.69	9.88	9.88	10.07	10.26
20	9.40	9.60	9.80	9.80	10.00	10.20	10.40	10.40	10.60	10.80
21	9.87	10.08	10.29	10.29	10.50	10.71	10.92	10.92	11.13	11.34
22	10.34	10.56	10.78	10.78	11.00	11.22	11.44	11.44	11.66	11.88
23	10.81	11.04	11.27	11.27	11.50	11.73	11.96	11.96	12.19	12.42
24	11.28	11.52	11.76	11.76	12.00	12.24	12.48	12.48	12.72	12.96
25	11.75	12.00	12.25	12.25	12.50	12.75	13.00	13.00	13.25	13.50
26	12.22	12.48	12.74	12.74	13.00	13.26	13.52	13.52	13.78	14.04
27	12.69	12.96	13.23	13.23	13.50	13.77	14.04	14.04	14.31	14.58
28	13.16	13.44	13.72	13.72	14.00	14.28	14.56	14.56	14.84	15.12
29	13.63	13.92	14.21	14.21	14.50	14.79	15.08	15.08	15.37	15.66
30	14.10	14.40	14.70	14.70	15.00	15.30	15.60	15.60	15.90	16.20
31	14.57	14.88	15.19	15.19	15.50	15.81	16.12	16.12	16.43	16.74
32	15.04	15.36	15.68	15.68	16.00	16.32	16.64	16.64	16.96	17.28
33	15.51	15.84	16.17	16.17	16.50	16.83	17.16	17.16	17.49	17.82
34	15.98	16.32	16.66	16.66	17.00	17.34	17.68	17.68	18.02	18.36
35	16.45	16.80	17.15	17.15	17.50	17.85	18.20	18.20	18.55	18.90

14 Inches Diameter.

Depth	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	0.55	0.55	0.56	0.57	0.58	0.59	0.59	0.60	0.61	0.62
2	1.10	1.10	1.12	1.14	1.16	1.18	1.18	1.20	1.22	1.24
3	1.65	1.65	1.68	1.71	1.74	1.77	1.77	1.80	1.83	1.86
4	2.20	2.20	2.24	2.28	2.32	2.36	2.36	2.40	2.44	2.48
5	2.75	2.75	2.80	2.85	2.90	2.95	2.95	3.00	3.05	3.10
6	3.30	3.30	3.36	3.42	3.48	3.54	3.54	3.60	3.66	3.72
7	3.85	3.85	3.92	3.99	4.06	4.13	4.13	4.20	4.27	4.34
8	4.40	4.40	4.48	4.56	4.64	4.72	4.72	4.80	4.88	4.96
9	4.95	4.95	5.04	5.13	5.22	5.31	5.31	5.40	5.49	5.58
10	5.50	5.50	5.60	5.70	5.80	5.90	5.90	6.00	6.10	6.20
11	6.05	6.05	6.16	6.27	6.38	6.49	6.49	6.60	6.71	6.82
12	6.60	6.60	6.72	6.84	6.96	7.08	7.08	7.20	7.32	7.44
13	7.15	7.15	7.28	7.41	7.54	7.67	7.67	7.80	7.93	8.06
14	7.70	7.70	7.84	7.98	8.12	8.26	8.26	8.40	8.54	8.68
15	8.25	8.25	8.40	8.55	8.70	8.85	8.85	9.00	9.15	9.30
16	8.80	8.80	8.96	9.12	9.28	9.44	9.44	9.60	9.76	9.92
17	9.35	9.35	9.52	9.69	9.86	10.03	10.03	10.20	10.37	10.54
18	9.90	9.90	10.08	10.26	10.44	10.62	10.62	10.80	10.98	11.16
19	10.45	10.45	10.64	10.83	11.02	11.21	11.21	11.40	11.59	11.78
20	11.00	11.00	11.20	11.40	11.60	11.80	11.80	12.00	12.20	12.40
21	11.55	11.55	11.76	11.97	12.18	12.39	12.39	12.60	12.81	13.02
22	12.10	12.10	12.32	12.54	12.76	12.98	12.98	13.20	13.42	13.64
23	12.65	12.65	12.88	13.11	13.34	13.57	13.57	13.80	14.03	14.26
24	13.20	13.20	13.44	13.68	13.92	14.16	14.16	14.40	14.64	14.88
25	13.75	13.75	14.00	14.25	14.50	14.75	14.75	15.00	15.25	15.50
26	14.30	14.30	14.56	14.82	15.08	15.34	15.34	15.60	15.86	16.12
27	14.85	14.85	15.12	15.39	15.66	15.93	15.93	16.20	16.47	16.74
28	15.40	15.40	15.68	15.96	16.24	16.52	16.52	16.80	17.08	17.36
29	15.95	15.95	16.24	16.53	16.82	17.11	17.11	17.40	17.69	17.98
30	16.50	16.50	16.80	17.10	17.40	17.70	17.70	18.00	18.30	18.60
31	17.05	17.05	17.36	17.67	17.98	18.29	18.29	18.60	18.91	19.22
32	17.60	17.60	17.92	18.24	18.56	18.88	18.88	19.20	19.52	19.84
33	18.15	18.15	18.48	18.81	19.14	19.47	19.47	19.80	20.13	20.46
34	18.70	18.70	19.04	19.38	19.72	20.06	20.06	20.40	20.74	21.08
35	19.25	19.25	19.60	19.95	20.30	20.65	20.65	21.00	21.35	21.70

Cylinders in Ale Gallons.

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15 Inches Diameter.

Depth	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	0.63	0.63	0.64	0.65	0.66	0.67	0.68	0.69	0.70	0.70
2	1.26	1.26	1.28	1.30	1.32	1.34	1.36	1.38	1.40	1.40
3	1.89	1.89	1.92	1.95	1.98	2.01	2.04	2.07	2.10	2.10
4	2.52	2.52	2.56	2.60	2.64	2.68	2.72	2.76	2.80	2.80
5	3.15	3.15	3.20	3.25	3.30	3.35	3.40	3.45	3.50	3.50
6	3.78	3.78	3.84	3.90	3.96	4.02	4.08	4.14	4.20	4.20
7	4.41	4.41	4.48	4.55	4.62	4.69	4.76	4.83	4.90	4.90
8	5.04	5.04	5.12	5.20	5.28	5.36	5.44	5.52	5.60	5.60
9	5.67	5.67	5.76	5.85	5.94	6.03	6.12	6.21	6.30	6.30
10	6.30	6.30	6.40	6.50	6.60	6.70	6.80	6.90	7.00	7.00
11	6.93	6.93	7.04	7.15	7.26	7.37	7.48	7.59	7.70	7.70
12	7.56	7.56	7.68	7.80	7.92	8.04	8.16	8.28	8.40	8.40
13	8.19	8.19	8.32	8.45	8.58	8.71	8.84	8.97	9.10	9.10
14	8.82	8.82	8.96	9.10	9.24	9.38	9.52	9.66	9.80	9.80
15	9.45	9.45	9.60	9.75	9.90	10.05	10.20	10.35	10.50	10.50
16	10.08	10.08	10.24	10.40	10.56	10.72	10.88	11.04	11.20	11.20
17	10.71	10.71	10.88	11.05	11.22	11.39	11.56	11.73	11.90	11.90
18	11.34	11.34	11.52	11.70	11.88	12.06	12.24	12.42	12.60	12.60
19	11.97	11.97	12.16	12.35	12.54	12.73	12.92	13.11	13.30	13.30
20	12.60	12.60	12.80	13.00	13.20	13.40	13.60	13.80	14.00	14.00
21	13.23	13.23	13.44	13.65	13.86	14.07	14.28	14.49	14.70	14.70
22	13.86	13.86	14.08	14.30	14.52	14.74	14.96	15.18	15.40	15.40
23	14.49	14.49	14.72	14.95	15.18	15.41	15.64	15.87	16.10	16.10
24	15.12	15.12	15.36	15.60	15.84	16.08	16.32	16.56	16.80	16.80
25	15.75	15.75	16.00	16.25	16.50	16.75	17.00	17.25	17.50	17.50
26	16.38	16.38	16.64	16.90	17.16	17.42	17.68	17.94	18.20	18.20
27	17.01	17.01	17.28	17.55	17.82	18.09	18.36	18.63	18.90	18.90
28	17.64	17.64	17.92	18.20	18.48	18.76	19.04	19.32	19.60	19.60
29	18.27	18.27	18.56	18.85	19.14	19.43	19.72	20.01	20.30	20.30
30	18.90	18.90	19.20	19.50	19.80	20.10	20.40	20.70	21.00	21.00
31	19.53	19.53	19.84	20.15	20.46	20.77	21.08	21.39	21.70	21.70
32	20.16	20.16	20.48	20.80	21.12	21.44	21.76	22.08	22.40	22.40
33	20.79	20.79	21.12	21.45	21.78	22.11	22.44	22.77	23.10	23.10
34	21.42	21.42	21.76	22.10	22.44	22.78	23.12	23.46	23.80	23.80
35	22.05	22.05	22.40	22.75	23.10	23.45	23.80	24.15	24.50	24.50

16 Inches Diameter.

Depth	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	0.71	0.72	0.73	0.74	0.75	0.76	0.77	0.78	0.79	0.80
2	1.42	1.44	1.46	1.48	1.50	1.52	1.54	1.56	1.58	1.60
3	2.13	2.16	2.19	2.22	2.25	2.28	2.31	2.34	2.37	2.40
4	2.84	2.88	2.92	2.96	3.00	3.04	3.08	3.12	3.16	3.20
5	3.55	3.60	3.65	3.70	3.75	3.80	3.85	3.90	3.95	4.00
6	4.26	4.32	4.38	4.44	4.50	4.56	4.62	4.68	4.74	4.80
7	4.97	5.04	5.11	5.18	5.25	5.32	5.39	5.46	5.53	5.60
8	5.68	5.76	5.84	5.92	6.00	6.08	6.16	6.24	6.32	6.40
9	6.39	6.48	6.57	6.66	6.75	6.84	6.93	7.02	7.11	7.20
10	7.10	7.20	7.30	7.40	7.50	7.60	7.70	7.80	7.90	8.00
11	7.81	7.92	8.03	8.14	8.25	8.36	8.47	8.58	8.69	8.80
12	8.52	8.64	8.76	8.88	9.00	9.12	9.24	9.36	9.48	9.60
13	9.23	9.36	9.49	9.62	9.75	9.88	10.01	10.14	10.27	10.40
14	9.94	10.08	10.22	10.36	10.50	10.64	10.78	10.92	11.06	11.20
15	10.65	10.80	10.95	11.10	11.25	11.40	11.55	11.70	11.85	12.00
16	11.36	11.52	11.68	11.84	12.00	12.16	12.32	12.48	12.64	12.80
17	12.07	12.24	12.41	12.58	12.75	12.92	13.09	13.26	13.43	13.60
18	12.78	12.96	13.14	13.32	13.50	13.68	13.86	14.04	14.22	14.40
19	13.49	13.68	13.87	14.06	14.25	14.44	14.63	14.81	15.01	15.20
20	14.40	14.40	14.60	14.80	15.00	15.20	15.40	15.60	15.80	16.00
21	14.91	15.12	15.33	15.54	15.75	15.96	16.17	16.38	16.59	16.80
22	15.62	15.84	16.06	16.28	16.50	16.72	16.94	17.16	17.38	17.60
23	16.33	16.56	16.79	17.02	17.25	17.48	17.71	17.94	18.17	18.40
24	17.04	17.28	17.52	17.76	18.00	18.24	18.48	18.72	18.96	19.20
25	17.75	18.00	18.25	18.50	18.75	19.00	19.25	19.50	19.75	20.00
26	18.46	18.72	18.98	19.24	19.50	19.76	20.02	20.28	20.54	20.80
27	19.17	19.44	19.71	19.98	20.25	20.52	20.79	21.06	21.33	21.60
28	19.88	20.16	20.44	20.72	21.00	21.28	21.56	21.84	22.12	22.40
29	20.59	20.88	21.17	21.46	21.75	22.04	22.33	22.62	22.91	23.20
30	21.30	21.60	21.90	22.20	22.50	22.80	23.10	23.40	23.70	24.00
31	22.01	22.32	22.63	22.94	23.25	23.56	23.87	24.18	24.49	24.80
32	22.72	23.04	23.36	23.68	24.00	24.32	24.64	24.96	25.28	25.60
33	23.43	23.76	24.09	24.42	24.75	25.08	25.41	25.74	26.07	26.40
34	24.14	24.48	24.82	25.16	25.50	25.84	26.18	26.52	26.86	27.20
35	24.85	25.20	25.55	25.90	26.25	26.60	26.95	27.30	27.65	28.00

Cylinders in Ale Gallons.

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17 Inches Diameter.

Depth	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	0.80	0.81	0.82	0.83	0.84	0.85	0.86	0.87	0.88	0.89
2	1.60	1.62	1.64	1.66	1.68	1.70	1.72	1.74	1.76	1.78
3	2.40	2.43	2.46	2.49	2.52	2.55	2.58	2.61	2.64	2.67
4	3.20	3.24	3.28	3.32	3.36	3.40	3.44	3.48	3.52	3.56
5	4.00	4.05	4.10	4.15	4.20	4.25	4.30	4.35	4.40	4.45
6	4.80	4.86	4.92	4.98	5.04	5.10	5.16	5.22	5.28	5.34
7	5.60	5.67	5.74	5.81	5.88	5.95	6.02	6.09	6.16	6.23
8	6.40	6.48	6.56	6.64	6.72	6.80	6.88	6.96	7.04	7.12
9	7.20	7.29	7.38	7.47	7.56	7.65	7.74	7.83	7.92	8.01
10	8.00	8.10	8.20	8.30	8.40	8.50	8.60	8.70	8.80	8.90
11	8.80	8.91	9.02	9.13	9.24	9.35	9.46	9.57	9.68	9.79
12	9.60	9.72	9.84	9.96	10.08	10.20	10.32	10.44	10.56	10.68
13	10.40	10.53	10.66	10.79	10.92	11.05	11.18	11.31	11.44	11.57
14	11.20	11.34	11.48	11.62	11.76	11.90	12.04	12.18	12.32	12.46
15	12.00	12.15	12.30	12.45	12.60	12.75	12.90	13.05	13.20	13.35
16	12.80	12.96	13.12	13.28	13.44	13.60	13.76	13.92	14.08	14.24
17	13.60	13.77	13.94	14.11	14.28	14.45	14.62	14.79	14.96	15.13
18	14.40	14.58	14.76	14.94	15.12	15.30	15.48	15.66	15.84	16.01
19	15.20	15.39	15.58	15.77	15.96	16.15	16.34	16.53	16.72	16.91
20	16.00	16.20	16.40	16.60	16.80	17.00	17.20	17.40	17.60	17.80
21	16.80	17.01	17.22	17.43	17.64	17.85	18.06	18.27	18.48	18.69
22	17.60	17.81	18.04	18.26	18.48	18.70	18.92	19.14	19.36	19.58
23	18.40	18.63	18.86	19.09	19.32	19.55	19.78	20.01	20.24	20.47
24	19.20	19.44	19.68	19.92	20.16	20.40	20.64	20.88	21.12	21.36
25	20.00	20.25	20.50	20.75	21.00	21.25	21.50	21.75	22.00	22.25
26	20.80	21.06	21.32	21.58	21.84	22.10	22.36	22.62	22.88	23.14
27	21.60	21.87	22.14	22.41	22.68	22.95	23.22	23.49	23.76	24.03
28	22.40	22.68	22.96	23.24	23.52	23.80	24.08	24.36	24.64	24.92
29	23.20	23.49	23.78	24.07	24.36	24.65	24.94	25.23	25.52	25.81
30	24.00	24.30	24.60	24.90	25.20	25.50	25.80	26.10	26.40	26.70
31	24.80	25.11	25.42	25.73	26.04	26.35	26.66	26.97	27.28	27.59
32	25.60	25.92	26.24	26.56	26.88	27.20	27.52	27.84	28.16	28.48
33	26.40	26.73	27.06	27.39	27.72	28.05	28.38	28.71	29.04	29.37
34	27.20	27.54	27.88	28.22	28.56	28.90	29.24	29.58	29.92	30.26
35	28.00	28.35	28.70	29.05	29.40	29.75	30.10	30.45	30.80	31.15

18 Inches Diameter.

Depth	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	0.90	0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99
2	1.80	1.82	1.84	1.86	1.88	1.90	1.92	1.94	1.96	1.98
3	2.70	2.73	2.76	2.79	2.82	2.85	2.88	2.91	2.94	2.97
4	3.60	3.64	3.68	3.72	3.76	3.80	3.84	3.88	3.92	3.96
5	4.50	4.55	4.60	4.65	4.70	4.75	4.80	4.85	4.90	4.95
6	5.40	5.46	5.52	5.58	5.64	5.70	5.76	5.82	5.88	5.94
7	6.30	6.37	6.44	6.51	6.58	6.65	6.72	6.79	6.86	6.93
8	7.20	7.28	7.36	7.44	7.52	7.60	7.68	7.76	7.84	7.92
9	8.10	8.19	8.28	8.37	8.46	8.55	8.64	8.73	8.82	8.91
10	9.00	9.10	9.20	9.30	9.40	9.50	9.60	9.70	9.80	9.90
11	9.90	10.01	10.12	10.23	10.34	10.45	10.56	10.67	10.78	10.89
12	10.80	10.92	11.04	11.16	11.28	11.40	11.52	11.64	11.76	11.88
13	11.70	11.83	11.96	12.09	12.22	12.35	12.48	12.61	12.74	12.87
14	12.60	12.74	12.88	13.02	13.16	13.30	13.44	13.58	13.72	13.86
15	13.50	13.65	13.80	13.95	14.10	14.25	14.40	14.55	14.70	14.85
16	14.40	14.56	14.72	14.88	15.04	15.20	15.36	15.52	15.68	15.84
17	15.30	15.47	15.64	15.81	15.98	16.15	16.32	16.49	16.66	16.83
18	16.20	16.38	16.56	16.74	16.92	17.10	17.28	17.46	17.64	17.82
19	17.10	17.29	17.48	17.67	17.86	18.05	18.24	18.43	18.62	18.81
20	18.00	18.20	18.40	18.60	18.80	19.00	19.20	19.40	19.60	19.80
21	18.90	19.11	19.32	19.53	19.74	19.95	20.16	20.37	20.58	20.79
22	19.80	20.02	20.24	20.46	20.68	20.90	21.12	21.34	21.56	21.78
23	20.70	20.93	21.16	21.39	21.62	21.85	22.08	22.31	22.54	22.77
24	21.60	21.84	22.08	22.32	22.56	22.80	23.04	23.28	23.52	23.76
25	22.50	22.75	23.00	23.25	23.50	23.75	24.00	24.25	24.50	24.75
26	23.40	23.66	23.92	24.18	24.44	24.70	24.96	25.22	25.48	25.74
27	24.30	24.57	24.84	25.11	25.38	25.65	25.92	26.19	26.46	26.73
28	25.20	25.48	25.76	26.04	26.32	26.60	26.88	27.16	27.44	27.72
29	26.10	26.39	26.68	26.97	27.26	27.55	27.84	28.13	28.42	28.71
30	27.00	27.30	27.60	27.90	28.20	28.50	28.80	29.10	29.40	29.70
31	27.90	28.21	28.52	28.83	29.14	29.45	29.76	30.07	30.38	30.69
32	28.80	29.12	29.44	29.76	30.08	30.40	30.72	31.04	31.36	31.68
33	29.70	30.03	30.36	30.69	31.02	31.35	31.68	32.01	32.34	32.67
34	30.60	30.94	31.28	31.62	31.96	32.30	32.64	32.98	33.32	33.66
35	31.50	31.85	32.20	32.55	32.90	33.25	33.60	33.95	34.30	34.65

Cylinders in Ale Gallons.

28

19 Inches Diameter.

Depth	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	1.01	1.02	1.03	1.04	1.05	1.06	1.07	1.08	1.09	1.1
2	2.02	2.04	2.06	2.08	2.10	2.12	2.14	2.16	2.18	2.2
3	3.03	3.06	3.09	3.12	3.15	3.18	3.21	3.24	3.27	3.3
4	4.04	4.08	4.12	4.16	4.20	4.24	4.28	4.32	4.36	4.4
5	5.05	5.10	5.15	5.20	5.25	5.30	5.35	5.40	5.45	5.5
6	6.06	6.12	6.18	6.24	6.30	6.36	6.42	6.48	6.54	6.6
7	7.07	7.14	7.21	7.28	7.35	7.42	7.49	7.56	7.63	7.7
8	8.08	8.16	8.24	8.32	8.40	8.48	8.56	8.64	8.72	8.8
9	9.09	9.18	9.27	9.36	9.45	9.54	9.63	9.72	9.81	9.9
10	10.10	10.20	10.30	10.40	10.50	10.60	10.70	10.80	10.90	11.0
11	11.11	11.22	11.33	11.44	11.55	11.66	11.77	11.88	11.99	12.1
12	12.12	12.24	12.36	12.48	12.60	12.72	12.84	12.96	13.08	13.2
13	13.13	13.26	13.39	13.52	13.65	13.78	13.91	14.04	14.17	14.3
14	14.14	14.28	14.42	14.56	14.70	14.84	14.98	15.12	15.26	15.4
15	15.15	15.30	15.45	15.60	15.75	15.90	16.05	16.20	16.35	16.5
16	16.16	16.32	16.48	16.64	16.80	16.96	17.12	17.28	17.44	17.6
17	17.17	17.34	17.51	17.68	17.85	18.02	18.19	18.36	18.53	18.7
18	18.18	18.36	18.54	18.72	18.90	19.08	19.26	19.44	19.62	19.8
19	19.19	19.38	19.57	19.76	19.95	20.14	20.33	20.52	20.71	20.9
20	20.20	20.40	20.60	20.80	21.00	21.20	21.40	21.60	21.80	22.0
21	21.21	21.42	21.63	21.84	22.05	22.26	22.47	22.68	22.89	23.1
22	22.22	22.44	22.66	22.88	23.10	23.32	23.54	23.76	23.98	24.2
23	23.23	23.46	23.69	23.92	24.15	24.38	24.61	24.84	25.07	25.3
24	24.24	24.48	24.72	24.96	25.20	25.44	25.68	25.92	26.16	26.4
25	25.25	25.50	25.75	26.00	26.25	26.50	26.75	27.00	27.25	27.5
26	26.26	26.52	26.78	27.04	27.30	27.56	27.82	28.08	28.34	28.6
27	27.27	27.54	27.81	28.08	28.35	28.62	28.89	29.16	29.43	29.7
28	28.28	28.56	28.84	29.12	29.40	29.68	29.96	30.24	30.52	30.8
29	29.29	29.58	29.87	30.16	30.45	30.74	31.03	31.32	31.61	31.9
30	30.30	30.60	30.90	31.20	31.50	31.80	32.10	32.40	32.70	33.0
31	31.31	31.62	31.93	32.24	32.55	32.86	33.17	33.48	33.79	34.1
32	32.32	32.64	32.96	33.28	33.60	33.92	34.24	34.56	34.88	35.2
33	33.33	33.66	33.99	34.32	34.65	34.98	35.31	35.64	35.97	36.3
34	34.34	34.68	35.02	35.36	35.70	36.04	36.38	36.72	37.06	37.4
35	35.35	35.70	36.05	36.40	36.75	37.10	37.45	37.80	38.15	38.5

10 Inches Diameter.

Depth	0	1	2	3	4	5	6	7	8	9
1	1.11	1.13	1.14	1.15	1.16	1.17	1.18	1.19	1.20	1.22
2	2.21	2.26	2.28	2.30	2.32	2.34	2.36	2.38	2.40	2.44
3	3.33	3.39	3.42	3.45	3.48	3.51	3.54	3.57	3.60	3.66
4	4.44	4.52	4.56	4.60	4.64	4.68	4.72	4.76	4.80	4.88
5	5.55	5.65	5.70	5.75	5.80	5.85	5.90	5.95	6.00	6.10
6	6.66	6.78	6.84	6.90	6.96	7.02	7.08	7.14	7.20	7.32
7	7.77	7.91	7.98	8.05	8.12	8.19	8.26	8.33	8.40	8.54
8	8.88	9.04	9.12	9.20	9.28	9.36	9.44	9.52	9.60	9.76
9	9.99	10.17	10.26	10.35	10.44	10.53	10.62	10.71	10.80	10.98
10	11.10	11.30	11.40	11.50	11.60	11.70	11.80	11.90	12.00	12.20
11	12.21	12.43	12.54	12.65	12.76	12.87	12.98	13.09	13.20	13.42
12	13.32	13.56	13.68	13.80	13.92	14.04	14.16	14.28	14.40	14.64
13	14.43	14.69	14.82	14.95	15.08	15.21	15.34	15.47	15.60	15.86
14	15.54	15.82	15.96	16.10	16.24	16.38	16.52	16.66	16.80	17.08
15	16.65	16.95	17.10	17.25	17.40	17.55	17.70	17.85	18.00	18.30
16	17.76	18.08	18.24	18.40	18.56	18.72	18.88	19.04	19.20	19.52
17	18.87	19.21	19.38	19.55	19.72	19.89	20.06	20.23	20.40	20.74
18	19.98	20.34	20.52	20.70	20.88	21.06	21.24	21.42	21.60	21.96
19	21.09	21.47	21.66	21.85	22.04	22.23	22.42	22.61	22.80	23.18
20	22.20	22.60	22.80	23.00	23.20	23.40	23.60	23.80	24.00	24.30
21	23.31	23.73	23.94	24.15	24.36	24.57	24.78	24.99	25.20	25.62
22	24.42	24.86	25.08	25.30	25.52	25.74	25.96	26.18	26.40	26.84
23	25.53	25.99	26.22	26.45	26.68	26.91	27.14	27.37	27.60	28.06
24	26.64	27.12	27.36	27.60	27.84	28.08	28.32	28.56	28.80	29.28
25	27.75	28.25	28.50	28.75	29.00	29.25	29.50	29.75	30.00	30.50
26	28.86	29.38	29.64	29.90	30.16	30.42	30.68	30.94	31.20	31.72
27	29.97	30.51	30.78	31.05	31.32	31.59	31.86	32.13	32.40	32.94
28	31.08	31.64	31.92	32.20	32.48	32.76	33.04	33.32	33.60	34.16
29	32.19	32.77	33.06	33.35	33.64	33.93	34.22	34.51	34.80	35.38
30	33.30	33.90	34.20	34.50	34.80	35.10	35.40	35.70	36.00	36.60
31	34.41	35.03	35.34	35.65	35.96	36.27	36.58	36.89	37.20	37.82
32	35.52	36.16	36.48	36.80	37.12	37.44	37.76	38.08	38.40	39.04
33	36.63	37.29	37.62	37.95	38.28	38.61	38.94	39.27	39.60	40.26
34	37.74	38.42	38.76	39.10	39.44	39.78	40.12	40.46	40.80	41.48
35	38.85	39.55	39.90	40.25	40.60	40.95	41.30	41.65	42.00	42.70

Cylinders in Ale Gallons.

285

21 Inches Diameter.

Depth	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	1.23	1.24	1.25	1.26	1.28	1.29	1.30	1.31	1.32	1.34
2	2.46	2.48	2.50	2.52	2.56	2.58	2.60	2.62	2.64	2.68
3	3.69	3.72	3.75	3.78	3.84	3.87	3.90	3.93	3.96	4.02
4	4.92	4.96	5.00	5.04	5.12	5.16	5.20	5.24	5.28	5.36
5	6.15	6.20	6.25	6.30	6.40	6.45	6.50	6.55	6.60	6.70
6	7.38	7.44	7.50	7.56	7.68	7.74	7.80	7.86	7.92	8.04
7	8.61	8.68	8.75	8.82	8.96	9.03	9.10	9.17	9.24	9.38
8	9.84	9.92	10.00	10.08	10.24	10.32	10.40	10.48	10.56	10.72
9	11.07	11.16	11.25	11.34	11.52	11.61	11.70	11.79	11.88	12.06
10	12.30	12.40	12.50	12.60	12.80	12.90	13.00	13.10	13.20	13.40
11	13.53	13.64	13.75	13.86	14.08	14.19	14.30	14.41	14.52	14.74
12	14.76	14.88	15.00	15.12	15.36	15.48	15.60	15.72	15.84	16.08
13	15.99	16.12	16.25	16.38	16.64	16.77	16.90	17.03	17.16	17.42
14	17.22	17.36	17.50	17.64	17.92	18.06	18.20	18.34	18.48	18.76
15	18.45	18.60	18.75	18.90	19.20	19.35	19.50	19.65	19.80	20.10
16	16.68	19.84	20.00	20.16	20.48	20.64	20.80	20.96	21.12	21.44
17	20.91	21.08	21.25	21.42	21.76	21.93	22.10	22.27	22.44	22.78
18	22.14	22.32	22.50	22.68	23.04	23.22	23.40	23.58	23.76	24.12
19	23.37	23.56	23.75	23.94	24.32	24.51	24.70	24.89	25.08	25.46
20	24.60	24.80	25.00	25.20	25.60	25.80	26.00	26.20	26.40	26.80
21	25.83	26.04	26.25	26.46	26.88	27.09	27.30	27.51	27.72	28.14
22	27.06	27.28	27.50	27.72	28.16	28.38	28.60	28.82	29.04	29.48
23	28.29	28.52	28.75	28.98	29.44	29.67	29.90	30.13	30.36	30.82
24	29.52	29.76	30.00	30.24	30.72	30.96	31.20	31.44	31.68	32.16
25	30.75	31.00	31.25	31.50	32.00	32.25	32.50	32.75	33.00	33.50
26	31.98	32.24	32.50	32.76	33.28	33.54	33.80	34.06	34.32	34.84
27	33.21	33.48	33.75	34.02	34.56	34.83	35.10	35.37	35.64	36.18
28	34.44	34.72	35.00	35.28	35.84	36.12	36.40	36.68	36.96	37.52
29	35.67	35.96	36.25	36.54	37.12	37.41	37.70	37.99	38.28	38.86
30	36.90	37.20	37.50	37.80	38.40	38.70	39.00	39.30	39.60	40.20
31	38.13	38.44	38.75	39.06	39.68	39.99	40.30	40.61	40.92	41.54
32	39.36	39.68	40.00	40.32	40.96	41.28	41.60	41.92	42.24	42.88
33	40.59	40.92	41.25	41.58	42.24	42.57	42.90	43.23	43.56	44.22
34	41.82	42.16	42.50	42.84	43.52	43.86	44.20	44.54	44.88	45.56
35	42.05	42.40	42.75	43.10	43.80	44.15	44.50	44.85	45.20	45.90

22 Inches Diameter.

Depth	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	1.35	1.36	1.37	1.38	1.40	1.41	1.42	1.44	1.45	1.46
2	2.70	2.72	2.74	2.76	2.80	2.82	2.84	2.88	2.90	2.92
3	4.05	4.08	4.11	4.14	4.20	4.23	4.26	4.32	4.35	4.38
4	5.40	5.44	5.48	5.52	5.60	5.64	5.68	5.76	5.80	5.84
5	6.75	6.80	6.85	6.90	7.00	7.05	7.10	7.20	7.25	7.30
6	8.10	8.16	8.22	8.28	8.40	8.46	8.52	8.64	8.70	8.76
7	9.45	9.52	9.59	9.66	9.80	9.87	9.94	10.08	10.15	10.22
8	10.80	10.88	10.96	11.04	11.20	11.28	11.36	11.52	11.60	11.68
9	12.15	12.24	12.33	12.42	12.60	12.69	12.78	12.96	13.05	13.14
10	13.50	13.60	13.70	13.80	14.00	14.10	14.20	14.40	14.50	14.60
11	14.85	14.96	15.07	15.18	15.40	15.51	15.62	15.84	15.95	16.06
12	16.20	16.32	16.44	16.56	16.80	16.92	17.04	17.28	17.40	17.52
13	17.55	17.68	17.81	17.94	18.20	18.33	18.46	18.72	18.85	18.98
14	18.90	19.04	19.18	19.32	19.60	19.74	19.88	20.16	20.30	20.44
15	20.25	20.40	20.55	20.70	21.00	21.15	21.30	21.60	21.75	21.90
16	21.60	21.76	21.92	22.08	22.40	22.56	22.72	23.04	23.20	23.36
17	22.95	23.12	23.29	23.46	23.80	23.97	24.14	24.48	24.65	24.82
18	24.30	24.48	24.66	24.84	25.20	25.38	25.56	25.92	26.10	26.28
19	25.65	25.84	26.03	26.22	26.60	26.79	26.98	27.36	27.55	27.74
20	27.00	27.20	27.40	27.60	28.00	28.20	28.40	28.80	29.00	29.20
21	28.35	28.56	28.77	28.98	29.40	29.61	29.82	30.24	30.45	30.66
22	29.70	29.92	30.14	30.36	30.80	31.02	31.24	31.68	31.90	32.12
23	31.05	31.28	31.51	31.74	32.20	32.43	32.66	33.12	33.35	33.58
24	32.40	32.64	32.88	33.12	33.60	33.84	34.08	34.56	34.80	35.04
25	33.75	34.00	34.25	34.50	35.00	35.25	35.50	36.00	36.25	36.50
26	35.10	35.36	35.62	35.88	36.40	36.66	36.92	37.44	37.70	37.96
27	36.45	36.72	36.99	37.26	37.80	38.07	38.34	38.88	39.15	39.42
28	37.80	38.08	38.36	38.64	39.20	39.48	39.76	40.32	40.60	40.88
29	39.15	39.44	39.73	40.02	40.60	40.89	41.18	41.76	42.05	42.34
30	40.50	40.80	41.10	41.40	42.00	42.30	42.60	43.20	43.50	43.80
31	41.85	42.16	42.47	42.78	43.40	43.71	44.02	44.64	44.95	45.26
32	43.20	43.52	43.84	44.16	44.80	45.12	45.44	46.08	46.40	46.72
33	44.55	44.88	45.21	45.54	46.20	46.53	46.86	47.52	47.85	48.18
34	45.90	46.24	46.58	46.92	47.60	47.94	48.28	48.96	49.30	49.64
35	47.25	47.60	47.95	48.30	49.00	49.35	49.70	50.40	50.75	51.10

Cylinders in Ale Gallons.

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23 Inches Diameter.

Depth	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	1.47	1.49	1.50	1.51	1.52	1.54	1.55	1.56	1.58	1.59
2	2.94	2.98	3.00	3.02	3.04	3.08	3.10	3.12	3.16	3.18
3	4.41	4.47	4.50	4.53	4.56	4.62	4.65	4.68	4.74	4.77
4	5.88	5.96	6.00	6.04	6.08	6.16	6.20	6.24	6.32	6.36
5	7.35	7.45	7.50	7.55	7.60	7.70	7.75	7.80	7.90	7.95
6	8.82	8.94	9.00	9.06	9.12	9.24	9.30	9.36	9.48	9.54
7	10.29	10.43	10.50	10.57	10.64	10.78	10.85	10.92	11.06	11.13
8	11.76	11.92	12.00	12.08	12.16	12.32	12.40	12.48	12.64	12.72
9	13.23	13.41	13.50	13.59	13.68	13.86	13.95	14.04	14.22	14.31
10	14.70	14.90	15.00	15.10	15.20	15.40	15.50	15.60	15.80	15.90
11	16.17	16.39	16.50	16.61	16.72	16.94	17.05	17.16	17.38	17.49
12	17.64	17.88	18.00	18.12	18.24	18.48	18.60	18.72	18.96	19.08
13	19.11	19.37	19.50	19.63	19.76	20.02	20.15	20.28	20.54	20.67
14	20.58	20.86	21.00	21.14	21.28	21.56	21.70	21.84	22.12	22.26
15	22.05	22.35	22.50	22.65	22.80	23.10	23.25	23.40	23.70	23.85
16	23.52	23.84	24.00	24.16	24.32	24.64	24.80	24.96	25.28	25.44
17	25.99	26.33	26.50	26.67	26.84	26.18	26.35	26.52	26.86	27.03
18	26.46	26.82	27.00	27.18	27.36	27.72	27.90	28.08	28.44	28.62
19	27.93	28.31	28.50	28.69	28.88	29.26	29.45	29.64	30.02	30.21
20	29.40	29.80	30.00	30.20	30.40	30.80	31.00	31.20	31.60	31.80
21	30.87	31.29	31.50	31.71	31.92	32.34	32.55	32.76	33.18	33.39
22	32.34	32.78	33.00	33.22	33.44	33.88	34.10	34.32	34.76	34.98
23	33.81	34.27	34.50	34.73	34.96	35.42	35.65	35.88	36.34	36.57
24	35.28	35.76	36.00	36.24	36.48	36.96	37.20	37.44	37.92	38.16
25	36.75	37.25	37.50	37.75	38.00	38.50	38.75	39.00	39.50	39.75
26	38.22	38.74	39.00	39.26	39.52	40.04	40.30	40.56	41.08	41.34
27	39.69	40.23	40.50	40.77	41.04	41.58	41.85	42.12	42.66	42.93
28	41.16	41.72	42.00	42.28	42.56	43.12	43.40	43.68	44.24	44.52
29	42.63	43.21	43.50	43.79	44.08	44.66	44.95	45.24	45.82	46.11
30	44.10	44.70	45.00	45.30	45.60	46.20	46.50	46.80	47.40	47.70
31	45.57	46.19	46.50	46.81	47.12	47.74	48.05	48.36	48.98	49.29
32	47.04	47.68	48.00	48.32	48.64	49.28	49.60	49.92	50.56	50.88
33	48.51	49.17	49.50	49.83	50.16	50.82	51.15	51.48	52.14	52.47
34	49.98	50.66	51.00	51.34	51.68	52.36	52.70	53.04	53.72	54.06
35	51.45	52.15	52.50	52.85	53.20	53.90	54.25	54.60	55.30	55.65

24 Inches Diameter.

Depth	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	1.60	1.62	1.63	1.64	1.66	1.67	1.69	1.70	1.71	1.73
2	3.20	3.24	3.26	3.28	3.32	3.34	3.38	3.40	3.42	3.46
3	4.80	4.86	4.89	4.92	4.98	5.01	5.07	5.10	5.13	5.19
4	6.40	6.48	6.52	6.56	6.64	6.68	6.76	6.80	6.84	6.92
5	8.00	8.10	8.15	8.20	8.30	8.35	8.45	8.50	8.55	8.65
6	9.60	9.72	9.78	9.84	9.96	10.02	10.14	10.20	10.26	10.38
7	11.20	11.34	11.41	11.48	11.62	11.69	11.83	11.90	11.97	12.11
8	12.80	12.96	13.04	13.12	13.28	13.36	13.52	13.60	13.68	13.84
9	14.40	14.58	14.67	14.76	14.94	15.03	15.21	15.30	15.39	15.57
10	16.00	16.20	16.30	16.40	16.60	16.70	16.90	17.00	17.10	17.30
11	17.60	17.82	17.93	18.04	18.26	18.37	18.59	18.70	18.81	19.03
12	19.20	19.44	19.56	19.68	19.92	20.04	20.28	20.40	20.52	20.76
13	20.80	21.06	21.19	21.32	21.58	21.71	21.97	22.10	22.23	22.49
14	22.40	22.68	22.82	22.96	23.24	23.38	23.66	23.80	23.94	24.22
15	24.00	24.30	24.45	24.60	24.90	25.05	25.35	25.50	25.65	25.95
16	25.60	25.92	26.08	26.24	26.56	26.72	27.04	27.20	27.36	27.68
17	27.20	27.54	27.71	27.88	28.22	28.39	28.73	28.90	29.07	29.41
18	28.80	29.16	29.34	29.52	29.88	30.06	30.42	30.60	30.78	31.14
19	30.40	30.78	30.97	31.16	31.54	31.73	32.11	32.30	32.49	32.87
20	32.00	32.40	32.60	32.80	33.20	33.40	33.80	34.00	34.20	34.60
21	33.60	34.02	34.23	34.44	34.86	35.07	35.49	35.70	35.91	36.33
22	35.20	35.64	35.86	36.08	36.52	36.74	37.18	37.40	37.62	38.06
23	36.80	37.26	37.49	37.72	38.18	38.41	38.87	39.10	39.33	39.79
24	38.40	38.88	39.12	39.36	39.84	40.08	40.56	40.80	41.04	41.52
25	40.00	40.50	40.75	41.00	41.50	41.75	42.25	42.50	42.75	43.25
26	41.60	42.12	42.38	42.64	43.16	43.42	43.94	44.20	44.46	44.98
27	43.20	43.74	44.01	44.28	44.82	45.09	45.63	45.90	46.17	46.71
28	44.80	45.36	45.64	45.92	46.48	46.76	47.32	47.60	47.88	48.44
29	46.40	46.98	47.27	47.56	48.14	48.43	49.01	49.30	49.59	50.17
30	48.00	48.60	48.90	49.20	49.80	50.10	50.70	51.00	51.30	51.90
31	49.60	50.22	50.53	50.84	51.46	51.77	52.39	52.70	53.01	53.63
32	51.20	51.84	52.16	52.48	53.12	53.44	54.08	54.40	54.72	55.36
33	52.80	53.46	53.79	54.12	54.78	55.11	55.77	56.10	56.43	57.09
34	54.40	55.08	55.42	55.76	56.44	56.78	57.46	57.80	58.14	58.82
35	56.00	56.70	57.05	57.40	58.10	58.45	59.15	59.50	59.85	60.55

Cylinders in Ale Gallons.

289

25 Inches Diameter.

Depth	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	1.74	1.75	1.77	1.78	1.80	1.81	1.83	1.84	1.85	1.87
2	3.48	3.50	3.54	3.56	3.60	3.62	3.66	3.68	3.70	3.74
3	5.22	5.25	5.31	5.34	5.40	5.43	5.49	5.52	5.55	5.61
4	6.96	7.00	7.08	7.12	7.20	7.24	7.32	7.36	7.40	7.48
5	8.70	8.75	8.85	8.90	9.00	9.05	9.15	9.20	9.25	9.35
6	10.44	10.50	10.62	10.68	10.80	10.86	10.98	11.04	11.10	11.22
7	12.18	12.25	12.39	12.46	12.60	12.67	12.81	12.88	12.95	13.09
8	13.92	14.00	14.16	14.24	14.40	14.48	14.64	14.72	14.80	14.96
9	15.66	15.75	15.93	16.02	16.20	16.29	16.47	16.56	16.65	16.83
10	17.40	17.50	17.70	17.80	18.00	18.10	18.30	18.40	18.50	18.70
11	19.14	19.25	19.47	19.58	19.80	19.91	20.13	20.24	20.35	20.57
12	20.88	21.00	21.24	21.36	21.60	21.72	21.95	22.08	22.20	22.44
13	22.62	22.75	23.01	23.14	23.40	23.53	23.79	23.92	24.05	24.31
14	24.36	24.50	24.78	24.92	25.20	25.34	25.62	25.76	25.90	26.18
15	26.10	26.25	26.55	26.70	27.00	27.15	27.45	27.60	27.75	28.05
16	27.84	28.00	28.32	28.48	28.80	28.96	29.28	29.44	29.60	29.92
17	29.58	29.75	30.09	30.26	30.60	30.77	31.11	31.28	31.45	31.79
18	31.32	31.50	31.86	32.04	32.40	32.58	32.94	33.12	33.30	33.66
19	33.06	33.25	33.63	33.82	34.20	34.39	34.77	34.96	35.15	35.53
20	34.80	35.00	35.40	35.60	36.00	36.20	36.60	36.80	37.00	37.40
21	36.54	36.75	37.17	37.38	37.80	38.01	38.43	38.64	38.85	39.27
22	38.28	38.50	38.94	39.16	39.60	39.82	40.26	40.48	40.70	41.14
23	40.02	40.25	40.71	40.94	41.40	41.63	42.09	42.32	42.55	43.01
24	41.76	42.00	42.48	42.72	43.20	43.44	43.92	44.16	44.40	44.88
25	43.50	43.75	44.25	44.50	45.00	45.25	45.75	46.00	46.25	46.75
26	45.24	45.50	46.02	46.28	46.80	47.06	47.58	47.84	48.10	48.62
27	46.98	47.25	47.79	48.06	48.60	48.87	49.41	49.68	49.95	50.49
28	48.72	49.00	49.56	49.84	50.40	50.68	51.24	51.52	51.80	52.36
29	50.46	50.75	51.33	51.62	52.20	52.49	53.07	53.36	53.65	54.23
30	52.20	52.50	53.10	53.40	54.00	54.30	54.9	55.20	55.50	56.10
31	53.94	54.25	54.87	55.18	55.80	56.11	56.73	57.04	57.35	57.97
32	55.68	56.00	56.64	56.96	57.60	57.92	58.56	58.88	59.20	59.84
33	57.42	57.75	58.41	58.74	59.40	59.73	60.39	60.72	61.05	61.71
34	59.16	59.50	60.18	60.52	61.20	61.54	62.22	62.56	62.90	63.58
35	60.90	61.25	61.95	62.30	63.00	63.35	64.05	64.40	64.75	65.45

26 Inches Diameter.

Depth	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	1.88	1.90	1.91	1.93	1.94	1.96	1.97	1.99	2.00	2.02
2	3.76	3.80	3.82	3.86	3.88	3.92	3.94	3.98	4.00	4.04
3	5.64	5.70	5.73	5.79	5.82	5.88	5.91	5.97	6.00	6.06
4	7.52	7.60	7.64	7.72	7.76	7.84	7.88	7.96	8.00	8.08
5	9.40	9.50	9.55	9.65	9.70	9.80	9.85	9.95	10.00	10.10
6	11.28	11.40	11.46	11.58	11.64	11.76	11.82	11.94	12.00	12.12
7	13.16	13.30	13.37	13.51	13.58	13.72	13.79	13.93	14.00	14.14
8	15.04	15.20	15.28	15.44	15.52	15.68	15.76	15.92	16.00	16.16
9	16.92	17.10	17.19	17.37	17.46	17.64	17.73	17.91	18.00	18.18
10	18.80	19.00	19.10	19.30	19.40	19.60	19.70	19.90	20.00	20.20
11	20.68	20.90	21.01	21.23	21.34	21.56	21.67	21.89	22.00	22.22
12	22.56	22.80	22.92	23.16	23.28	23.52	23.64	23.88	24.00	24.24
13	24.44	24.70	24.83	25.09	25.22	25.48	25.61	25.87	26.00	26.26
14	26.32	26.60	26.74	27.02	27.16	27.44	27.58	27.86	28.00	28.28
15	28.20	28.50	28.65	28.95	29.10	29.40	29.55	29.85	30.00	30.30
16	30.08	30.40	30.56	30.88	31.04	31.36	31.52	31.84	32.00	32.32
17	31.96	32.30	32.47	32.81	32.98	33.32	33.49	33.83	34.00	34.34
18	33.84	34.20	34.38	34.73	34.92	35.28	35.46	35.82	36.00	36.36
19	35.72	36.10	36.29	36.67	36.86	37.24	37.43	37.81	38.00	38.38
20	37.60	38.00	38.20	38.60	38.80	39.20	39.40	39.80	40.00	40.40
21	39.48	39.90	40.11	40.53	40.74	41.16	41.37	41.79	42.00	42.42
22	41.36	41.80	42.02	42.46	42.68	43.12	43.34	43.78	44.00	44.44
23	43.24	43.70	43.93	44.39	44.62	45.08	45.31	45.77	46.00	46.46
24	45.12	45.60	45.84	46.32	46.56	47.04	47.28	47.76	48.00	48.48
25	47.00	47.50	47.75	48.25	48.50	49.00	49.25	49.75	50.00	50.50
26	48.88	49.40	49.66	50.18	50.44	50.96	51.22	51.74	52.00	52.52
27	50.76	51.30	51.57	52.11	52.38	52.92	53.19	53.73	54.00	54.54
28	52.64	53.20	53.48	54.04	54.32	54.88	55.16	55.72	56.00	56.56
29	54.52	55.10	55.39	55.97	56.26	56.84	57.13	57.71	58.00	58.58
30	56.40	57.00	57.30	57.90	58.20	58.80	59.10	59.70	60.00	60.60
31	58.28	58.90	59.21	59.83	60.14	60.76	61.07	61.69	62.00	62.62
32	60.16	60.80	61.12	61.76	62.08	62.72	63.04	63.68	64.00	64.64
33	62.04	62.70	63.03	63.69	64.02	64.68	65.01	65.67	66.00	66.66
34	63.92	64.60	64.94	65.62	65.96	66.64	66.98	67.66	68.00	68.68
35	65.80	66.50	66.85	67.55	67.90	68.60	68.95	69.65	70.00	70.70

Cylinders in Ale Gallons.

291

27 Inches Diameter.

Depth	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	2.03	2.05	2.06	2.08	2.09	2.11	2.12	2.14	2.15	2.17
2	4.06	4.10	4.12	4.16	4.18	4.22	4.24	4.28	4.30	4.34
3	6.09	6.15	6.18	6.24	6.27	6.33	6.36	6.42	6.45	6.51
4	8.12	8.20	8.24	8.32	8.36	8.44	8.48	8.56	8.60	8.68
5	10.15	10.25	10.30	10.40	10.45	10.55	10.60	10.70	10.75	10.85
6	12.18	12.30	12.36	12.48	12.54	12.66	12.72	12.84	12.90	13.02
7	14.21	14.35	14.42	14.56	14.63	14.77	14.84	14.98	15.05	15.19
8	16.24	16.40	16.48	16.64	16.72	16.88	16.96	17.12	17.20	17.36
9	18.27	18.45	18.54	18.72	18.81	18.99	19.08	19.26	19.35	19.53
10	20.30	20.50	20.60	20.80	20.90	21.10	21.20	21.40	21.50	21.70
11	22.33	22.55	22.66	22.88	22.99	23.21	23.32	23.54	23.65	23.87
12	24.36	24.60	24.72	24.96	25.08	25.32	25.44	25.68	25.80	26.04
13	26.39	26.65	26.78	27.04	27.17	27.43	27.56	27.82	27.95	28.21
14	28.42	28.70	28.84	29.12	29.26	29.54	29.68	29.96	30.10	30.38
15	30.45	30.75	30.90	31.20	31.35	31.65	31.80	32.10	32.25	32.55
16	32.48	32.80	32.96	33.28	33.44	33.76	33.92	34.24	34.40	34.72
17	34.51	34.85	35.02	35.36	35.53	35.87	36.04	36.38	36.55	36.89
18	36.54	36.90	37.08	37.44	37.62	37.98	38.16	38.52	38.70	39.06
19	38.57	38.95	39.14	39.52	39.71	40.09	40.28	40.66	40.85	41.23
20	40.60	41.00	41.20	41.60	41.80	42.20	42.40	42.80	43.00	43.40
21	42.63	43.05	43.26	43.68	43.89	44.31	44.52	44.94	45.15	45.57
22	44.66	45.10	45.32	45.76	45.98	46.42	46.64	47.08	47.30	47.74
23	46.69	47.15	47.38	47.84	48.07	48.53	48.76	49.22	49.45	49.91
24	48.72	49.20	49.44	49.92	50.16	50.64	50.88	51.36	51.60	52.08
25	50.75	51.25	51.50	52.00	52.25	52.75	53.00	53.50	53.75	54.25
26	52.78	53.30	53.56	54.08	54.34	54.86	55.12	55.64	55.90	56.42
27	54.81	55.35	55.62	56.16	56.43	56.97	57.24	57.78	58.05	58.59
28	56.84	57.40	57.68	58.24	58.52	59.08	59.36	59.92	60.20	60.76
29	58.87	59.45	59.74	60.32	60.61	61.19	61.48	62.06	62.35	62.93
30	60.90	61.50	61.80	62.40	62.70	63.30	63.60	64.20	64.50	65.10
31	62.93	63.55	63.86	64.48	64.79	65.41	65.72	66.34	66.65	67.27
32	64.96	65.60	65.92	66.56	66.88	67.52	67.84	68.48	68.80	69.44
33	66.99	67.65	67.98	68.64	68.97	69.63	69.96	70.62	70.95	71.61
34	69.02	69.70	70.04	70.72	71.06	71.74	72.08	72.76	73.10	73.78
35	71.05	71.75	72.10	72.80	73.15	73.85	74.20	74.90	75.25	75.95

Cylinders in Ale Gallons.

28 Inches Diameter.

Depth	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	2.18	2.20	2.21	2.23	2.25	2.26	2.28	2.29	2.31	2.33
2	4.36	4.40	4.42	4.46	4.50	4.52	4.56	4.58	4.62	4.66
3	6.54	6.60	6.63	6.69	6.75	6.78	6.84	6.87	6.93	6.99
4	8.72	8.80	8.84	8.92	9.00	9.04	9.12	9.16	9.24	9.32
5	10.90	11.00	11.05	11.15	11.25	11.30	11.40	11.45	11.55	11.65
6	13.08	13.20	13.26	13.38	13.50	13.56	13.68	13.74	13.86	13.98
7	15.26	15.40	15.47	15.61	15.75	15.82	15.96	16.03	16.17	16.31
8	17.44	17.60	17.68	17.84	18.00	18.08	18.24	18.32	18.48	18.64
9	19.62	19.80	19.89	20.07	20.25	20.34	20.52	20.61	20.79	20.97
10	21.80	22.00	22.10	22.30	22.50	22.60	22.80	22.90	23.10	23.30
11	23.98	24.20	24.31	24.53	24.75	24.86	25.08	25.19	25.41	25.63
12	26.16	26.40	26.52	26.76	27.00	27.12	27.36	27.48	27.72	27.96
13	28.34	28.60	28.73	28.99	29.25	29.38	29.64	29.77	30.03	30.29
14	30.52	30.80	30.94	31.22	31.50	31.64	31.92	32.06	32.34	32.62
15	32.70	33.00	33.15	33.45	33.75	33.90	34.20	34.35	34.65	34.95
16	34.88	35.20	35.36	35.68	36.00	36.16	36.48	36.64	36.96	37.29
17	37.06	37.40	37.57	37.91	38.25	38.42	38.76	38.93	39.27	39.62
18	39.24	39.60	39.78	40.14	40.50	40.68	41.04	41.22	41.58	41.95
19	41.42	41.80	41.99	42.37	42.75	42.94	43.32	43.51	43.89	44.28
20	43.60	44.00	44.20	44.60	45.00	45.20	45.60	45.80	46.20	46.60
21	45.78	46.20	46.41	46.83	47.25	47.46	47.88	48.09	48.51	48.93
22	47.96	48.40	48.62	49.06	49.50	49.72	50.16	50.38	50.82	51.26
23	50.14	50.60	50.83	51.29	51.75	51.98	52.44	52.67	53.13	53.59
24	52.32	52.80	53.04	53.52	54.00	54.24	54.72	54.96	55.44	55.92
25	54.50	55.00	55.25	55.75	56.25	56.50	57.00	57.25	57.75	58.25
26	56.68	57.20	57.46	57.98	58.50	58.76	59.28	59.54	60.06	60.58
27	58.86	59.40	59.67	60.21	60.75	61.02	61.56	61.83	62.37	62.91
28	61.04	61.60	61.88	62.44	63.00	63.28	63.84	64.12	64.68	65.24
29	63.22	63.80	64.09	64.67	65.25	65.54	66.12	66.41	66.99	67.57
30	65.40	66.00	66.30	66.90	67.50	67.80	68.40	68.70	69.30	69.90
31	67.58	68.20	68.51	69.13	69.75	70.06	70.68	70.99	71.61	72.23
32	69.76	70.40	70.72	71.36	72.00	72.32	72.96	73.28	73.92	74.56
33	71.94	72.60	72.93	73.59	74.25	74.58	75.24	75.57	76.23	76.89
34	74.12	74.80	75.14	75.82	76.50	76.84	77.52	77.86	78.54	79.22
35	76.30	77.00	77.35	78.05	78.75	79.10	79.80	80.15	80.85	81.55

Cylinders in Ale Gallons.

293

29 Inches Diameter.

Depth	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	2.34	2.36	2.37	2.39	2.41	2.42	2.44	2.46	2.47	2.49
2	4.68	4.72	4.74	4.78	4.82	4.84	4.88	4.92	4.94	4.98
3	7.02	7.08	7.11	7.17	7.23	7.26	7.32	7.38	7.41	7.47
4	9.36	9.44	9.48	9.56	9.64	9.68	9.76	9.84	9.88	9.96
5	11.70	11.80	11.85	11.95	12.05	12.10	12.20	12.30	12.35	12.45
6	14.04	14.16	14.22	14.34	14.46	14.52	14.64	14.76	14.82	14.94
7	16.38	16.52	16.59	16.73	16.87	16.94	17.08	17.22	17.29	17.43
8	18.72	18.88	18.96	19.12	19.28	19.36	19.52	19.68	19.76	19.92
9	21.06	21.24	21.33	21.51	21.69	21.78	21.96	22.14	22.23	22.41
10	23.40	23.60	23.70	23.90	24.10	24.20	24.40	24.60	24.70	24.90
11	25.74	25.96	26.07	26.29	26.51	26.62	26.84	27.06	27.17	27.39
12	28.08	28.32	28.44	28.68	28.92	29.04	29.28	29.52	29.64	29.88
13	30.42	30.68	30.81	31.07	31.33	31.46	31.72	31.98	32.11	32.37
14	32.76	33.04	33.18	33.46	33.74	33.88	34.16	34.47	34.58	34.86
15	35.10	35.40	35.55	35.85	36.15	36.30	36.60	36.90	37.05	37.35
16	37.44	37.76	37.92	38.24	38.56	38.72	39.04	39.36	39.52	39.84
17	39.78	40.12	40.29	40.63	40.97	41.14	41.48	41.82	41.99	42.33
18	42.12	42.48	42.66	43.02	43.38	43.56	43.92	44.28	44.46	44.82
19	44.46	44.84	45.03	45.41	45.79	45.98	46.36	46.74	46.93	47.31
20	46.80	47.20	47.40	47.80	48.20	48.40	48.80	49.20	49.40	49.80
21	49.14	49.56	49.77	50.19	50.61	50.82	51.24	51.66	51.87	52.29
22	51.48	51.92	52.14	52.58	53.02	53.24	53.68	54.12	54.34	54.78
23	53.82	54.28	54.51	54.97	55.43	55.66	56.12	56.58	56.81	57.27
24	56.16	56.64	56.88	57.36	57.84	58.08	58.56	59.04	59.28	59.76
25	58.50	59.00	59.25	59.75	60.25	60.50	61.00	61.50	61.75	62.25
26	60.84	61.36	61.62	62.14	62.66	62.92	63.44	63.96	64.22	64.74
27	63.18	63.72	63.99	64.53	65.07	65.34	65.88	66.42	66.69	67.23
28	65.52	66.08	66.36	66.92	67.48	67.76	68.32	68.88	69.16	69.72
29	67.86	68.44	68.73	69.31	69.89	70.18	70.76	71.34	71.63	72.21
30	70.20	70.80	71.10	71.70	72.30	72.60	73.20	73.80	74.10	74.70
31	72.54	73.16	73.47	74.09	74.71	75.02	75.64	76.26	76.57	77.19
32	74.88	75.52	75.84	76.48	77.12	77.44	78.08	78.72	79.04	79.68
33	77.22	77.88	78.21	78.87	79.53	79.86	80.52	81.18	81.51	82.17
34	79.56	80.24	80.58	81.26	81.94	82.28	82.96	83.64	83.98	84.66
35	81.90	82.60	82.95	83.65	84.35	84.70	85.40	86.10	86.45	87.15

30 Inches Diameter.

Depth	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	2.51	2.52	2.54	2.56	2.57	2.59	2.61	2.62	2.64	2.66
2	5.02	5.04	5.08	5.12	5.14	5.18	5.22	5.24	5.28	5.32
3	7.53	7.56	7.62	7.68	7.71	7.77	7.83	7.86	7.92	7.98
4	10.04	10.08	10.16	10.24	10.28	10.36	10.44	10.48	10.56	10.64
5	12.55	12.60	12.70	12.80	12.85	12.95	13.05	13.10	13.20	13.30
6	15.06	15.12	15.24	15.36	15.42	15.54	15.66	15.72	15.84	15.96
7	17.57	17.64	17.78	17.92	17.99	18.13	18.27	18.34	18.48	18.62
8	20.08	20.16	20.32	20.48	20.56	20.72	20.88	20.96	21.12	21.28
9	22.59	22.68	22.86	23.04	23.13	23.31	23.49	23.58	23.76	23.94
10	25.10	25.20	25.40	25.60	25.70	25.90	26.10	26.20	26.40	26.60
11	27.61	27.72	27.94	28.16	28.27	28.49	28.71	28.82	29.04	29.26
12	30.12	30.24	30.48	30.72	30.84	31.08	31.32	31.44	31.68	31.92
13	32.63	32.76	33.02	33.28	33.41	33.67	33.93	34.06	34.32	34.58
14	35.14	35.28	35.56	35.84	35.98	36.26	36.54	36.68	36.96	37.24
15	37.65	37.80	38.10	38.40	38.55	38.85	39.15	39.30	39.60	39.90
16	40.16	40.32	40.64	40.96	41.12	41.44	41.76	41.92	42.24	42.56
17	42.67	42.84	43.18	43.52	43.69	44.03	44.37	44.54	44.88	45.22
18	45.18	45.36	45.72	46.08	46.26	46.62	46.98	47.16	47.52	47.88
19	47.69	47.88	48.26	48.64	48.83	49.21	49.69	49.78	50.16	50.54
20	50.20	50.40	50.80	51.20	51.40	51.80	52.20	52.40	52.80	53.20
21	52.71	52.92	53.34	53.76	53.97	54.39	54.81	55.02	55.44	55.86
22	55.22	55.44	55.88	56.32	56.54	56.98	57.42	57.64	58.08	58.52
23	57.73	57.96	58.42	58.88	59.11	59.57	60.03	60.26	60.72	61.18
24	60.24	60.48	60.96	61.44	61.68	62.16	62.64	62.88	63.36	63.84
25	62.75	63.00	63.50	64.00	64.25	64.75	65.25	65.50	66.00	66.50
26	65.26	65.52	66.04	66.56	66.82	67.34	67.86	68.12	68.64	69.16
27	67.77	68.04	68.58	69.12	69.39	69.93	70.47	70.74	71.28	71.82
28	70.28	70.56	71.12	71.68	71.96	72.52	73.08	73.36	73.92	74.48
29	72.79	73.08	73.76	74.24	74.53	75.11	75.69	75.98	76.56	77.14
30	75.30	75.60	76.20	76.80	77.10	77.70	78.30	78.60	79.20	79.80
31	77.81	78.12	78.74	79.36	79.67	80.29	80.91	81.22	81.84	82.46
32	80.32	80.64	81.28	81.92	82.24	82.88	83.52	83.84	84.46	85.12
33	82.83	83.16	83.82	84.48	84.81	85.47	86.13	86.46	87.10	87.78
34	85.34	85.68	86.36	87.04	87.38	88.06	88.74	89.08	89.74	90.44
35	87.85	88.20	88.90	89.60	89.95	90.65	91.35	91.70	92.40	93.10

Cylinders in Ale Gallons.

295

31 Inches Diameter.

Depth	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	2.68	2.69	2.71	2.73	2.75	2.76	2.78	2.80	2.82	2.83
2	5.36	5.38	5.42	5.46	5.50	5.52	5.56	5.60	5.64	5.66
3	8.04	8.07	8.13	8.19	8.25	8.28	8.34	8.40	8.46	8.49
4	10.72	10.76	10.84	10.92	11.00	11.04	11.12	11.20	11.28	11.32
5	13.40	13.45	13.55	13.65	13.75	13.80	13.90	14.00	14.10	14.15
6	16.08	16.14	16.26	16.38	16.50	16.56	16.68	16.80	16.92	16.98
7	18.76	18.83	18.97	19.11	19.25	19.32	19.46	19.60	19.74	19.81
8	21.44	21.52	21.68	21.84	22.00	22.08	22.24	22.40	22.56	22.64
9	24.12	24.21	24.39	24.57	24.75	24.84	25.02	25.20	25.38	25.47
10	26.80	26.90	27.10	27.30	27.50	27.60	27.80	28.00	28.20	28.30
11	29.48	29.59	29.81	30.03	30.25	30.36	30.58	30.80	31.02	31.13
12	32.16	32.28	32.52	32.76	33.00	33.12	33.36	33.60	33.84	33.96
13	34.84	34.97	35.23	35.49	35.75	35.88	36.14	36.40	36.66	36.79
14	37.52	37.66	37.94	38.22	38.50	38.64	38.92	39.20	39.48	39.62
15	40.20	40.35	40.65	40.95	41.25	41.40	41.70	42.00	42.30	42.45
16	42.88	43.04	43.36	43.68	44.00	44.16	44.48	44.80	45.12	45.28
17	45.56	45.73	46.07	46.41	46.75	46.92	47.26	47.60	47.94	48.11
18	48.24	48.42	48.78	49.14	49.50	49.68	50.04	50.40	50.76	50.94
19	50.92	51.11	51.49	51.87	52.25	52.44	52.82	53.20	53.58	53.77
20	53.60	53.80	54.20	54.60	55.00	55.20	55.60	56.00	56.40	56.60
21	56.28	56.49	56.91	57.33	57.75	57.96	58.38	58.80	59.22	59.43
22	58.96	59.18	59.62	60.06	60.50	60.72	61.16	61.60	62.04	62.26
23	61.64	61.87	62.33	62.79	63.25	63.48	63.94	64.40	64.86	65.09
24	64.32	64.56	65.04	65.52	66.00	66.24	66.72	67.20	67.68	67.92
25	67.00	67.25	67.75	68.25	68.75	69.00	69.50	70.00	70.50	70.75
26	69.68	69.94	70.46	70.98	71.50	71.76	72.28	72.80	73.32	73.58
27	72.36	72.63	73.17	73.71	74.25	74.52	75.06	75.60	76.14	76.41
28	75.04	75.32	75.88	76.44	77.00	77.28	77.84	78.40	78.96	79.24
29	77.72	78.01	78.59	79.17	79.75	80.04	80.62	81.20	81.78	82.07
30	80.40	80.70	81.30	81.90	82.50	82.80	83.40	84.00	84.60	84.90
31	83.08	83.39	84.01	84.63	85.25	85.56	86.18	86.80	87.42	87.73
32	85.76	86.08	86.72	87.36	88.00	88.32	88.96	89.60	90.24	90.56
33	88.44	88.77	89.43	90.09	90.75	91.08	91.74	92.40	93.06	93.35
34	91.12	91.46	92.14	92.82	93.50	93.84	94.52	95.20	95.88	96.22
35	93.80	94.15	94.85	95.55	96.25	96.60	97.30	98.00	98.70	99.05

32 Inches Diameter.

Depth	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	2.85	2.87	2.89	2.91	2.92	2.94	2.96	2.98	3.00	3.01
2	5.70	5.74	5.78	5.82	5.84	5.88	5.92	5.96	6.00	6.02
3	8.55	8.61	8.67	8.73	8.76	8.82	8.88	8.94	9.00	9.03
4	11.40	11.48	11.56	11.64	11.68	11.76	11.84	11.92	12.00	12.04
5	14.25	14.35	14.45	14.55	14.60	14.70	14.80	14.90	15.00	15.05
6	17.10	17.22	17.34	17.46	17.52	17.64	17.76	17.88	18.00	18.06
7	19.95	20.09	20.23	20.37	20.44	20.58	20.72	20.86	21.00	21.07
8	22.80	22.96	23.13	23.28	23.36	23.52	23.68	23.84	24.00	24.08
9	25.65	25.83	26.01	26.19	26.28	26.46	26.64	26.82	27.00	27.09
10	28.50	28.70	28.90	29.10	29.20	29.40	29.60	29.80	30.00	30.10
11	31.35	31.57	31.79	32.01	32.12	32.34	32.56	32.78	33.00	33.11
12	34.20	34.44	34.68	34.92	35.04	35.28	35.52	35.76	36.00	36.12
13	37.05	37.31	37.57	37.83	37.96	38.22	38.48	38.74	39.00	39.13
14	39.90	40.18	40.46	40.74	40.88	41.16	41.44	41.72	42.00	42.14
15	42.75	43.05	43.35	43.65	43.80	44.10	44.40	44.70	45.00	45.15
16	45.60	45.92	46.24	46.56	46.72	47.04	47.36	47.68	48.00	48.16
17	48.45	48.79	49.13	49.47	49.64	49.98	50.32	50.66	51.00	51.17
18	51.30	51.66	52.02	52.38	52.56	52.92	53.28	53.64	54.00	54.18
19	54.15	54.53	54.91	55.29	55.48	55.86	56.24	56.62	57.00	57.19
20	57.00	57.40	57.80	58.20	58.40	58.80	59.20	59.60	60.00	60.20
21	59.85	60.27	60.69	61.11	61.32	61.74	62.16	62.58	63.00	63.21
22	62.70	63.14	63.58	64.02	64.24	64.68	65.12	65.56	66.00	66.22
23	65.55	66.01	66.47	66.93	67.16	67.62	68.08	68.54	69.00	69.23
24	68.40	68.88	69.36	69.84	70.08	70.56	71.04	71.52	72.00	72.24
25	71.25	71.75	72.25	72.75	73.00	73.50	74.00	74.50	75.00	75.25
26	74.10	74.62	75.14	75.66	75.92	76.44	76.96	77.48	78.00	78.26
27	76.95	77.49	78.03	78.57	78.84	79.38	79.92	80.46	81.00	81.27
28	79.80	80.36	80.92	81.48	81.76	82.32	82.88	83.44	84.00	84.28
29	82.65	83.23	83.81	84.39	84.68	85.26	85.84	86.42	87.00	87.29
30	85.50	86.10	86.70	87.30	87.60	88.20	88.80	89.40	90.00	90.30
31	88.35	88.97	89.59	90.21	90.52	91.14	91.76	92.38	93.00	93.31
32	91.20	91.84	92.48	93.12	93.44	94.08	94.72	95.36	96.00	96.32
33	94.05	94.71	95.37	96.03	96.36	97.02	97.68	98.34	99.00	99.33
34	96.90	97.58	98.26	98.94	99.28	99.96	100.64	101.32	102.00	102.34
35	99.75	100.45	101.15	101.85	102.20	102.90	103.60	104.30	105.00	105.35

Cylinders in Ale Gallons.

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Depth	33 Inches Diameter.									
	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	3.03	3.05	3.07	3.09	3.11	3.13	3.14	3.16	3.18	3.20
2	6.06	6.10	6.14	6.18	6.22	6.26	6.28	6.32	6.36	6.40
3	9.09	9.15	9.21	9.27	9.33	9.39	9.42	9.48	9.54	9.60
4	12.12	12.20	12.28	12.36	12.44	12.52	12.56	12.64	12.72	12.80
5	15.15	15.25	15.35	15.45	15.55	15.65	15.70	15.80	15.90	16.00
6	18.18	18.30	18.42	18.54	18.66	18.78	18.84	18.96	19.08	19.20
7	21.21	21.35	21.49	21.63	21.77	21.91	21.98	22.12	22.26	22.40
8	24.24	24.40	24.56	24.72	24.88	25.04	25.12	25.28	25.44	25.60
9	27.27	27.45	27.63	27.81	27.99	28.17	28.26	28.44	28.62	28.80
10	30.30	30.50	30.70	30.90	31.10	31.30	31.40	31.60	31.80	32.00
11	33.33	33.55	33.77	33.99	34.21	34.43	34.54	34.76	34.98	35.20
12	36.36	36.60	36.84	37.08	37.32	37.56	37.68	37.92	38.16	38.40
13	39.39	39.65	39.91	40.17	40.43	40.69	40.82	41.08	41.34	41.60
14	42.42	42.70	42.98	43.26	43.54	43.82	43.96	44.24	44.52	44.80
15	45.45	45.75	46.05	46.35	46.65	46.95	47.10	47.40	47.70	48.00
16	48.48	48.80	49.12	49.44	49.76	50.08	50.24	50.56	50.88	51.20
17	51.51	51.85	52.19	52.53	52.87	53.21	53.38	53.72	54.06	54.40
18	54.54	54.90	55.26	55.62	55.98	56.34	56.52	56.88	57.24	57.60
19	57.57	57.95	58.33	58.71	59.09	59.47	59.66	60.04	60.42	60.80
20	60.60	61.00	61.40	61.80	62.20	62.60	62.80	63.20	63.60	64.00
21	63.63	64.05	64.47	64.89	65.31	65.73	65.94	66.36	66.78	67.20
22	66.66	67.10	67.54	67.98	68.42	68.86	69.08	69.52	69.96	70.40
23	69.69	70.15	70.61	71.07	71.53	71.99	72.22	72.68	73.14	73.60
24	72.72	73.20	73.68	74.16	74.64	75.12	75.36	75.84	76.22	76.80
25	75.75	76.25	76.75	77.25	77.75	78.25	78.50	79.00	79.50	80.00
26	78.78	79.30	79.82	80.34	80.86	81.38	81.64	82.16	82.68	83.20
27	81.81	82.35	82.89	83.43	83.97	84.51	84.78	85.32	85.86	86.40
28	84.84	85.40	85.96	86.52	87.08	87.64	87.92	88.48	89.04	89.60
29	87.87	88.45	89.03	89.61	90.19	90.77	91.06	91.64	92.22	92.80
30	90.90	91.50	92.10	92.70	93.30	93.90	94.20	94.80	95.40	96.00
31	93.93	94.55	95.17	95.79	96.41	97.03	97.34	97.96	98.58	99.20
32	96.96	97.60	98.24	98.88	99.52	100.16	100.48	101.12	101.76	102.40
33	99.99	100.65	101.31	101.97	102.63	103.29	103.62	104.28	104.94	105.60
34	103.02	103.70	104.38	105.06	105.74	106.42	106.76	107.44	108.12	108.80
35	106.75	106.75	107.45	108.15	108.85	109.55	109.90	110.60	111.30	112.00

Cylinders in Ale Gallons.

34 Inches Diameter.

0	.1	.2	.3	.4	.5	.6	.7	8.	.9
3.22	3.24	3.26	3.28	3.30	3.31	3.33	3.35	3.37	3.39
6.44	6.48	6.52	6.56	6.60	6.62	6.66	6.70	6.74	6.78
9.66	9.72	9.78	9.84	9.90	9.93	9.99	10.05	10.11	10.17
12.88	12.96	13.04	13.12	13.20	13.24	13.32	13.40	13.48	13.56
16.10	16.20	16.30	16.40	16.50	16.55	16.65	16.75	16.85	16.95
19.32	19.44	19.56	19.68	19.80	19.86	19.98	20.10	20.22	20.34
22.54	22.68	22.82	22.96	23.10	23.17	23.31	23.45	23.59	23.73
25.76	25.92	26.08	26.24	26.40	26.48	26.64	26.80	26.96	27.12
28.98	29.16	29.34	29.52	29.70	29.79	29.97	30.15	30.33	30.51
32.20	32.40	32.60	32.80	33.00	33.10	33.30	33.50	33.70	33.90
35.42	35.64	35.86	36.08	36.30	36.41	36.63	36.85	37.07	37.29
38.64	38.88	39.12	39.36	39.60	39.72	39.96	40.20	40.44	40.68
41.86	42.12	42.38	42.64	42.90	43.03	43.29	43.55	43.81	44.07
45.08	45.36	45.64	45.92	46.20	46.34	46.62	46.90	47.18	47.46
48.30	48.60	48.90	49.20	49.50	49.65	49.95	50.25	50.55	50.85
51.52	51.84	52.16	52.48	52.80	52.96	53.28	53.60	53.92	54.24
54.74	55.08	55.42	55.76	56.10	56.27	56.61	56.95	57.29	57.63
57.96	58.32	58.68	59.04	59.40	59.58	59.94	60.30	60.66	61.02
61.18	61.56	61.94	62.32	62.70	62.89	63.27	63.65	64.03	64.41
64.40	64.80	65.20	65.60	66.00	66.20	66.60	67.00	67.40	67.80
67.62	68.04	68.46	68.88	69.30	69.51	69.93	70.35	70.77	71.19
70.84	71.28	71.72	72.16	72.60	72.82	73.26	73.70	74.14	74.58
74.06	74.52	74.98	75.44	75.90	76.13	76.59	77.05	77.51	77.97
77.28	77.76	78.24	78.72	79.20	79.44	79.92	80.40	80.88	81.36
80.50	81.00	81.50	82.00	82.50	82.75	83.25	83.75	84.25	84.75
83.72	84.24	84.76	85.28	85.80	86.06	86.58	87.10	87.62	88.14
86.94	87.48	88.02	88.56	89.10	89.37	89.91	90.45	90.99	91.53
90.16	90.72	91.28	91.84	92.40	92.68	93.24	93.80	94.36	94.92
93.78	93.96	94.54	95.12	95.70	95.99	96.57	97.15	97.73	98.31
96.60	97.20	97.80	98.40	99.00	99.30	99.90	100.50	101.10	101.70
99.82	100.44	101.06	101.68	102.30	102.61	103.23	103.85	104.47	105.09
103.04	103.68	104.32	104.96	105.60	105.92	106.56	107.20	107.84	108.48
106.26	106.92	107.58	108.24	108.90	109.23	109.89	110.55	111.21	111.87
109.48	110.16	110.84	111.52	112.20	112.54	113.22	113.90	114.58	115.26
112.70	113.40	114.10	114.80	115.50	115.85	116.55	117.25	117.95	118.65

Cylinders in Ale Gallons.

299

Depth	35 Inches Diameter.									
	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	3.41	3.43	3.45	3.47	3.49	3.51	3.53	3.55	3.57	3.59
2	6.82	6.86	6.90	6.94	6.98	7.02	7.06	7.10	7.14	7.18
3	10.23	10.29	10.35	10.41	10.47	10.53	10.59	10.65	10.71	10.77
4	13.64	13.72	13.80	13.88	13.96	14.04	14.12	14.20	14.28	14.36
5	17.05	17.15	17.25	17.35	17.45	17.55	17.65	17.75	17.85	17.95
6	20.46	20.58	20.70	20.82	20.94	21.06	21.18	21.30	21.42	21.54
7	23.87	24.01	24.15	24.29	24.43	24.56	24.71	24.85	24.99	25.13
8	27.28	27.44	27.60	27.76	27.92	28.07	28.24	28.40	28.56	28.71
9	30.69	30.87	31.05	31.23	31.41	31.58	31.77	31.95	32.13	32.31
10	34.10	34.30	34.50	34.70	34.90	35.10	35.30	35.50	35.70	35.90
11	37.51	37.73	37.95	38.17	38.39	38.61	38.83	39.05	39.27	39.49
12	40.92	41.16	41.40	41.64	41.88	42.12	42.36	42.60	42.84	43.08
13	44.33	44.59	44.85	45.11	45.37	45.63	45.89	46.15	46.41	46.67
14	47.74	48.02	48.30	48.58	48.86	49.14	49.42	49.70	49.98	50.26
15	51.15	51.45	51.75	52.05	52.35	52.65	52.95	53.25	53.55	53.85
16	54.56	54.88	55.20	55.52	55.84	56.16	56.48	56.80	57.12	57.44
17	57.97	58.31	58.65	58.99	59.33	59.67	60.01	60.35	60.69	61.03
18	61.38	61.74	62.10	62.46	62.82	63.18	63.54	63.90	64.26	64.62
19	64.79	65.17	65.55	65.93	66.31	66.69	67.07	67.45	67.83	68.21
20	68.20	68.60	69.00	69.40	69.80	70.20	70.60	71.00	71.40	71.80
21	71.61	72.03	72.45	72.87	73.29	73.71	74.13	74.55	74.97	75.39
22	75.02	75.46	75.90	76.34	76.78	77.22	77.66	78.10	78.54	78.98
23	78.43	78.89	79.37	79.81	80.27	80.73	81.19	81.65	82.11	82.57
24	81.84	82.32	82.80	83.28	83.76	84.24	84.72	85.20	85.68	86.16
25	85.25	85.75	86.25	86.75	87.25	87.75	88.25	88.75	89.25	89.75
26	88.66	89.18	89.70	90.22	90.75	91.26	91.78	92.30	92.82	93.34
27	92.07	92.61	93.15	93.69	94.24	94.77	95.31	95.85	96.39	96.93
28	95.48	96.04	96.60	97.16	97.72	98.28	98.84	99.40	99.96	100.52
29	98.89	99.47	100.05	100.63	101.21	101.79	102.37	102.95	103.53	104.11
30	102.30	102.90	103.50	104.10	104.70	105.30	105.90	106.50	107.10	107.70
31	105.71	106.33	106.95	107.57	108.19	108.81	109.43	110.05	110.67	111.29
32	109.12	109.76	110.40	111.04	111.68	112.32	112.96	113.60	114.24	114.88
33	112.53	113.19	113.85	114.51	115.17	115.83	116.49	117.15	117.81	118.47
34	115.94	116.62	117.30	117.98	118.66	119.34	120.02	120.70	121.38	122.06
35	119.35	120.05	120.75	121.45	122.15	122.85	123.55	124.25	124.95	125.65

Depth	36 Inches Diameter.									
	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	3.61	3.63	3.65	3.67	3.69	3.71	3.73	3.75	3.77	3.79
2	7.22	7.26	7.30	7.34	7.38	7.42	7.46	7.50	7.54	7.58
3	10.83	10.89	10.95	11.01	11.07	11.13	11.19	11.25	11.31	11.37
4	14.44	14.52	14.60	14.68	14.76	14.84	14.92	15.00	15.08	15.16
5	18.05	18.15	18.25	18.35	18.45	18.55	18.65	18.75	18.85	18.95
6	21.66	21.78	21.90	22.02	22.14	22.26	22.38	22.50	22.62	22.74
7	25.27	25.41	25.55	25.69	25.83	25.97	26.11	26.25	26.39	26.53
8	28.88	29.04	29.20	29.36	29.52	29.68	29.84	30.00	30.16	30.32
9	32.49	32.67	32.85	33.03	33.21	33.39	33.57	33.75	33.93	34.11
10	36.10	36.30	36.50	36.70	36.90	37.10	37.30	37.50	37.70	37.90
11	39.71	39.93	40.15	40.37	40.59	40.81	41.03	41.25	41.47	41.69
12	43.32	43.56	43.80	44.04	44.28	44.52	44.76	45.00	45.24	45.48
13	46.93	47.19	47.45	47.71	47.97	48.23	48.49	48.75	49.01	49.27
14	50.54	50.82	51.10	51.38	51.66	51.94	52.22	52.50	52.78	53.06
15	54.15	54.45	54.75	55.05	55.35	55.65	55.95	56.25	56.55	56.85
16	57.76	58.08	58.40	58.72	59.04	59.36	59.68	60.00	60.32	60.64
17	61.37	61.71	62.05	62.39	62.73	63.07	63.41	63.75	64.09	64.43
18	64.98	65.34	65.70	66.06	66.42	66.78	67.14	67.50	67.86	68.22
19	68.59	68.97	69.35	69.73	70.11	70.49	70.87	71.25	71.63	72.01
20	72.20	72.60	73.00	73.40	73.80	74.20	74.60	75.00	75.40	75.80
21	75.81	76.23	76.65	77.07	77.49	77.91	78.33	78.75	79.17	79.59
22	79.42	79.86	80.30	80.74	81.18	81.62	82.06	82.50	82.94	83.38
23	83.03	83.49	83.95	84.41	84.87	85.33	85.79	86.25	86.71	87.17
24	86.64	87.12	87.60	88.08	88.56	89.04	89.52	90.00	90.48	90.96
25	90.25	90.75	91.25	91.75	92.25	92.75	93.25	93.75	94.25	94.75
26	93.86	94.38	94.90	95.42	95.94	96.46	96.98	97.50	98.02	98.54
27	97.47	98.01	98.55	99.09	99.63	100.17	100.71	101.25	101.79	102.33
28	101.08	101.64	102.20	102.76	103.32	103.88	104.44	105.00	105.56	106.12
29	104.69	105.27	105.85	106.43	107.01	107.59	108.17	108.75	109.33	109.91
30	108.30	108.90	109.50	110.10	110.70	111.30	111.90	112.50	113.10	113.70
31	111.91	112.53	113.15	113.77	114.39	115.01	115.63	116.25	116.87	117.49
32	115.52	116.16	116.80	117.44	118.08	118.72	119.36	120.00	120.64	121.28
33	119.13	119.79	120.45	121.11	121.77	122.43	123.09	123.75	124.41	125.07
34	122.74	123.42	124.10	124.78	125.46	126.14	126.82	127.50	128.18	128.86
35	126.35	127.05	127.75	128.45	129.15	129.85	130.55	131.25	131.95	132.65

Cylinders in Ale Gallons.

301

37 Inches Diameter.

Depth	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	3.81	3.83	3.85	3.87	3.90	3.92	3.94	3.96	3.98	4.00
2	7.62	7.66	7.70	7.74	7.80	7.84	7.88	7.92	7.96	8.00
3	11.43	11.49	11.55	11.61	11.70	11.76	11.82	11.88	11.94	12.00
4	15.24	15.32	15.40	15.48	15.60	15.68	15.76	15.84	15.92	16.00
5	19.05	19.15	19.25	19.35	19.50	19.60	19.70	19.80	19.90	20.00
6	22.86	22.98	23.10	23.22	23.40	23.52	23.64	23.76	23.88	24.00
7	26.67	26.81	26.95	27.09	27.30	27.44	27.58	27.72	27.86	28.00
8	30.48	30.64	30.80	30.96	31.20	31.36	31.52	31.68	31.84	32.00
9	34.29	34.47	34.65	34.83	35.10	35.28	35.46	35.64	35.82	36.00
10	38.10	38.30	38.50	38.70	39.00	39.20	39.40	39.60	39.80	40.00
11	41.91	42.13	42.35	42.57	42.90	43.12	43.34	43.56	43.78	44.00
12	45.72	45.96	46.20	46.44	46.80	47.04	47.28	47.52	47.76	48.00
13	49.53	49.79	50.05	50.31	50.70	50.96	51.22	51.48	51.74	52.00
14	53.34	53.62	53.90	54.18	54.60	54.88	55.16	55.44	55.72	56.00
15	57.15	57.45	57.75	58.05	58.50	58.80	59.10	59.40	59.70	60.00
16	60.96	61.28	61.60	61.92	62.40	62.72	63.04	63.36	63.68	64.00
17	64.77	65.11	65.45	65.79	66.30	66.64	66.98	67.32	67.66	68.00
18	68.58	68.94	69.30	69.66	70.20	70.56	70.92	71.28	71.64	72.00
19	72.39	72.77	73.15	73.53	74.10	74.48	74.86	75.24	75.62	76.00
20	76.20	76.70	77.00	77.40	78.00	78.40	78.80	79.20	79.60	80.00
21	80.01	80.43	80.85	81.27	81.90	82.32	82.74	83.16	83.58	84.00
22	83.82	84.26	84.70	85.14	85.80	86.24	86.68	87.12	87.56	88.00
23	87.63	88.09	88.55	89.01	89.70	90.16	90.62	91.08	91.54	92.00
24	91.44	91.92	92.40	92.88	93.60	94.08	94.56	95.04	95.52	96.00
25	95.25	95.75	96.25	96.75	97.50	98.00	98.50	99.00	99.50	100.00
26	99.06	99.58	100.10	100.62	101.40	101.92	102.44	102.96	103.48	104.00
27	102.87	103.41	103.95	104.49	105.30	105.84	106.38	106.92	107.46	108.00
28	106.68	107.24	107.80	108.36	109.20	109.76	110.32	110.88	111.44	112.00
29	110.49	111.07	111.65	112.23	113.10	113.68	114.26	114.84	115.42	116.00
30	114.30	114.90	115.50	116.10	117.00	117.60	118.20	118.80	119.40	120.00
31	118.11	118.73	119.35	119.97	120.90	121.52	122.14	122.76	123.38	124.00
32	121.92	122.56	123.20	123.84	124.80	125.44	126.08	126.72	127.36	128.00
33	125.73	126.39	127.05	127.71	128.70	129.36	130.02	130.68	131.34	132.00
34	129.54	130.22	130.90	131.58	132.60	133.28	133.96	134.64	135.32	136.00
35	133.35	134.05	134.75	135.45	136.50	137.20	137.90	138.60	139.30	140.00

*Cylinders in Ale Gallons.**38 Inches Diameter.*

0	.1	.2	.3	.4	.5	.6	.7	.8	.9
4.02	4.04	4.06	4.09	4.11	4.13	4.15	4.17	4.19	4.21
8.04	8.08	8.12	8.18	8.22	8.26	8.30	8.34	8.38	8.42
12.06	12.12	12.18	12.27	12.33	12.39	12.45	12.51	12.57	12.63
16.08	16.16	16.24	16.36	16.44	16.52	16.60	16.68	16.76	16.84
20.10	20.20	20.30	20.45	20.55	20.65	20.75	20.85	20.95	21.05
24.12	24.24	24.36	24.54	24.66	24.78	24.90	25.02	25.14	25.26
28.14	28.28	28.42	28.63	28.77	28.91	29.05	29.19	29.33	29.47
32.16	32.32	32.48	32.72	32.88	33.04	33.20	33.36	33.52	33.68
36.18	36.36	36.54	36.81	36.99	37.17	37.35	37.53	37.71	37.89
40.20	40.40	40.60	40.90	41.10	41.30	41.50	41.70	41.90	42.10
44.22	44.44	44.66	44.99	45.21	45.43	45.65	45.87	46.09	46.31
48.24	48.48	48.72	49.08	49.32	49.56	49.80	50.04	50.28	50.52
52.26	52.52	52.78	53.17	53.43	53.69	53.95	54.21	54.47	54.73
56.28	56.56	56.84	57.26	57.54	57.82	58.10	58.38	58.66	58.94
60.30	60.60	60.90	61.35	61.65	61.95	62.25	62.55	62.85	63.15
64.32	64.64	64.96	65.44	65.76	66.08	66.40	66.72	67.04	67.36
68.34	68.68	69.02	69.53	69.87	70.21	70.55	70.89	71.23	71.57
72.36	72.72	73.08	73.62	73.98	74.34	74.70	75.06	75.42	75.78
76.38	76.76	77.14	77.71	78.09	78.47	78.85	79.23	79.61	79.99
80.40	80.80	81.20	81.80	82.20	82.60	83.00	83.40	83.80	84.20
84.42	84.84	85.26	85.89	86.31	86.73	87.15	87.57	87.99	88.41
88.44	88.88	89.32	89.98	90.42	90.86	91.30	91.74	92.18	92.62
92.46	92.92	93.38	94.07	94.53	94.99	95.45	95.91	96.37	96.83
96.48	96.96	97.44	98.16	98.64	99.12	99.60	100.08	100.56	101.04
00.50	101.00	101.50	102.25	102.75	103.25	103.75	104.25	104.75	105.25
04.52	105.04	105.56	106.34	106.86	107.38	107.90	108.42	108.94	109.46
08.54	109.08	109.62	110.43	110.97	111.51	112.05	112.59	113.13	113.67
12.56	113.12	113.68	114.52	115.08	115.64	116.20	116.76	117.32	117.88
16.58	117.16	117.74	118.61	119.19	119.77	120.35	120.93	121.51	122.09
20.60	121.20	121.80	122.70	123.30	123.90	124.50	125.10	125.70	126.30
24.62	125.24	125.86	126.79	127.41	128.03	128.65	129.27	129.89	130.51
28.64	129.28	129.92	130.88	131.52	132.16	132.80	133.44	134.08	134.72
32.66	133.32	133.98	134.97	135.63	136.29	136.95	137.61	138.27	138.93
36.68	137.36	138.04	139.06	139.74	140.42	141.10	141.78	142.46	143.14
40.70	141.40	142.10	143.15	143.85	144.55	145.25	145.95	146.65	147.35

Cylinders in Ale Gallons.

303

39 Inches Diameter.

Depth	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	4.24	4.26	4.28	4.30	4.32	4.35	4.37	4.39	4.41	4.43
2	8.48	8.52	8.56	8.60	8.64	8.70	8.74	8.78	8.82	8.86
3	12.72	12.78	12.84	12.90	12.96	13.05	13.11	13.17	13.23	13.29
4	16.96	17.04	17.12	17.20	17.28	17.40	17.48	17.56	17.64	17.72
5	21.20	21.30	21.40	21.50	21.60	21.75	21.85	21.95	22.05	22.15
6	25.44	25.56	25.68	25.80	25.92	26.10	26.22	26.34	26.46	26.58
7	29.68	29.82	29.96	30.10	30.24	30.45	30.59	30.73	30.87	31.01
8	33.92	34.08	34.24	34.40	34.56	34.80	34.96	35.12	35.28	35.44
9	38.16	38.34	38.52	38.70	38.88	39.15	39.33	39.51	39.69	39.87
10	42.40	42.60	42.80	43.00	43.20	43.50	43.70	43.90	44.10	44.30
11	46.64	46.86	47.08	47.30	47.52	47.85	48.07	48.29	48.51	48.73
12	50.88	51.12	51.36	51.60	51.84	52.20	52.44	52.68	52.92	53.16
13	55.12	55.38	55.64	55.90	56.16	56.55	56.81	57.07	57.33	57.59
14	59.36	59.64	59.92	60.20	60.48	60.90	61.18	61.46	61.74	62.02
15	63.60	63.90	64.20	64.50	64.80	65.25	65.55	65.85	66.15	66.45
16	67.84	68.16	68.48	68.80	69.12	69.60	69.92	70.24	70.56	70.88
17	72.08	72.42	72.76	73.10	73.44	73.95	74.29	74.63	74.97	75.31
18	76.32	76.68	77.04	77.40	77.76	78.30	78.66	79.02	79.38	79.74
19	80.56	80.94	81.32	81.70	82.08	82.65	83.03	83.41	83.79	84.17
20	84.80	85.20	85.60	86.00	86.40	87.00	87.40	87.80	88.20	88.60
21	89.04	89.46	89.88	90.30	90.72	91.35	91.77	92.19	92.61	93.03
22	93.28	93.72	94.16	94.60	95.04	95.70	96.14	96.58	97.02	97.46
23	97.52	97.98	98.44	98.90	99.36	100.05	100.51	100.97	101.43	101.89
24	101.76	102.24	102.72	103.20	103.68	104.40	104.88	105.36	105.84	106.32
25	106.00	106.50	107.00	107.50	108.00	108.75	109.25	109.75	110.25	110.75
26	110.24	110.76	111.28	111.80	112.32	113.10	113.62	114.14	114.66	115.18
27	114.48	115.02	115.56	116.10	116.64	117.45	117.99	118.53	119.07	119.61
28	118.72	119.28	119.84	120.40	120.96	121.80	122.36	122.92	123.48	124.04
29	122.96	123.54	124.12	124.70	125.28	126.15	126.73	127.31	127.89	128.47
30	127.20	127.80	128.40	129.00	129.60	130.50	131.10	131.70	132.30	132.90
31	131.44	132.06	132.68	133.30	133.92	134.85	135.47	136.09	136.71	137.33
32	135.68	136.32	136.96	137.60	138.24	139.20	139.84	140.48	141.12	141.76
33	139.92	140.58	141.24	141.90	142.56	143.55	144.21	144.87	145.53	146.19
34	144.16	144.84	145.52	146.20	146.88	147.90	148.58	149.26	149.94	150.62
35	148.40	149.10	149.80	150.50	151.20	152.25	152.95	153.65	154.35	155.05

Depth	40 Inches Diameter.									
	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	4.46	4.48	4.50	4.52	4.55	4.57	4.59	4.61	4.64	4.66
2	8.92	8.96	9.00	9.04	9.10	9.14	9.18	9.22	9.28	9.32
3	13.38	13.44	13.50	13.56	13.65	13.71	13.77	13.83	13.92	13.98
4	17.84	17.92	18.00	18.08	18.20	18.28	18.36	18.44	18.56	18.64
5	22.30	22.40	22.50	22.60	22.75	22.85	22.95	23.05	23.20	23.30
6	26.76	26.88	27.00	27.12	27.30	27.42	27.54	27.66	27.84	27.96
7	31.22	31.36	31.50	31.64	31.85	31.99	32.13	32.27	32.48	32.62
8	35.68	35.84	36.00	36.16	36.40	36.56	36.72	36.88	37.12	37.28
9	40.14	40.32	40.50	40.68	40.95	41.13	41.31	41.49	41.76	41.94
10	44.60	44.80	45.00	45.20	45.50	45.70	45.90	46.10	46.40	46.60
11	49.06	49.28	49.50	49.72	50.05	50.27	50.49	50.71	51.04	51.26
12	53.52	53.76	54.00	54.24	54.60	54.84	55.08	55.32	55.68	55.92
13	57.98	58.24	58.50	58.76	59.15	59.41	59.67	59.93	60.32	60.58
14	62.44	62.72	63.00	63.28	63.70	63.98	64.26	64.54	64.96	65.24
15	66.90	67.20	67.50	67.80	68.25	68.55	68.85	69.15	69.60	69.90
16	71.36	71.68	72.00	72.32	72.80	73.12	73.44	73.76	74.24	74.56
17	75.82	76.16	76.50	76.84	77.35	77.69	78.03	78.37	78.88	79.22
18	80.28	80.64	81.00	81.36	81.90	82.26	82.62	82.98	83.52	83.88
19	84.74	85.12	85.50	85.88	86.45	86.83	87.21	87.59	88.16	88.54
20	89.20	89.60	90.00	90.40	91.00	91.40	91.80	92.20	92.80	93.20
21	93.66	94.08	94.50	94.92	95.55	95.97	96.39	96.81	97.44	97.86
22	98.12	98.56	99.00	99.44	100.10	100.54	100.98	101.42	102.08	102.52
23	102.58	103.04	103.50	103.96	104.65	105.11	105.57	106.03	106.72	107.18
24	107.04	107.52	108.00	108.48	109.20	109.68	110.16	110.64	111.36	111.84
25	111.50	112.00	112.50	113.00	113.75	114.25	114.75	115.25	116.00	116.50
26	115.96	116.48	117.00	117.52	118.30	118.82	119.34	119.86	120.64	121.16
27	120.42	120.96	121.50	122.04	122.85	123.39	123.93	124.47	125.28	125.82
28	124.88	125.44	126.00	126.56	127.40	127.96	128.52	129.08	129.92	130.48
29	129.34	129.92	130.50	131.08	131.95	132.53	133.11	133.69	134.56	135.14
30	133.80	134.40	135.00	135.60	136.50	137.10	137.70	138.30	139.20	139.80
31	138.26	138.88	139.50	140.12	141.05	141.67	142.29	142.91	143.84	144.46
32	142.72	143.36	144.00	144.64	145.60	146.24	146.88	147.52	148.48	149.12
33	147.18	147.84	148.50	149.16	150.15	150.81	151.47	152.13	153.12	153.78
34	151.64	152.32	153.00	153.68	154.70	155.38	156.06	156.74	157.76	158.44
35	156.10	156.80	157.50	158.20	159.25	159.95	160.65	161.35	162.40	163.10

A
TABLE
TO CONVERT
GALLONS
INTO
Barrels, Firkins and Gallons
OF
ALE or BEER.
And the Converse from 1 to 100 BARRELS.

X

Ale.

Bar.	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$
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1	32	40	48	56
2	64	72	80	88
3	96	104	112	120
4	128	136	144	152
5	160	168	176	184

6	192	200	208	216
7	224	232	240	248
8	256	264	272	280
9	288	296	304	312
10	320	328	336	344

11	352	360	368	376
12	384	392	400	408
13	416	424	432	440
14	448	456	464	472
15	480	488	496	504

16	512	520	528	536
17	544	552	560	568
18	576	584	592	600
19	608	616	624	632
20	640	648	656	664

21	672	680	688	696
22	704	712	720	728
23	736	744	752	760
24	768	776	784	792
25	800	808	816	824

26	832	840	848	856
27	864	872	880	888
28	896	904	912	920
29	928	936	944	952
30	960	968	976	984

31	992	1000	1008	1016
32	1024	1032	1040	1048
33	1056	1064	1072	1080
34	1088	1096	1104	1112
35	1120	1128	1136	1144

Beer.

Bar.	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$
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1	36	45	54	63
2	72	81	90	99
3	108	117	126	135
4	144	153	162	171
5	180	189	198	207

6	216	225	234	243
7	252	261	270	279
8	288	297	306	315
9	324	333	342	351
10	360	369	378	387

11	396	405	414	423
12	432	441	450	459
13	468	477	486	495
14	504	513	522	531
15	540	549	558	567

16	576	585	594	603
17	612	621	630	639
18	648	657	666	675
19	684	693	702	711
20	720	729	738	747

21	756	765	774	783
22	792	801	810	819
23	828	837	846	855
24	864	873	882	891
25	900	909	918	927

26	936	945	954	963
27	972	981	990	999
28	1008	1017	1026	1035
29	1044	1053	1062	1071
30	1080	1089	1098	1107

31	1116	1125	1134	1143
32	1152	1161	1170	1179
33	1188	1197	1206	1215
34	1224	1233	1242	1251
35	1260	1269	1278	1287

Alc.

Bar. | 0 | $\frac{1}{4}$ | $\frac{1}{2}$ | $\frac{3}{4}$

36 | 1152 | 1160 | 1168 | 1176
37 | 1184 | 1192 | 1200 | 1208
38 | 1216 | 1224 | 1232 | 1240
39 | 1248 | 1256 | 1264 | 1272
40 | 1280 | 1288 | 1296 | 1304

41 | 1312 | 1320 | 1328 | 1336
42 | 1344 | 1352 | 1360 | 1368
43 | 1376 | 1384 | 1392 | 1400
44 | 1408 | 1416 | 1424 | 1432
45 | 1440 | 1448 | 1456 | 1464

46 | 1472 | 1480 | 1488 | 1496
47 | 1504 | 1512 | 1520 | 1528
48 | 1536 | 1544 | 1552 | 1560
49 | 1568 | 1576 | 1584 | 1592
50 | 1600 | 1608 | 1616 | 1624

51 | 1632 | 1640 | 1648 | 1656
52 | 1664 | 1672 | 1680 | 1688
53 | 1696 | 1704 | 1712 | 1720
54 | 1728 | 1736 | 1744 | 1752
55 | 1760 | 1768 | 1776 | 1784

56 | 1792 | 1800 | 1808 | 1816
57 | 1824 | 1832 | 1840 | 1848
58 | 1856 | 1864 | 1872 | 1880
59 | 1888 | 1896 | 1904 | 1912
60 | 1920 | 1928 | 1936 | 1944

61 | 1952 | 1960 | 1968 | 1976
62 | 1984 | 1992 | 2000 | 2008
63 | 2016 | 2024 | 2032 | 2040
64 | 2048 | 2056 | 2064 | 2072
65 | 2080 | 2088 | 2096 | 2104

66 | 2112 | 2120 | 2128 | 2136
67 | 2144 | 2152 | 2160 | 2168
68 | 2176 | 2184 | 2192 | 2200
69 | 2208 | 2216 | 2224 | 2232
70 | 2240 | 2248 | 2256 | 2264

Beer.

Bar. | 0 | $\frac{1}{4}$ | $\frac{1}{2}$ | $\frac{3}{4}$

36 | 1296 | 1305 | 1314 | 1323
37 | 1332 | 1341 | 1350 | 1359
38 | 1368 | 1375 | 1384 | 1393
39 | 1404 | 1413 | 1422 | 1431
40 | 1440 | 1449 | 1458 | 1467

41 | 1476 | 1485 | 1494 | 1503
42 | 1512 | 1521 | 1530 | 1539
43 | 1548 | 1557 | 1566 | 1575
44 | 1584 | 1593 | 1602 | 1611
45 | 1620 | 1629 | 1638 | 1647

46 | 1656 | 1665 | 1674 | 1683
47 | 1692 | 1701 | 1710 | 1719
48 | 1728 | 1737 | 1746 | 1755
49 | 1764 | 1773 | 1782 | 1791
50 | 1800 | 1809 | 1818 | 1827

51 | 1836 | 1845 | 1854 | 1863
52 | 1872 | 1881 | 1890 | 1899
53 | 1908 | 1917 | 1926 | 1935
54 | 1944 | 1953 | 1962 | 1971
55 | 1980 | 1989 | 1998 | 2007

56 | 2016 | 2025 | 2034 | 2043
57 | 2052 | 2061 | 2070 | 2079
58 | 2088 | 2097 | 2106 | 2115
59 | 2124 | 2133 | 2142 | 2151
60 | 2160 | 2169 | 2178 | 2187

61 | 2196 | 2205 | 2214 | 2223
62 | 2232 | 2241 | 2250 | 2259
63 | 2268 | 2277 | 2286 | 2295
64 | 2304 | 2313 | 2322 | 2331
65 | 2340 | 2349 | 2358 | 2367

66 | 2376 | 2385 | 2394 | 2403
67 | 2412 | 2421 | 2430 | 2439
68 | 2448 | 2457 | 2466 | 2475
69 | 2484 | 2493 | 2502 | 2511
70 | 2520 | 2529 | 2538 | 2547

Alle.

Beer.

Bar. | 0 | 1 | 2 | 3

71	2572	2580	2588	2596
72	2304	2312	2320	2328
73	2336	2344	2352	2360
74	2368	2376	2384	2392
75	2400	2408	2416	2424

76	2432	2440	2448	2456
77	2464	2472	2480	2488
78	2496	2504	2512	2520
79	2528	2536	2544	2552
80	2560	2568	2576	2584

81	2592	2600	2608	2616
82	2624	2632	2640	2648
83	2656	2664	2672	2680
84	2688	2696	2704	2712
85	2720	2728	2736	2744

86	2752	2760	2768	2776
87	2784	2792	2800	2808
88	2816	2824	2832	2840
89	2848	2856	2864	2872
90	2880	2888	2896	2904

91	2912	2920	2928	2936
92	2944	2952	2960	2968
93	2976	2984	2992	3000
94	3008	3016	3024	3032
95	3040	3048	3056	3064

96	3072	3080	3088	3096
97	3104	3112	3120	3128
98	3136	3144	3152	3160
99	3168	3176	3184	3192
100	3200	3208	3216	3224

Bar. | 0 | 1 | 2 | 3

71	2556	2564	2572	2580
72	2592	2600	2608	2616
73	2624	2632	2640	2648
74	2664	2672	2680	2688
75	2700	2708	2716	2724

76	2736	2744	2752	2760
77	2772	2780	2788	2796
78	2808	2816	2824	2832
79	2844	2852	2860	2868
80	2880	2888	2896	2904

81	2916	2924	2932	2940
82	2952	2960	2968	2976
83	2988	2996	3004	3012
84	3024	3032	3040	3048
85	3060	3068	3076	3084

86	3096	3104	3112	3120
87	3132	3140	3148	3156
88	3168	3176	3184	3192
89	3204	3212	3220	3228
90	3240	3248	3256	3264

91	3276	3284	3292	3300
92	3312	3320	3328	3336
93	3348	3356	3364	3372
94	3384	3392	3400	3408
95	3420	3428	3436	3444

96	3456	3464	3472	3480
97	3492	3500	3508	3516
98	3528	3536	3544	3552
99	3564	3572	3580	3588
100	3600	3608	3616	3624

FINIS.

89

75

